

1 **LOCAL SENSITIVITY ANALYSIS OF FORECAST UNCERTAINTY IN A RANDOM-**  
2 **UTILITY-BASED MULTI-REGIONAL INPUT-OUTPUT MODEL**

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16  
17 The following is a pre-print, the final publication can be found in  
18 *Journal of the Transportation Research Forum* 55 (2): 49-70, 2016.  
19

20 **ABSTRACT:** Transportation systems are critical to regional economies and quality of life. The  
21 Random-Utility-Based Multiregional Input-Output Model (RUBMRIO) for trade and travel  
22 choices is used here to appreciate the distributed nature of commodity flow patterns across the  
23 U.S.'s 3,109 contiguous counties and 12 industry sectors, for rail and truck operations. This  
24 paper demonstrates the model's sensitivity to various inputs using the method of local sensitivity  
25 analysis with interactions (LSAI). This work simulates both individual effects as well as  
26 interaction effects of model inputs on outputs by providing sensitivity indices of model outputs  
27 to variations of inputs under two scenarios. Model outputs include predictions of domestic and  
28 export trade flows, value of goods produced, labor expenditures, and household and industry  
29 consumption levels across the counties in the U.S. The LSAI technique allows transportation  
30 system operators to appreciate the roles of any model input and the associated uncertainty of  
31 outputs.

32 **KEYWORDS:** Random-Utility-Based Multiregional Input-Output Model (RUBMRIO),  
33 uncertainty propagation, local sensitivity analysis with interactions (LSAI), transport and land  
34 use models, trade modeling

35 **INTRODUCTION**

36 Transportation systems are critical to regional economies and planning. Their spatial structures  
37 and cost implications dramatically affect household and firm location choices, production levels,  
38 and trade patterns, in multiple ways. These choices manifest themselves in various forms of  
39 travel demand, impacting the operational performance of the transportation system. To recognize  
40 this critical interaction and enhance planning, policy, and investment decisions, integrated  
41 models of transportation and land use have been pursued.

42 Traditional Input-Output (IO) models are popular for simulating expenditure linkages between  
43 industries, and between producers and consumers (Leontief and Strout1963). These models are  
44 demand driven, in the sense that production levels adjust to meet both final and intermediate

1 demands. Spatial (or interregional, inter-zonal) IO (SIO) analysis extends the classical IO model  
2 to include spatial disaggregation when coupled with random utility theory for the distribution of  
3 productive input, such as MEPLAN (Hunt and Simmonds 1993; Abraham and Hunt 1999;  
4 Rodier et al. 2002; Clay and Johnston 2006), TRANUS (De la Barra et al. 1984; De la Barra  
5 2005; Modelistica 2007; Lefevre 2009), and PECAS (Hunt and Abraham 2003). These models  
6 can be made dynamic, by allowing the travel costs associated with freight and people (labor and  
7 customer) flows to affect location and land use decisions in the model's next iteration, along with  
8 network system changes (e.g., roadway expansions) and exogenous economic shocks (e.g.  
9 increases in export demands). Entropy concepts were then proposed, to establish a connection  
10 between SIO models, entropy-maximizing theory, and random-utility theory (Wilson 1970; Anas  
11 1984).

12 Isard (1960) firstly proposed the extension of the IO model to multiple regions, which may be  
13 referred to as Random-Utility-Based Multiregional Input-Output (RUBMRIO) models. These  
14 combine traditional SIO models with a multinomial logit (MNL) model for trade and travel  
15 choices to represent the distributed nature of commodity flow patterns. Hunt (1993) and De la  
16 Barra (2005) suggested the standard algorithm for the RUBMRIO model, which is usually solved  
17 by iteratively applying a set of equations. Each equation describes a key model variable.

18 Kockelman et al. (2005) developed a RUBMRIO model of Texas trade. Their RUBMRIO model  
19 described the production and trade patterns of 18 socio-economic sectors (including households  
20 and government) across Texas' 254 counties. Production and trade typically are driven by export  
21 demands at 31 key ports, while specific trade patterns respond to prices, measured in utility units  
22 and based on expected minimum transportation costs (represented by distance on a two-mode  
23 highway/railway network). Their applications considered network and corridor congestion and  
24 the multiplier effects of shifts in demand, by port and sector. Ruiz-Juri and Kockelman (2004)  
25 extended the RUBMRIO model to recognize land use constraints on production (and residence),  
26 to incorporate "domestic demands" by other U.S. states, to estimate vehicle trips resulted from  
27 monetary trades, and to capture the effects of the network congestion on trade and production  
28 decisions. Based on the above work, Huang and Kockelman (2008) extended the RUBMRIO  
29 model to characterize near-term production and trade patterns based on current settlement and  
30 earnings patterns, and to introduce dynamic features, which forecast the evolution of a region's  
31 trade patterns – from a state of short-term disequilibrium to longer-run scenarios. Du and  
32 Kockelman (2012) extended work by Kockelman et al. (2005) to a U.S.-level RUBMRIO model  
33 for trade patterns among the nation's 3,109 contiguous counties (excluding Hawaii and Alaska),  
34 across 20 socio-economic sectors, and two transportation modes. The applications anticipated  
35 trade and location choices resulting from a variety of scenarios, including changes in export  
36 demands and transport cost. A series of scenarios were carried out by changing the export  
37 demands in each of the 12 export-related sectors to forecast the effects of different export  
38 demands on the U.S. economy. Highway congestion effects and transport cost effects on U.S.  
39 trade and production patterns were illustrated by a rise (fall) in IH40 travel times and the  
40 marginal average cost of trucking.

41 In these studies, they mainly focused on how the effects of inputs (e.g. export demands of  
42 different commodities, the transport cost, and the network congestion) and parameters (e.g.  
43 technical coefficient) on outputs, such as the distribution of trade flows and production.  
44 Additionally, they only demonstrated the individual effect of every input on the outputs. In fact,  
45 the interaction effects across inputs may amplify or dampen individual effects of inputs on

1 outputs in complex and dynamic urban systems.

2 Thus, we used the local sensitivity analysis with interaction (LSAI) to evaluate the RUBMRIO  
 3 model by producing finite change sensitivity indices for the variation of inputs under different  
 4 scenarios. This feature is particularly appealing when the set of uncertain variables is especially  
 5 large since this procedure requires a relatively low number of model runs. This paper illustrates  
 6 how the local sensitivity analysis applies to the case of scenarios in transport and land use  
 7 models through an analysis of the RUBMRIO model, which simulates not only the individual  
 8 effect of each input but also all inputs' interaction effects. In this study, a RUBMRIO model is  
 9 developed for trade patterns among the 3,109 contiguous counties from the continental U.S.  
 10 across 12 socio-economic sectors and two transportation modes (truck and rail). The following  
 11 two scenarios are used: simultaneously increasing all foreign export demands (ED), transport  
 12 costs (TC) and travel times (TT) between counties (or from counties to export zones) by 20% as  
 13 Scenario 1, simultaneously decreasing all ED, TC and TT by 20% as Scenario 2. Applications of  
 14 the model anticipate changes (including individual effects and interaction effects) of domestic  
 15 trade flow, export trade flow, production (sum of domestic and export trade flows) and  
 16 consumption in the continental U.S. resulting from two scenarios. Thus, these scenarios include  
 17 increasing or decreasing ED, TC and TT between counties (or from counties to export zones)  
 18 by 20%, in order to forecast their effects on key metrics of the U.S. economy (including  
 19 production, consumption, and domestic trade flows in continental U.S. States).

## 20 **BRIEF INTRODUCTION TO THE RUBMRIO MODEL**

21 RUBMRIO is a transportation-economic model that simulates the flow of goods, labor, and  
 22 vehicles across a multiregional area (see Figure 1, and Du and Kockelman [2012]). RUBMRIO  
 23 simulates trade across zones of a region, as motivated by foreign and domestic ED, and  
 24 computes this trade within numerous economic sectors. IO relationships/tables are used to  
 25 anticipate consumption needs of commodity producers, and multinomial logit models distribute  
 26 commodity flows (across origin zones and shipment modes).

### 27 **The Utility of Trade Choices**

28 The application of the random utility theory for cost minimization, domestic trade flows (among  
 29 counties, as zones) and export flows (from counties to export zones) is based on the utility of  
 30 purchasing commodity  $m$  from zone  $j$  and transporting it via different transportation  
 31 modes (export it to zone  $k$ ). The utility function is composed of two items, including the price of  
 32 the commodity, as well as travel time and cost attributes between zones (rather than distance), as  
 33 shown in Equations (1)-(2).

$$34 \quad U_{ij}^m = -p_i^m + \lambda^m \ln[\sum_t \exp(\beta_{0,t}^m + \beta_{1,t}^m time_{ij,t} + \beta_{2,t}^m cost_{ij,t})] \quad (1)$$

$$35 \quad U_{ik}^m = -p_i^m + \lambda^m \ln[\sum_t \exp(\beta_{0,t}^m + \beta_{1,t}^m time_{ik,t} + \beta_{2,t}^m cost_{ik,t})] \quad (2)$$

36  $p_i^m$  is the sales price of commodity  $m$  in county/zone  $i$ ,  $time_{ij,t}$  and  $cost_{ij,t}$  represent the travel times  
 37 and costs between zones  $i$  and  $j$  via mode  $t$ . Parameters  $\beta_{0,t}^m$ ,  $\beta_{1,t}^m$ , and  $\beta_{2,t}^m$  were estimated using a  
 38 series of industry-specific nested logit specifications as described by Ben-Akiva and Lerman  
 39 (1985).

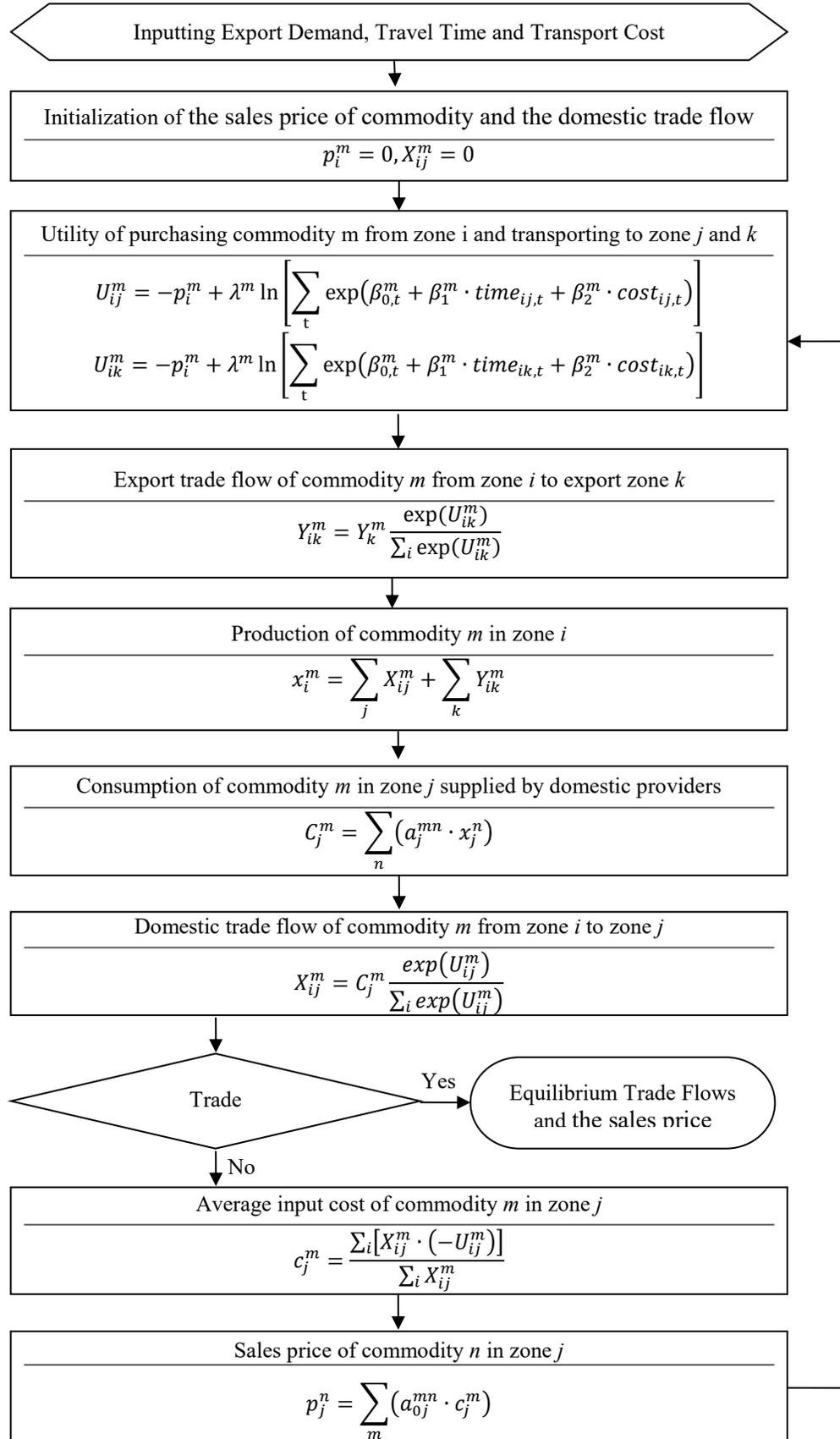
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## 1 **Production Function**

2 Sales price is a key factor influencing consumption of a commodity, purchase choices,  
3 production costs, and thus, trade patterns. In the RUBMRIO model, sales price (the cost of  
4 producing one unit of commodity  $n$  in zone  $j$ ) depends on the costs of purchasing raw materials,  
5 labor, and necessary services from other producers, including transport costs associated with the  
6 shipment of those inputs. The ultimate sales price of commodity by industry  $n$  from zone  $j$  is as  
7 follows:

$$8 \quad p_j^n = \sum_m a_{0j}^{mn} \times c_j^m \quad (3)$$

9 where  $a_{0j}^{mn}$  is the technical coefficient for producing commodity  $n$  in zone  $j$ .  $a_{0j}^{mn}$  means the dollar  
10 values of commodity  $m$  required to produce one unit of commodity  $n$  in zone  $j$ . Thus, they are all  
11 dimensionless because their units are in terms of dollar-per-dollar.

1  
2  
3

1 **FIGURE 1 RUBMRIO structure and solution algorithm.**

2 They can be calculated through a transactions table (input-output matrix of dollar flows between  
3 industries) by dividing each  $m, n$  cell's transaction by its corresponding column totally from the  
4 original IMPLAN transactions tables (Minnesota IMPLAN Group 1997) for total purchases, both  
5 local and imported.

6 The input costs  $c_j^m$ , shown in Equation (4), are a flow-weighted average of purchase price for  
7 commodity  $m$  in zone  $j$  and transport costs for commodity  $m$  from zone  $i$  to zone  $j$  (in units of  
8 disutility). The weights are domestic trade flows,  $X_{ij}^m$ .

$$9 \quad c_j^m = \frac{\sum_i [X_{ij}^m \cdot (-U_{ij}^m)]}{\sum_i X_{ij}^m} \quad (4)$$

11 **Trade Flows**

12 Domestic and export trade flows are calculated under an assumption of utility-maximizing/cost-  
13 minimizing behavior, which means consumers will choose producer(s) that can supply the lowest  
14 cost (including both the price and the transport cost) in order to maximize their utility and  
15 (or) minimize their costs. The unobserved heterogeneity of this choice, across producers and  
16 consumers, introduces the random elements, which leads to a nested logit model for origin and  
17 mode choices. The domestic trade flow,  $X_{ij}^m$ , and export trade flow,  $Y_{ik}^m$ , are computed using  
18 Equations (5) -(6):

$$19 \quad X_{ij}^m = C_j^m \frac{\exp(U_{ij}^m)}{\sum_i \exp(U_{ij}^m)} \quad (5)$$

$$20 \quad Y_{ik}^m = Y_k^m \frac{\exp(U_{ik}^m)}{\sum_i \exp(U_{ik}^m)} \quad (6)$$

21 where  $Y_k^m$  is the demand of export zone  $k$  for commodity  $m$ , and  $C_j^m$  is the total (dollar) amount of  
22 commodity  $m$  consumed in zone  $j$ , which can be obtained as follows:

$$23 \quad C_j^m = \sum_n a_j^{mn} x_j^n \quad (7)$$

24 Here,  $a_j^{mn}$  represents “local-purchase” technical coefficient for commodity  $m$  in zone  $j$ . Regional  
25 purchase coefficients (RPCs) bridge these two styles of technical coefficient matrices by  
26 representing the proportion of total demand for a commodity that is supplied by producers within  
27 the study area, rather than imported from abroad (MIG, 2011). This relationship between  $a_{0j}^{mn}$  and  
28  $a_j^{mn}$  is shown in Equation (8). Finally,  $x_i^m$  is the total production of commodity  $m$  in zone  $i$ , which  
29 is the sum of domestic and export flows “leaving” zone  $i$ , as shown in Equation (9).

$$30 \quad a_j^{mn} = \frac{a_{0j}^{mn} \times RPC^n}{\sum_m a_{0j}^{mn}} \quad (8)$$

$$x_i^m = \sum_j X_{ij}^m + \sum_k Y_{ik}^m \quad (9)$$

Equations (1) through (9) constitute the majority of the RUBMRIO model, and they are solved iteratively to achieve an equilibrium trade pattern, as described by Zhao and Kockelman (2004), who examined the existence and uniqueness of the equilibrium solution. After inputting Foreign Export Demand, Highway Distances and Railway Distances between Zones, Highway Distances and Railway Distances to Export, and Transport Cost between Zones and to Export, the iteration procedure begins with initial sales prices and the domestic trade flow at zero. The relative utilities of both domestic and export origin and mode choices are computed. Then, export demands are distributed among production zones to export according to the relative utilities. These export flows give rise to domestic demands and trade flows between counties on the basis of relative utilities. The total productions in zone  $i$  are multiplied by corresponding technical coefficients (following import/leakage considerations) in order to estimate the total consumption (set of inputs) required for purchase from domestic counties  $j$  (including zone  $i$  itself). Average input costs are computed as a flow-weighted average of utilities, and coupled with original technical coefficients to provide updated sales prices, which feedback for recalculating of all purchase utilities. This process leads to new iterations, until consecutive trade flows stabilize, achieving system equilibrium.

## 18 LOCAL SENSITIVITY ANALYSIS WITH INTERACTION (LSAI)

While building and using numerical simulation models, sensitivity analysis is an invaluable tool to study how uncertainty in the output of a mathematical model or system is apportioned to different sources of uncertainty in its inputs (Saltelli et al. 2008). Local sensitivity analysis is the assessment of the local impact of input factors' variation on model response by concentrating on the sensitivity in the vicinity of a set of input factors. Such sensitivity is often evaluated through gradients or partial derivatives of the output functions at these input factors, thus other inputs' are held constant when studying the local sensitivity of a specific input. Such approaches have been used in evaluating large environmental systems, including climate modeling, oceanography, and hydrology (Cacuci 2003, Castaings et al. 2007). Borgonovo et al. (2014) used Gravity-based Land Use Model (G-LUM) by Kockelman et al. (2008) to illustrate LSAI techniques and found that the outputs respond almost additively to variations in the model inputs over the given scenarios. Changes in the base year employment assumptions strongly influence future job and land use pattern predictions.

Here, the following mathematical model is used to denote the input-output mapping:

$$y = f(\mathbf{x}), f: \Omega_{\mathbf{x}} \rightarrow \mathbb{R} \quad (10)$$

where  $y$  is the output,  $\mathbf{x} = (x_1, x_2, \dots, x_l) \in \Omega_{\mathbf{x}} \subseteq \mathbb{R}^l$  is the vector of the inputs.  $l$  is the number of (groups of) inputs.

Therefore, the base-case output of the simulation  $y^0 = f(\mathbf{x}^0)$  can be obtained by the simulation with inputs to a base-case scenario,  $\mathbf{x}^0$ . Furthermore, the analyst can know the response of the inputs in each scenario by obtaining different outputs  $y^s = f(\mathbf{x}^s)$  ( $s = 1, 2, \dots, S$ ) through simulating the alternative scenarios. However, he/she has no information about the sources of change (Borgonovo et al. 2014). The analyst also cannot distinguish both the importance of each input and their individual and interaction effects on the output. Recent works have addressed those problems through the concept of sensitivity analysis setting (Borgonovo et al. 2014).

1 To identify the relative importance of changes in single input or of interactions between inputs,  
 2 we can use the following complete decomposition of any finite change in  $f(\mathbf{x})$  (Saltelli and  
 3 Tarantola 2002; Saltelli et al. 2004; Borgonovo et al. 2014):

$$4 \quad \Delta y = f(\mathbf{x}^1) - f(\mathbf{x}^0) = \sum_{k_1=1}^l \Delta_{k_1} f + \sum_{k_1 < k_2}^l \Delta_{k_1, k_2} f + \dots + \Delta_{1,2,\dots,l} f \quad (11)$$

5 with

$$6 \quad \begin{cases} \Delta_{k_1} f = f(x_{k_1}^1, \mathbf{x}_{\sim k_1}^0) - f(\mathbf{x}^0) \\ \Delta_{k_1, k_2} f = f(x_{k_1}^1, x_{k_2}^1, \mathbf{x}_{\sim (k_1, k_2)}^0) - \Delta_{k_1} f - \Delta_{k_2} f - f(\mathbf{x}^0) \end{cases} \quad (12)$$

7 and where  $(x_{k_1}^1, \mathbf{x}_{\sim k_1}^0)$  denotes that the  $k_1$ th element of the  $\mathbf{x}$  vector,  $x_{k_1}^1$  is set at the value it  
 8 assumes in Scenario 1, while all other variables are at their Scenario 0 values. Thus, the  
 9 change  $\Delta y$  induced by the change of the inputs can be decomposed into individual effects and  
 10 interaction effects of inputs. Based on such decomposition, finite-change sensitivity indices can  
 11 be computed as follows:

$$12 \quad \varphi_{k_1, k_2, \dots, k_r}^r = \Delta_{k_1, k_2, \dots, k_r} f \quad (13)$$

13 where  $k_1, k_2, \dots, k_r$  denotes a group of  $r$  indices ( $r \leq l$ ) and  $\varphi_{k_1, k_2, \dots, k_r}^r$  is the portion of  $\Delta y$  due to the  
 14 interaction of inputs corresponding to the selected indices.

15 Particularly, the first-order finite-change sensitivity indices are  $\varphi_{k_i}^1 = \Delta_i f$  ( $k_i = 1, 2, \dots, l$ ) and the  
 16 total-order indices of  $x_{k_i}$  are  $\varphi_{k_i}^T = \Delta_{k_i} f + \sum_{k_1 < k_2}^l \Delta_{k_1, k_2} f + \dots + \Delta_{1,2,\dots,l} f$ , where  $\varphi_{k_i}^T$  is the total  
 17 contribution of  $x_{k_i}$  to  $\Delta y$ , and is the sum of the individual contribution of  $x_{k_i}$ , plus all the  
 18 contributions due to the interaction of  $x_{k_i}$  with the remaining inputs. Thus, the index  $\varphi_{k_i}^l =$   
 19  $\varphi_{k_i}^T - \varphi_{k_i}^1$  represents the interaction effects associated with  $x_{k_i}$  (Borgonovo et al. 2014).

20 As discussed in the literature (Saltelli and Tarantola 2002; Saltelli et al. 2004), the sign of the  
 21 first-order indices ( $\varphi_{k_i}^1$ ) is the sign change in  $y$  due to the individual change in  $x_{k_i}$ . The sign of  
 22  $\varphi_{k_1, k_2, \dots, k_r}^r$  is the sign of the interaction effects between the inputs  $x_{k_1}, x_{k_2}$  and  $x_{k_r}$ . The total-order  
 23 indices ( $\varphi_{k_i}^T$ ) are the appropriate sensitivity measures, since they deliver not only the individual  
 24 importance of the inputs, but also account for interaction effects. The magnitudes of  $\varphi_{k_1, k_2, \dots, k_r}^r$   
 25 provide the natural sensitivity measures.

26

## 27 **SENSITIVITY ANALYSIS OF THE RUBMRIO MODEL**

28 In this section, the RUBMRIO model is used to anticipate changes of domestic trade flow, export  
 29 trade flow, production and consumption in the continental U.S. resulting from two scenarios:  
 30 simultaneously increasing and decreasing ED, TC and TT by 20%. First, the data acquisition and  
 31 parameters estimates are introduced. Then, the two scenarios are considered through analyzing  
 32 sensitivity indices and total-order indices. In this sensitivity analysis, one can obtain both  
 33 individual effects of each input and their interactions' effects. This reflects whether interaction  
 34 effects across inputs amplify or dampen individual effects.

## 35 **DATA ACQUISITION**

36 The primary data source is the U.S. Department of Transportation's Freight Analysis Framework  
 37 version 3 (FAF<sup>3</sup>) database of networks and flows between FAF regions (FAF 2007). FAF  
 38 integrates data from a variety of sources to create a comprehensive picture of freight movement

1 among states and major metropolitan areas by all modes of transport. With data from the U.S.  
 2 2007 Commodity Flow Survey and other sources; FAF<sup>3</sup> provides estimates for tonnage and value,  
 3 by commodity type, mode, origin, and destination for year 2007 flows. FAF<sup>3</sup>'s origin-destination-  
 4 commodity-mode (ODCM) annual freight flows matrix was used to estimate RUBMRIO's nested  
 5 logit model's origin and mode choice parameters, to calculate all export demands (by port and  
 6 industry), and evaluate RUBMRIO model predictions. Commodities are classified at the 2-digit  
 7 level of the Standard Classification of Transported Goods  
 8 (SCTG) <http://www.statcan.gc.ca/eng/subjects/standard/sctg/sctgclass>, and were aggregated to  
 9 the closest 12 economic sectors, according to the codes with a complete description of these  
 10 categories and their constituent parts shown in Table 1 with corresponding IMPLAN Code and  
 11 NAICS Code.

12 **TABLE 1 Description of Economic Sectors in RUBMRIO Model**

Sector	Description	SCTG Code	IMPLAN Code	NAICS Code
1	Agriculture, Forestry, Fishing and Hunting	1	1~19	11
2	Food, Beverage and Tobacco Product Manufacturing	2~9	41~74	311, 312
3	Mining	10~15	20~30	21
4	Petroleum and Coal Product Manufacturing	16~19	115~119	324
5	Chemicals, Plastics and Rubber Product Manufacturing	20~24	120~152	325, 326
6	Other Durable & Non-Durable Manufacturing	25~31, 39	75~114, 153~169, 295~304	313~316, 321~323, 327, 337
7	Primary Metal Manufacturing	32	170~180	331
8	Fabricated Metal Manufacturing	33	181~202	332
9	Machinery Manufacturing	34	203~233	333
10	Computer, Electronic Product and Electrical Equipment Manufacturing	35, 38	234~275	334, 335
11	Transportation Equipment Manufacturing	36, 37	276~294	336
12	Miscellaneous Manufacturing	40, 41, 43	305~318	339

13

14 FAF<sup>3</sup> flows are also broken down by eight modes of transportation including truck, rail, water,  
 15 air, multiple modes and mail, pipeline, other and unknown, no domestic mode. See  
 16 <http://faf.ornl.gov/fafweb/Data/FAF3ODCMOverview.pdf> for more details about these mode  
 17 and commodity classes. Considering that truck and rail modes carry 40.1% and 40.2%,  
 18 respectively, of the U.S.'s 3,344 billion ton-miles of traded commodities according to the 2007  
 19 Commodity Flow Survey  
 20 ([http://www.rita.dot.gov/bts/sites/rita.dot.gov/bts/files/publications/commodity\\_flow\\_survey/final\\_tables\\_december\\_2009/html/table\\_01b.html](http://www.rita.dot.gov/bts/sites/rita.dot.gov/bts/files/publications/commodity_flow_survey/final_tables_december_2009/html/table_01b.html)), the RUBMRIO model used here includes just  
 21

1 two modes - truck and rail - all other modes are excluded. Travel times and costs between  
 2 counties (and from counties to export zones) were computed for the county-to-county matrix  
 3 based on shortest-path distances over TransCAD's highway and railway network models. See  
 4 <https://www.census.gov/geo/reference/codes/cou.html> for the details about the 3,109 counties  
 5 from the continental U.S.

## 6 Estimation of Parameters

7 As introduced in Equations (1) and (2), parameters  $\lambda^m$ , and  $\beta^m$  reflect producers' and shippers'  
 8 attraction to an origin zone's size and sensitivity to travel times and costs of the two alternative  
 9 modes (highway and railway), for each commodity  $m$ . To estimate such parameters for the  
 10 nested logit model structure (with lower level for mode choice and upper level for origin choice),  
 11 FAF<sup>3</sup>'s dollar values of freight flows between 120 domestic zones were used, for the 12  
 12 economic sectors (as shown in Table 1). Each FAF record was used as a data point or  
 13 "observation", and its dollar value used as the "weight" factor in the logit's log-likelihood  
 14 function. In the lower layer of the nested logit model, mode choices were first estimated for each  
 15 of the 12 sectors. Travel times and costs between counties (and from counties to export zones)  
 16 are computed based on shortest-path distances over TransCAD's highway and railway networks.  
 17 For sector  $m$ , the probability of choosing transport mode  $t$  between origin  $i$  and destination  $j$  is as  
 18 follows:

$$19 \quad P_{t|ij}^{mn} = \frac{\exp(V_{ij,t}^m)}{\sum_s \exp(V_{ij,s}^m)} \quad (14)$$

20 where  $V_{ij,t}^m$ , the systematic (non-random) conditional indirect utility, is given by:

$$21 \quad V_{ij,t}^m = \beta_{0,t}^m + \beta_{1,t}^m \text{time}_{ij,t} + \beta_{2,t}^m \cos t_{ij,t} \quad (15)$$

22  $\beta'$ 's are mode choice parameters to be estimated. ( $\beta_{0,railway}$  was set to zero in order to permit  
 23 statistical identification of the other parameters.)

24 In the upper layer, the probability of a producer in zone  $i$  choosing commodity  $m$  from firms in  
 25 zone  $j$  is:

$$26 \quad P_{ij}^{mn} = \frac{\exp(V_{ij}^m)}{\sum_i \exp(V_{ij}^m)} \quad (16)$$

27 where  $V_{ij}^m$  is the expected maximum utility across mode alternatives plus the origin-size  
 28 attractiveness term, shown as follows:

$$29 \quad V_{ij}^m = \lambda^m \ln[\sum_t \exp(V_{ij,t}^m)] \quad (17)$$

30 Table 2 shows all parameter estimates for the origin and mode choice models by sector (Du and  
 31 Kockelman 2012). The correlated nature of cost and time variables, and use of assumed (rather  
 32 than actual) results, is presumably causing the negative coefficient estimates for several sectors.

1 Such situations appear more common for high-weight, low-time-value goods, with long-distance  
 2 transport relying on rail, rather than the faster mode of trucking.

3 **TABLE 2 Estimated Parameters for Nested Logit Models of Origin and Mode Choice**

Sector	Origin Choice Parameters		Mode Choice Parameters			
	$\lambda^m$	$\rho^2$ (Rho-Square)	$\beta_{0,tuck}^m$	$\beta_{1,t}^m$	$\beta_{2,t}^m$	$\rho^2$ (Rho-Square)
1	0.448	0.403	5.640	-4.010	-4.040	0.999
2	-1.430	0.242	5.600	1.810	0.464	0.772
3	-3.830	0.262	1.850	0.857	0.0761	0.109
4	1.010	0.493	1.670	-1.560	-3.410	0.755
5	0.801	0.206	1.420	-1.010	-1.120	0.486
6	1.090	0.081	5.540	1.540	0.575	0.562
7	1.690	0.130	1.430	-0.823	-1.280	0.817
8	0.173	0.16	3.180	-0.478	-0.741	0.936
9	0.339	0.224	-3.610	-8.500	-6.980	0.934
10	0.097	0.288	-1.590	-6.000	-4.160	0.613
11	-0.840	0.130	-3.470	-6.090	-5.270	0.825
12	0.805	0.272	2.830	-1.900	-1.960	0.926

4

5 Technical coefficients  $a^{mn}$  reflect production technology within counties and are very important  
 6 parameters in the RUBMRIO model. In this study, the technical coefficients are assumed to be  
 7 stable due to only considering the situation in the short run. Therefore, they are exogenous to the  
 8 model, based on IMPLAN's transaction tables derived from U.S. inter-industry accounts and  
 9 estimate the values of purchases at finer levels of resolution. RPCs describe the proportion of  
 10 local demand for a commodity that is purchased from local producers. Here, a constant RPC  
 11 value was used in all counties. These RPCs are generated by IMPLAN automatically, using a set  
 12 of econometric equations (MIG 2001).

### 13 Sensitivity Analysis of RUBMRIO Model via Two Scenarios

14 This section describes the scenario decomposition applied to RUBMRIO. The 3,109 counties  
 15 come from the continental U.S. states, as shown in Table 3.

16

**TABLE 3 Continental U.S. States and Counties**

No.	State	Abbreviation	#Counties	No.	State	Abbreviation	#Counties
1	Alabama	AL	67	26	Nebraska	NE	93
2	Arizona	AZ	15	27	Nevada	NV	17
3	Arkansas	AR	75	28	New Hampshire	NH	10
4	California	CA	58	29	New Jersey	NJ	21
5	Colorado	CO	64	30	New Mexico	NM	33
6	Connecticut	CT	8	31	New York	NY	62
7	Delaware	DE	3	32	North Carolina	NC	100

8	District of Columbia	DC	1	33	North Dakota	ND	53
9	Florida	FL	67	34	Ohio	OH	88
10	Georgia	GA	159	35	Oklahoma	OK	77
11	Idaho	ID	44	36	Oregon	OR	36
12	Illinois	IL	102	37	Pennsylvania	PA	67
13	Indiana	IN	92	38	Rhode Island	RI	5
14	Iowa	IA	99	39	South Carolina	SC	46
15	Kansas	KS	105	40	South Dakota	SD	66
16	Kentucky	KY	120	41	Tennessee	TN	95
17	Louisiana	LA	64	42	Texas	TX	254
18	Maine	ME	14	43	Utah	UT	29
19	Maryland	MD	26	44	Vermont	VT	14
20	Massachusetts	MA	14	45	Virginia	VA	134
21	Michigan	MI	83	46	Washington	WA	39
22	Minnesota	MN	87	47	West Virginia	WV	55
23	Mississippi	MS	82	48	Wisconsin	WI	72
24	Missouri	MO	115	49	Wyoming	WY	23
25	Montana	MT	56	Total number of counties			3109

1

2 RUBMRIO's three major inputs are as follows:

- 3 a) Foreign Export Demand (ED): the foreign export flows via 106 export zones, across 12  
4 economic sectors. ED is assumed to be the only source of final demand, which must be  
5 satisfied by the U.S. counties.
- 6 b) Transport Costs (TC): travel costs between each pair of counties (or from counties to export  
7 zones). We vary travel costs between each pair of counties. TC is the key component of most  
8 any trade model, and can rise or fall relatively quickly in response to changing energy prices,  
9 labor costs, shipping regulations, and interest rates (which affect the real price of vehicle  
10 capital).
- 11 c) Travel Times (TT): the travel time between each pair of counties (or from counties to export  
12 zones). As a key component of the utility functions, transport time affects trade flow patterns,  
13 local production and consumption.

14 The base case scenario used here,  $x^0 = (ED^0, TC^0, TT^0)$ , is based on data used in Du and  
15 Kockelman (2012). The RUBMRIO model is used to examine the different scenarios' effects on  
16 the distributions of trade flows and production by simulating those alternative scenarios, after  
17 first changing ED in each of the 12 export-related sectors, changing Interstate Highway (IH) 40's  
18 TT by 10%, and changing the marginal average time of trucking by 20% up and then down, each  
19 factor one at a time (Du and Kockelman 2012). In this paper, one can consider the two

1 distinctive scenarios  $x^1 = (ED^1, TC^1, TT^1)$  (simultaneously increasing ED, TC and TT by 20%)  
 2 and  $x^2 = (ED^2, TC^2, TT^2)$  (simultaneously decreasing ED, TC and TT by 20%). Therefore, the  
 3 change of each model output resulted from  $x^0$  to  $x^1$  (or  $x^2$ ) can be decomposed into eight terms,  
 4 which account for the individual effect in ED, TC and TT, their interaction effects in pairs, and  
 5 in the residual term that contains their overall and residual interaction. Thus, the following  
 6 sensitivity indices can be obtained:  $\varphi_{ED}^1, \varphi_{TC}^1, \varphi_{TT}^1, \varphi_{ED,TC}^2, \varphi_{ED,TT}^2, \varphi_{TC,TT}^2, \varphi_{ED,TC,TT}^3$  and total-  
 7 order indices

$$8 \quad \begin{cases} \varphi_{ED}^T = \varphi_{ED}^1 + \varphi_{ED,TC}^2 + \varphi_{ED,TT}^2 + \varphi_{ED,TC,TT}^3 \\ \varphi_{TC}^T = \varphi_{TC}^1 + \varphi_{ED,TC}^2 + \varphi_{TC,TT}^2 + \varphi_{ED,TC,TT}^3 \\ \varphi_{TT}^T = \varphi_{TT}^1 + \varphi_{ED,TT}^2 + \varphi_{TC,TT}^2 + \varphi_{ED,TC,TT}^3 \end{cases} \quad (18)$$

9 Simultaneously increasing (or decreasing) ED, and TC and TT by 20% will have different first-  
 10 order effects, interaction effects and total-order effects on domestic trade flow (D for short),  
 11 export trade flow (E for short), production (P for short) and consumption (C for short) in  
 12 counties, where production is the sum of D and E. To obtain each state's overall effect estimate,  
 13 we summed all county-level effects across each continental U.S. state. Hence, we record 20  
 14 states with largest increase and decrease of effects on domestic trade flows in these two scenarios  
 15 in Tables 4 through 9. This paper records 10 states with largest and smallest changes in domestic  
 16 trade flows by the first-order and total-order effects of ED, because ED has the same sign with  
 17 different magnitude of first-order and total-order effects on domestic trade flows, production and  
 18 consumption in every state under each scenario. Apart from the first-order and total-order effects  
 19 of ED, other effects on the domestic trade flows may be negative and positive in different states  
 20 under each scenario. This paper records 10 states with negative and positive changes (where five  
 21 states with largest and five states with smallest) in domestic trade flows by the first-order and  
 22 total-order effects of TC and TT, and interaction effects under each scenario.

23 **TABLE 4 Scenario 1's First-order Effects**

24

The First-order effects of ED			The First-order effects of TC			The First-order effects of TT					
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
DC	165	47	188	VA	11414 8	435 0	10326 8	VA	-96449	641 9	-84894
DE	2720	415	2796	KY	-56319	217 8	-54369	KY	-34360	-638	-31778
RI	6491	1085	6778	NC	-55806	-447	-54591	NC	-31924	138 9	-28722
NH	10622	958	10679	GA	-50009	157 8	-48141	GA	-29637	132	-28313
M A	15350	1922	15627	KS	-47976	220 8	-45821	FL	-4224	443	-3873
ME	17331	2197	17645	WI	-2548	-144	-3194	RI	-1579	3	-1378

NV	18431	1639	18535	VT	-1886	-48	-1810	DE	-1448	-2	-1319
CT	22510	3730	23124	RI	-1822	55	-1688	ME	-1232	129	-1237
OR	25721	3222	26286	DE	-1444	8	-1363	MD	-980	272	-1017
VT	30841	4628	32302	DC	-98	-2	-94	DC	-94	1	-84
MI	17277 0	1839 7	17250 0	NH	3051	-14	2803	MA	24	185	7
AL	18074 9	1160 1	18108 3	NV	4506	-106	4155	NM	897	382	823
NC	18306 7	2879 1	18854 8	MO	4830	- 173 0	539	NJ	907	539	1793
NY	19566 6	2299 5	19969 9	AL	8081	844	10089	MS	1041	-396	1177
M O	28103 6	2486 2	27214 5	AZ	8688	-77	8595	PA	2064	194 5	2052
CA	34204 3	9666	33122 7	AR	28212	-616	26441	W Y	48815	107 3	46891
NE	35046 2	9918	33892 6	W Y	43631	383	42526	AR	53090	310	51705
CO	35680 7	1236 2	35134 9	CO	10365 0	-731	10818 6	CA	11286 4	-266	10955 5
TX	38561 8	5640 9	40175 6	CA	11221 7	- 183 7	10976 9	CO	17408 0	128 9	17209 0
VA	63061 2	7939 0	64659 7	NE	13704 3	- 102 3	13752 3	NE	17454 0	212	16321 0

1 **Note:** Simultaneously increasing all ED, TC and TT by 20% as Scenario 1.

2 The first-order effects of ED are positive on all of these outputs. That is to say, an increase in ED  
3 corresponds to an increase in domestic trade flows, export trade flows, production, and  
4 consumption. Table 4 reports the 20 states with the largest and smallest changes in domestic  
5 trade flows by the first-order effects of ED. Table 4 shows ED has the strongest first-order  
6 effects on VA's domestic trade flows, export trade flows, production and consumption. Increases  
7 in TX's domestic and export trade flows, production, and consumption resulting from a 20%  
8 increase in ED are almost half of increase in VA's, although TX exhibits the second strongest  
9 first-order ED effects. The reason is that VA At the same time, ED has almost no first-order  
10 effects on the small region/district of DC (with predicted changes in domestic trade flows, export  
11 trade flows, production and consumption of just \$165, \$47, \$212, and \$188, respectively).  
12 Compared with DC, DE (a very small state) exhibits the second weakest ED effects (with values  
13 of \$2,720, \$415, \$3,136, and \$2,796, respectively).

14 As opposed to ED, TC and TT have positive or negative effects on domestic and export trade  
15 flows, production, and consumption in different states under Scenario 1. Table 4 displays five  
16 states with both negative and positive changes in domestic trade flows via TC's and TT's first-  
17 order effects. VA suffers the strongest negative effects to its domestic trade flows (falling

1 \$114,148) when increasing TC by 20%, but with VA's export trade flows predicted to rise by  
2 \$4,350 (the most of any shown state). KY, NC, and GA follow VA in decreasing order of  
3 domestic trade flow impacts: -\$56,319, -\$55,806, and -\$50,009, respectively. VA, KY, NC, and  
4 GA exhibit the strongest negative effects on their production and consumption due to increasing  
5 TC by 20%. However, among states with increasing domestic trade flows, NE, CA, and CO  
6 exhibit the biggest increase of domestic trade flows, with values of \$137,043, \$112,217, and  
7 \$103,650, respectively. TC also has the strongest positive effect on their production and  
8 consumption although their export trade flows decrease because of increasing TC. Increasing TC  
9 has almost null (positive or negative) effects on export trade flows in DC, DE, NH and VT  
10 because the (negative or positive) changes of their export trade flows are less than \$50. TT has  
11 the strongest negative effects on VA's domestic trade flows, decreasing by \$96,449 compared  
12 with \$34,360 (the second decreasing of export trade flows in KY) and has the strongest positive  
13 effects on export trade flows in VA, increasing by \$6,419. VA, KY, NC, and GA have the  
14 strongest negative effect on their production and consumption due to increasing TT by 20%.  
15 However, when increasing TT by 20%, CO, NE, and CA obtain the biggest increase of domestic  
16 trade flows, production and consumption although CA's export trade flow decreases by \$266  
17 because of increasing TT. Increasing TT has almost null (positive or negative) effects on export  
18 trade flows in DC, DE, and RI because the (negative or positive) changes of their export trade  
19 flows are less than \$5.

20 To sum up, TC or TT has the same sign with different magnitude of first-order effect on  
21 domestic trade flows, production and consumption in VA, KY, NC, GA, CO, NE, and CA. ED is  
22 the most influential factor on all outputs compared with TC and TT. In some states, increasing  
23 TC and TT have completely opposite effects on domestic trade flows, export trade flows,  
24 production and consumption.

25

1

TABLE 5 Scenario 1's Interaction Effects

ED&TC			ED&TT			TC&TT			ED&TC&TT						
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
VA	-22830	870	-20654	VA	-19290	1284	-16979	VA	-106322	-3633	-104440	VA	-21264	-727	-20888
KY	-11264	-436	-10874	KY	-6872	-128	-6356	AL	-47868	81	-47273	AL	-9574	16	-9455
NC	-11161	-89	-10918	NC	-6385	278	-5744	KY	-21770	-355	-19375	KY	-4354	-71	-3875
GA	-10002	-316	-9628	GA	-5927	26	-5663	WI	-10872	-138	-10178	WI	-2174	-28	-2036
KS	-9595	-442	-9164	IN	-3927	-95	-3664	NC	-10738	-432	-10092	NC	-2148	-86	-2018
WI	-510	-29	-639	RI	-316	1	-276	OR	-672	45	-633	OR	-134	9	-127
VT	-377	-10	-362	DE	-290	0	-264	MS	-302	55	-9	MS	-60	11	-2
RI	-364	11	-338	ME	-246	26	-247	DE	-267	1	-220	DE	-53	0	-44
DE	-289	2	-273	MD	-196	54	-203	NH	-47	33	-17	NH	-9	7	-3
DC	-20	0	-19	DC	-19	0	-17	DC	-37	7	-29	DC	-7	1	-6
NH	610	-3	561	MA	5	37	1	WY	119	-194	-27	WY	24	-39	-5
NV	901	-21	831	NM	179	76	165	ME	138	148	182	ME	28	30	36
MO	966	-346	108	NJ	181	108	359	MD	310	213	309	MD	62	43	62
AL	1616	169	2018	MS	208	-79	235	FL	517	413	563	FL	103	83	113
AZ	1738	-15	1719	PA	413	389	410	CT	584	211	655	CT	117	42	131
AR	5642	-123	5288	WY	9763	215	9378	MO	19019	1085	14295	MO	3804	217	2859
WY	8726	77	8505	AR	10618	62	10341	AR	19688	198	19001	AR	3938	40	3800
CO	20730	-146	21637	CA	22573	-53	21911	MT	20743	508	23178	MT	4149	102	4636
CA	22443	-367	21954	CO	34816	258	34418	TX	21532	3853	20955	TX	4306	771	4191
NE	27409	-205	27505	NE	34908	42	32642	CO	24523	1303	29241	CO	4905	261	5848

2 **Note:** Simultaneously increasing all ED, TC and TT by 20% as Scenario 1.

3

4

1  
2 The interaction effects for domestic trade flows can be negative or positive across different states.  
3 Table 5 shows 10 states with both negative and positive changes (where five states with largest  
4 and five states with smallest) in domestic trade flows for Scenario 1's interaction effects. Table 5  
5 show that all 4 types of interaction effects (ED&TC, ED&TT, TC&TT, ED&TC&TT) are most  
6 strongly negative in the case of Virginia's (VA's) domestic trade flows and consumption, with  
7 values of \$22,830 and -\$20,654 (for ED&TC effects on domestic flows and consumption), -  
8 \$19,290 and -\$16,979 (for ED&TT effects), -\$106,322 and -\$104,440 (for TC&TT effects), and -  
9 \$21,264 and -\$20,888 (for ED&TC&TT effects). In other words, VA is estimated to experience  
10 the largest losses of domestic trade flows and consumption when ED, TC, and TT are all  
11 increased together, by 20%. However, ED&TC and ED&TT have the biggest positive interaction  
12 effects on VA's export trade flows, with values \$870 and \$1,284, while TC&TT and  
13 ED&TC&TT are anticipated to have the greatest negative interaction effects (of -\$3,633 and -  
14 \$724, respectively) on VA's export trade flows. Thus, increasing ED and TC, combined with ED  
15 and TT will lead to the biggest increase of VA's export trade flows while increasing TC and TT,  
16 combined with ED, TC and TT will induce the biggest decrease of VA's export trade flows. KY,  
17 NC, GA, and KS are the next four states that follow VA, in terms of domestic trade flow losses  
18 and consumption reductions, thanks to the negative interaction effects between ED and TC, as  
19 well as interaction effects between ED and TT.

20 The states of AL, KY, WI, and NC are expected to follow VA in terms of lowered domestic  
21 trade flows and consumption due to the negative interaction effects between TC and TT, as well  
22 as interaction effects among ED, TC and TT. VA and AL are expected to experience the greatest  
23 negative interaction effects on domestic trade flows, production and consumption, when TC and  
24 TT rise together and/or ED, TC and TT rise together. However, VA's changes in domestic trade  
25 flows, production and consumption more than double those of AL. NE, CA, and CO are  
26 estimated to experience the greatest increases in domestic trade flows, including production and  
27 consumption values over \$20,000, although their export trade flows are expected to fall, under  
28 interaction effects between ED and TC. The interaction effects between ED and TT also trigger  
29 the greatest increases in domestic trade flows, including production and consumption values over  
30 \$20,000 in NE, CO, CA. However, CA's export trade flows are nearly unchanged, falling by just  
31 \$53, while NE's and CO's export trade flows are projected to rise by \$42 and \$258, thanks to  
32 interaction effects between ED and TT. CO, TX, and MT are predicted to experience the greatest  
33 increases in domestic trade flows, as well as production and consumption, and their export trade  
34 flows also rise, thanks to interaction effects between ED and TC, and among ED, TC and TT.  
35 Essentially, trade, production and consumption are able to shift in a variety of ways across a set  
36 of networked states and regions; so it is valuable to have a model like RUBMRIO to anticipate  
37 those movements, and techniques like LSAI to appreciate the sources of variations in model  
38 outputs.

39 The negative or positive changes of domestic trade flows in other states are all less than \$9,000.  
40 Interactions between ED and TC have negligible (under \$100) effects on export trade flows in 10  
41 of the above-20 states. Thirteen (13) states of the 20 states exhibit negligible export-flow change  
42 from interactions effects between ED and TT. Six (6) of the 20 states have negative changes in  
43 export flows when TC and TT interactions are considered, and 15 have negligible export-flow  
44 effects from interactions across ED, TC and TT. Meaningfully, domestic trade flow effects from  
45 interactions between TC and TT and among ED, TC and TT all share the same signs/direction,

1 but with different magnitudes, in each of the 20 states.

2  
3

**TABLE 6 Scenario 1's Total-order Effects**

Total-order Effects of ED			Total-order Effects of TC			Total-order Effects of TT					
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
DC	119	48	147	VA	- 26456 4	860	- 24924 9	VA	- 24332 5	3343	- 22720 0
DE	2089	417	2216	KY	-93706	- 303 9	-88494	AL	-71297	1140	-70464
RI	5665	1080	6015	NC	-79852	- 105 5	-77619	KY	-67356	-1191	-61384
NH	12351	981	12311	GA	-61781	- 135 5	-58511	NC	-51195	1148	-46576
M A	14767	1972	15060	KS	-58686	- 262 4	-55517	GA	-37334	697	-34718
ME	16225	2222	16565	ME	-5154	-5	-4996	M N	-1980	974	-1592
CT	20017	3858	20788	M A	-3527	75	-3407	UT	-1373	1353	-308
NV	21280	1642	21215	RI	-3062	-30	-2920	ME	-1313	333	-1265
OR	24644	3160	25234	DE	-2053	11	-1899	M D	-804	581	-850
M D	30683	3707	30929	DC	-161	6	-147	DC	-157	10	-136
NC	16337 3	2889 3	16986 7	VT	64	398	286	TN	179	-51	2922
MI	16996 6	1828 9	16888 4	NJ	1056	401	-29	MS	886	-410	1402
AL	17048 2	1196 0	17135 7	NH	3605	22	3343	PA	1119	2503	721
NY	19567 0	2292 8	20010 8	SD	6316	-890	4729	OR	1771	181	1868
M O	28778 5	2490 2	27943 0	NV	7340	-76	6855	M A	1777	357	1627
CA	38601 5	9355	37445 5	MT	53423	-118	56128	TX	69602	1061 1	68913
TX	38812 5	5775 0	40395 8	AR	57480	-502	54530	AR	87333	610	84848
NE	41424	9848	40094	CA	12839	-	12790	CA	12917	338	12764

	6		4		8	154 7	3		3		6
CO	41725 8	1273 4	41325 2	CO	15380 8	687	16491 2	NE	21824 9	805	20707 7
VA	56722 8	8081 7	58807 7	NE	17325 4	-677	17625 3	CO	23832 3	3111	24159 7

**Note:** Simultaneously increasing all ED, TC and TT by 20% as Scenario 1.

Table 6 shows the total-order effects of ED, TC and TT on domestic and export trade flows and consumption in the 20 continental U.S. states listed. Similar to ED's first-order effects, ED's total-order effects are all positive on these outputs in all states - and ED is expected to have the strongest total-order effect on VA's domestic trade flows, export trade flows and consumption. However, in VA, ED's total-order effects are less than its first-order effects on domestic trade flows and consumption. TX exhibits the second strongest total-order effects for ED on export trade flows and production (when summing domestic and export trade flows), and ED has its next-strongest total-order effects on domestic trade flow and consumption in CO. Washington D.C. and DE, as very small regions, exhibit the weakest total-order effects of ED on domestic trade flows, export trade flows and consumption with values.

The strongest negative total-effects of TC on domestic trade flows and consumption happen in VA, although the total-effects of TC on export trade flows is positive. KY, NC, and GA follow VA in negative total-effects of TC on domestic trade flows and consumption with negative total-effects of TC on export trade flows. NE, CO, and CA have the strongest total-order effects of TC on domestic trade flows and consumption while the total-order of TC on export trade flows is positive in CO and are negative in CA and NE. TC has almost null (positive or negative) total-order effects on export trade flows in DC, DE, MA, ME, NH, NV, and RI because the (negative or positive) changes of their export trade flows are less than \$100. The largest decrease resulted from the total-order effect of TT on the domestic trade flows and consumption also happen in VA, which is the same as the first-order effect of TT. However, AL has the second strongest total-order effects of TT, as its domestic trade flows and consumption, while its export trade flows increases. Similarly, CA, CO, and NE have the strongest positive total-order effect of TT on domestic trade flows and consumption. The biggest increase of export trade flows happens in TX with \$10,611 compared to \$3,343, which is the second largest increase of export trade flows in VA. TT has almost null (positive or negative) total-order effects on export trade flows in DC and TN because the (negative or positive) changes of their export trade flows are less than \$100.

**TABLE 7 Scenario 2's First-order Effects**

	First-order effects of ED				First-order effects of TC				First-order effects of TT		
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
VA	- 63061 2	- 7939 0	- 64659 7	NE	- 19352 8	- 693	- 19087 1	NE	- 17688 3	- -85	- 17056 5
TX	- 38561 8	- 5640 9	- 40175 6	CO	- 15655 0	- 430	- 15904 0	CO	- 15280 6	- -694	- 15425 2

CO	- 35680 7	- 1236 2	- 35134 9	CA	- 14963 3	149 5	- 14450 9	CA	- 10954 0	392	- 10720 5
NE	- 35046 2	-9918	- 33892 6	MT	-78084	124	-78060	MO	-63347	- 213 8	-64246
CA	- 34204 3	-9666	- 33122 7	W Y	-60586	- 108 6	-59035	W Y	-60095	- 163 8	-58918
M O	- 28103 6	- 2486 2	- 27214 5	W V	-8536	-752	-6147	NY	-6441	- 133 0	-8525
NY	- 19566 6	- 2299 5	- 19969 9	MS	-7775	90	-6173	NH	-4746	-105	-4569
NC	- 18306 7	- 2879 1	- 18854 8	W A	-7460	- 301 7	-9239	NJ	-4651	-887	-4898
AL	- 18074 9	- 1160 1	- 18108 3	NV	-5820	14	-5489	OR	-2322	-63	-2238
MI	- 17277 0	- 1839 7	- 17250 0	NH	-4022	-66	-3816	PA	-337	- 186 6	-476
VT	-30841	-4628	-32302	DC	109	-1	103	MA	47	-205	17
OR	-25721	-3222	-26286	NJ	480	-236	941	RI	58	-147	-146
CT	-22510	-3730	-23124	DE	1257	-47	1179	DC	92	-2	84
NV	-18431	-1639	-18535	RI	1656	-118	1395	DE	640	-78	561
ME	-17331	-2197	-17645	VT	1770	-78	1506	MD	1382	-385	1218
M A	-15350	-1922	-15627	NC	57664	- 151 9	54104	ID	35861	101 2	34357
NH	-10622	-958	-10679	GA	61866	823	58839	TX	36517	- 168 9	28745
RI	-6491	-1085	-6778	KY	64426	140 6	61551	IN	36729	718	34628
DE	-2720	-415	-2796	TX	69416	349	63565	GA	49358	-184	46525
DC	-165	-47	-188	VA	83199	- 872 7	69003	VA	71359	- 941 4	55997

1 **Note:** Simultaneously decreasing all ED, TC and TT by 20% as Scenario 2.

2

3 As shown in Table 7, the first-order effects of ED are negative on all of these outputs. In other

1 words, a decrease in ED leads to reductions in domestic trade flows, export trade flows and  
2 consumption, as expected. Table 7 reports the 20 states with the largest and smallest changes in  
3 domestic trade flows, via ED's first-order effects. Table 4 shows ED's strongest first-order  
4 effects are on VA's domestic trade flows, export trade flows and consumption. TX, CO, NE and  
5 CA follow, with domestic trade flow and consumption losses all below-\$300,000 and export  
6 trade flows losses below-\$9,000.

7 Different with ED's rather consistently directed effects, TC and TT changes lead to a variety of  
8 changes in domestic and export trade flows, production, and consumption, across different states,  
9 under Scenario 2. ED is the most influential factor, overall, but TC and TT lie directly in the  
10 transportation infrastructure and operations domains, so they are of great interest to  
11 transportation policymakers and system managers. Table 7 reports five states with both negative  
12 and positive changes in domestic trade flows due to TC's and TT's first-order effects. NE is  
13 estimated/predicted to exhibit the greatest losses in domestic trade flows and consumption when  
14 TC or TT fall (by 20%), yet negligible export trade flow effects (just -\$85). CO and CA are next  
15 in terms of domestic trade flow and consumption losses, from TC or TT's first-order effects.  
16 Consistent with other evaluations, discussed above, TC and TT show the strongest positive first-  
17 order effects on VA's domestic trade flows and consumption, with TX coming in second for  
18 TC's effects and GA coming in second for TT's first-order effects on domestic trade flow and  
19 consumption.

20 Lower TC is predicted to have negligible effects on export trade flows in DC, DE, MS, NH, NV,  
21 and VT (all less than \$100, in absolute terms). And lower TT values have almost no effect on  
22 export trade flows in DC, DE, NE and OH and on domestic trade flows in DC, MA, RI.

23

1

**TABLE 8 Scenario 2's Interaction Effects**

ED&TC			ED&TT			TC&TT			ED,TC&TT						
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
VA	-16640	1745	-13801	VA	-14272	1883	-11199	VA	-46365	565	-42591	NE	-8565	-102	-8874
TX	-13883	-70	-12713	GA	-9872	37	-9305	AL	-32040	298	-31924	MT	-5771	-83	-6029
KY	-12885	-281	-12310	IN	-7346	-144	-6926	KY	-25142	-260	-23535	CO	-5326	-229	-5837
GA	-12373	-165	-11768	TX	-7303	338	-5749	GA	-18567	715	-16914	MO	-3616	-120	-2740
NC	-11533	304	-10821	ID	-7172	-202	-6871	TX	-17030	1001	-14924	AR	-3443	-53	-3292
VT	-354	16	-301	MD	-276	77	-244	MD	-1074	167	-966	NH	-253	-7	-239
RI	-331	24	-279	DE	-128	16	-112	RI	-945	-75	-876	VT	-251	-24	-246
DE	-251	9	-236	DC	-18	0	-17	FL	-677	356	-585	OR	-143	-13	-134
NJ	-96	47	-188	RI	-12	29	29	DE	-56	35	-42	MA	-35	-15	-29
DC	-22	0	-21	MA	-9	41	-3	DC	-28	4	-24	NM	-17	-61	-16
NH	804	13	763	PA	67	373	95	NM	83	303	79	DC	6	-1	5
NV	1164	-3	1098	OR	464	13	448	MA	176	77	146	DE	11	-7	8
WA	1492	603	1848	NJ	930	177	980	OR	715	63	671	FL	135	-71	117
MS	1555	-18	1235	NH	949	21	914	VT	1256	120	1228	RI	189	15	175
WV	1707	150	1229	NY	1288	266	1705	NH	1267	35	1196	MD	215	-33	193
WY	12117	217	11807	WY	12019	328	11784	AR	17215	264	16461	TX	3406	-200	2985
MT	15617	-25	15612	MO	12669	428	12849	MO	18081	598	13698	GA	3713	-143	3383
CA	29927	-299	28902	CA	21908	-78	21441	CO	26629	1147	29184	KY	5028	52	4707
CO	31310	-86	31808	CO	30561	139	30850	MT	28855	414	30146	AL	6408	-60	6385
NE	38706	-139	38174	NE	35377	17	34113	NE	42823	512	44372	VA	9273	-113	8518

2 **Note:** Simultaneously decreasing all ED, TC and TT by 20% as Scenario 2.

1  
2 Domestic trade flows effects for each pair of ED, TC and TT input assumptions, and across all  
3 three sets of inputs, vary in direction across different states. Table 8 record five states with both  
4 negative and positive effects, for the largest and smallest changes in domestic trade flows by  
5 interaction effects under Scenario 2. Table 8 shows how interaction effects between each pair of  
6 ED, TC and TT input assumptions are greatest for VA's domestic trade flows and consumption,  
7 while NE offers the biggest losses in domestic flows and consumption estimates. However,  
8 ED&TC, ED&TT, and TC&TT pairs have the biggest *positive* interaction effects on NE's  
9 domestic flows and consumption, while ED&TC&TT has the biggest positive interaction effects  
10 on VA's domestic flows and consumption.. TX follows VA in decreasing of domestic trade  
11 flows and consumption, while CO follows NE. However, ED and TC have almost no interaction  
12 effects (all less than \$100, in magnitude) on TX, CO and nine other states' export trade flows  
13 (among the 20 shown here). GA follows VA in decreasing of domestic trade flows and  
14 consumption ,while CO follows NE in increasing of domestic trade flows and consumption by  
15 interaction effects between ED and TT.AL follows VA in decreasing of domestic trade flows and  
16 consumption while MT follows NE in increasing of domestic trade flows and consumption by  
17 interaction effects between TC and TT. MT follows NE in decreasing of domestic trade flows  
18 and consumption while AL follows VA in increasing of domestic trade flows and consumption  
19 by interaction effects between ED, TC and TT.

20 **TABLE 9 Scenario 2's Total-order Effects**  
21

	Total-order Effects of ED				Total-order Effects of TC				Total-order Effects of TT		
	D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)		D(\$)	E(\$)	C(\$)
VA	-	-	-	NE	-	-	-	NE	-	-	-
	652,25	75,87	663,07		120,56	964	117,20		107,24	341	100,95
	0	5	9		4		0		8		5
TX	-	-	-	CA	-	1,71	-	CO	-	-	-
	403,39	56,34	417,23		115,15	1	110,60		100,94	362	100,05
	9	1	3		6		5		2		4
CO	-	-	-	CO	-	1,26	-	CA	-	-	-
	300,26	12,53	294,52		103,93	1	103,88		83,082	830	80,762
	1	8	7		7		5				
CA	-	-	-	AL	-	-532	-	W	-	-	-
	291,34	10,17	282,13		43,095		43,583		39,460	1,29	38,744
	6	2	5						2		
NE	-	-	-	W	-	-	-	M	-	-	-
	284,94	10,14	275,51		39,853	-851	38,837		36,213	1,23	40,438
	5	2	4						2		
M	-	-	-	AZ	-	48	-	OR	-1,285	0	-1,254
O	261,87	24,42	253,51		12,022		11,594				
	2	9	9								
NY	-	-	-	W	-8,795	-696	-7,793	OH	-1,246	-616	-2,121
	197,63	22,98	200,87								
	9	7	5	V							
NC	-	-	-	W	-8,395	-	-9,444	RI	-710	-178	-817

	197,554	28,075	201,763	A		2,117					
MI	-168,017	-18,166	-168,415	NV	-3,600	41	-3,386	SD	-536	-127	-1,047
AL	-167,425	-11,148	-167,334	NH	-2,204	-24	-2,096	SC	-503	-286	-794
AZ	-28,613	-1,110	-27,859	DC	65	3	64	DC	52	2	48
OR	-26,502	-3,281	-27,045	RI	569	-155	415	MA	179	-102	130
CT	-24,865	-3,596	-25,301	DE	961	-10	909	MD	247	-175	201
ME	-19,113	-2,225	-19,356	MS	1,899	452	2,873	DE	467	-34	415
NV	-16,116	-1,631	-16,342	VT	2,421	33	2,187	NJ	803	-159	171
MA	-16,032	-1,881	-16,251	KY	31,427	917	30,413	TX	15,590	-550	11,057
NH	-9,122	-931	-9,241	GA	34,639	1,230	33,541	IN	18,033	652	17,182
RI	-6,645	-1,016	-6,852	KS	36,850	1,014	34,507	ID	19,187	801	18,535
DE	-3,089	-397	-3,136	NC	38,855	-885	36,464	VA	19,995	-7,079	10,725
DC	-200	-47	-221	TX	41,909	1,080	38,913	GA	24,633	425	23,689

1 **Note:** Simultaneously decreasing all ED, TC and TT by 20% as Scenario 2.

2

3 Table 9 shows the total-order effects of ED, TC and TT on domestic and export trade flows,  
4 production and consumption in the continental U.S. Similar to ED's first-order effects, ED's  
5 total-order effects are negative on all states' outputs when ED is lowered. In contrast, TC and TT  
6 have much more complex total-order effects, moving in both negative and positive directions for  
7 domestic trade flows, export trade flows, production and consumption across states.

8 Table 9 shows the total-order effects of ED, TC and TT on domestic and export trade flows and  
9 consumption in the 20 continental U.S. states under Scenario 2. Similar to ED's first-order  
10 effects, ED's total-order effects are all negative on these outputs in all states, and ED's strongest  
11 total-order effect are on VA's domestic trade flows, export trade flows and consumption.  
12 However, in VA, ED's total-order effects are smaller than ED's first-order effects were, on  
13 domestic trade flows and consumption, yet larger for export trade flow effects.

14 The strongest negative total-effects of TC and TT on domestic trade flows and consumption  
15 happen in NE, although the total-effects of TC and TT on export trade flows are positive.

1 By comparing the results under these two scenarios, one can conclude that first-order effects of  
2 ED are symmetric from the first-order of ED in Tables 4 and 7 because ED has the opposite  
3 signs of first-order effects with the same magnitudes on domestic trade flows, export trade flows  
4 and consumption in 20 states. Other effects (excluding the first-order effects of ED) are not all  
5 symmetric, so the signs and/or magnitudes of the same effects under different scenarios differ  
6 across Tables 4 through 9.

## 7 **CONCLUSIONS AND EXTENSIONS**

8 This paper uses the technique of LSAI to produce sensitivity indices for the variation of outputs,  
9 due to finite variations in model inputs to a complex model of production, consumption and trade  
10 flows across 3,109 U.S. counties. The work illustrates how LSAI applies to the RUBMRIO  
11 model of land use and transport, by simulating both the individual effect of every input and the  
12 interaction effects of inputs on outputs. More importantly, the work analyzes changes in  
13 production (via domestic trade flows and export demands) and consumption across the  
14 continental U.S.'s counties, tracking trade patterns among 12 socio-economic sectors and two  
15 freight modes (truck and rail).

16 LSAI offers a valuable set of relationships to enable policymakers, planners and carriers to  
17 quickly predict trade flows by producers' location choices and production levels. LSAI offers the  
18 individual effects of inputs and their interaction effects on many types of models' outputs. LSAI  
19 enables analysts to clearly identify keydrivers for model predictions, and the magnitude and  
20 direction of changes in outputs, due to input changes and their interaction effects, which amplify  
21 or dampen individual effects of inputs.

22 Under scenarios developed here, LSAI techniques show how export demands (ED) are more  
23 important for accurately anticipating and quantifying U.S. trade flows than are transport costs  
24 and travel times (TC and TT). As expected, TC and TT effects typically carry the same sign or  
25 direction, with different magnitudes of first-order effect on domestic trade flows, production and  
26 consumption in most states (e.g. KY by the first-order effects under Scenario 1). However,  
27 changes in TC and TT have opposing effects on outputs in some states. Tracking various inputs'  
28 effects helps policymakers, businesses, and carriers pursue more optimal policies, operations,  
29 and investments.

30 This type of LSAI investigation can be extended by varying export demands (ED) in each  
31 market/industry sector, and varying transport cost and travel time (TC and TT) values by route,  
32 link, and mode. The number of required simulations for LSAI application rise exponentially with  
33 the number of variable inputs and parameters, if one wishes to compute all interaction effects.  
34 Thus, the standard approach of many Monte Carlo simulations remains an important option. The  
35 use of congested network assignment for travel time and cost feedbacks (which vary by route,  
36 and by time of day and day of week), and application of Bayesian Melding (which allows for  
37 dynamic forecasting, over time, but requires knowledge of intermediate-period outputs, for  
38 comparison) may provide useful extensions.

## 39 **ACKNOWLEDGEMENTS**

40 The authors thank several anonymous reviewers for their content suggestions, the China  
41 Scholarship Council's State Scholarship Fund for financial support of the first author's research  
42 at the University of Texas at Austin, and Amy Banker and Scott Schauer-West for their editing  
43 support.

1 **REFERENCES**

- 2 Abraham J.E. and J.D.Hunt. Firm Location in the MEPLAN Model of Sacramento.  
3 *Transportation Research Record 1685*, (1993):187-197.
- 4 Anas, A. Discrete Choice Theory and the General Equilibrium of Employment, Housing, and  
5 Travel Networks in a Lowry-Type Model of the Urban Economy. *Environment and Planning*  
6 16(A), (1984): 1489-1502.
- 7 Ben-Akiva, M. and S.R. Lerman. *Discrete Choice Analysis: Theory and Application to Travel*  
8 *Demand*. MIT Press, Cambridge, Massachusetts, 1985.
- 9 Borgonovo, E., M. Percoco, R. Polizzi, K.Kockelman, and L.Cavalli.Sensitivity Analysis of a  
10 Gravity-Based Land Use Model: The Importance of Scenarios (2014).  
11 <https://marcopercoco.files.wordpress.com/2014/11/glum-final.pdf>
- 12 Cacuci, D. G. *Sensitivity and Uncertainty Analysis, vol. 1, Theory*, Chapman and Hall, Boca  
13 Raton, Fla, 2003.
- 14 Castaings, W., D. Dartus, F. X. LeDimet, et al. Sensitivity Analysis and Parameter Estimation  
15 for the Distributed Modeling of Infiltration Excess Overland Flow. *Hydrology and Earth System*  
16 *Sciences* 4, (2007): 363-405.
- 17 Clay, M.J. and R.Johnston. Multivariate Uncertainty Analysis of an Integrated Land Use and  
18 Transportation Model: MEPLAN. *Transportation Research Part D: Transport and Environment*  
19 11(3), (2006): 191–203.
- 20 De la Barra, T. *Integrated Land Use and Transport Modeling: Decision Chains and Hierarchies*.  
21 Cambridge University Press, New York, 2005.
- 22 De la Barra, T.B. PÁrez, N.Vera, “TRANUS-J: Putting Large Models into Small Computers.”  
23 *Environment and Planning B: Planning and Design* 11, (1984): 87–101.
- 24 Du, X.C. and K.M. Kockelman. Tracking Transportation and Industrial Production Across a  
25 Nation: Applications of RUBMRIO Model for U.S. Trade Patterns. *Transportation Research*  
26 *Record* No. 2269, (2012): 99-109.
- 27 FAF.FAF<sup>3</sup> Network Database and Flow Assignment,  
28 2007.[http://ops.fhwa.dot.gov/freight/freight\\_analysis/faf/](http://ops.fhwa.dot.gov/freight/freight_analysis/faf/)  
29
- 30 Huang, T. andK.M. Kockelman. The Introduction of Dynamic Features in a Random-Utility-  
31 Based Multiregional Input-Output Model of Trade, Production, and Location Choice. *Journal of*  
32 *the Transportation Research Forum* 47(1), (2008): 23-42.
- 33 Hunt, J.D. “A Description of the MEPLAN Framework for Land Use and Transport Interaction  
34 Modeling.” *The 73rd Annual Transportation Research Board Meetings* January 9-13,  
35 Washington, D.C, 1993.
- 36 Hunt, J.D. and J.E.Abraham. Design and Application of the PECAS Land Use Modeling System.  
37 *Presented at the Computers in Urban Planning and Urban Management Conference*, Sendai,  
38 Japan, (2003).
- 39 Hunt, J.D. and M.H.Echenique. Experiences in the Application of the MEPLAN Framework for  
40 Land Use and Transportation Interaction Modeling. *Proc. 4<sup>th</sup> National Conference on the*

- 1 *Application of Transportation Planning Methods*, Daytona Beach, FL, (1993): 723-754.
- 2 Hunt J.D. and D.C. Simmonds. Theory and Application of an Integrated Land-use and Transport  
3 Modelling Framework. *Environment and Planning B* 20, (1993): 221-244.
- 4 Isard, W. *Methods of Regional Analysis: An Introduction to Regional Science*. M.I.T. Press  
5 Cambridge, MA: and Wiley, New York, 1960.
- 6 Kockelman, K. Gravity-based Land Use Model (G-LUM) Website. University of Texas at  
7 Austin, 2008. Available at [http://www.ce.utexas.edu/prof/kockelman/G-](http://www.ce.utexas.edu/prof/kockelman/G-LUM_Website/homepage.htm)  
8 [LUM\\_Website/homepage.htm](http://www.ce.utexas.edu/prof/kockelman/G-LUM_Website/homepage.htm).
- 9 Kockelman, K.M., L.Jin, Y.Zhao, and N.Ruiz-Juri. Tracking Land Use, Transport, and Industrial  
10 Production Using Random-Utility Based Multiregional Input-Output Models: Applications for  
11 Texas Trade. *Journal of Transport Geography* 13(3), (2005): 275-286.
- 12 Lefevre, B. Long-Term Energy Consumptions of Urban Transportation: A Prospective  
13 Simulation of Transport-Land Uses Policies in Bangalore. *Energy Policy* 37(3), (2009): 940-953.
- 14 Leontief, W.W. and A. Strout. "Multiregional Input-Output Analysis." eds. T., De la Barna, W.I.  
15 Abraham, Z. *Kenessey Structural Interdependence and Economic Development*, New York:  
16 Macmillan, (1963):119-150.
- 17 MIG, Inc. *Calculate the IMPLAN RPCs*,  
18 2011.[http://implan.com/V4/index.php?option=com\\_docman&task=doc\\_download&gid=125&Ite](http://implan.com/V4/index.php?option=com_docman&task=doc_download&gid=125&Itemid=7)  
19 [mid=7](http://implan.com/V4/index.php?option=com_docman&task=doc_download&gid=125&Itemid=7)
- 20 Minnesota IMPLAN Group, Inc. *IMPLAN Professional: Social Accounting and Impact Analysis*  
21 *Software*, 1997.
- 22 Modelistica. *TRANUS: Integrated Land Use and Transport Modeling System*. Modelistica  
23 Company, 2007.
- 24 Rodier, C.J., J.E. Abraham, R.A.Johnston, and D.Hunt. A Comparison of Highway and Travel  
25 Demand Management Alternatives Using an Integrated Land Use and Transportation Model in  
26 the Sacramento Region *Presented at the Transportation Research Board Annual Meeting*,  
27 Washington, DC, 2002.
- 28 Ruiz-Juri, N. and K.M. Kockelman. Extending the Random-Utility-Based Multiregional Input-  
29 Output Model: Incorporating Land-Use Constraints, Domestic Demand and Network Congestion  
30 in a Model of Texas. *Proceedings of the 83rd Annual Meeting of Transportation Research Board*,  
31 Washington, D.C, 2004.
- 32 Saltelli, A.M.Ratto, T.Andres, et al. *Global Sensitivity Analysis: The Primer*. John Wiley &  
33 Sons, 2008.
- 34 Saltelli, A.S.Tarantola. On the Relative Importance of Input Factors in Mathematical Models:  
35 Safety Assessment for Nuclear Waste Disposal. *Journal of the American Statistical Association*  
36 97(459), (2002): 702-709.
- 37 Saltelli, A. S. Tarantola, F. Campolongo, M. Ratto. *Sensitivity Analysis in Practice: A Guide to*  
38 *Assessing Scientific Models*. John Wiley & Sons, New York, USA, 2004.
- 39 Wilson, A. G. Interregional Commodity Flows: Entropy Maximizing Procedures. *Geographical*  
40 *Analysis* 2, (1970): 255-282.

- 1 Zhao, Y. and K.M Kockelman. The Random-Utility-Based Multiregional Input-Output Model:
- 2 Solution Existence and Uniqueness. *Transportation Research Part B: Methodological* 38(9),
- 3 (2004): 789-807.