

**Empirical Investigation of the Continuous Logit for
Departure Time Choice using Bayesian Methods**

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ABSTRACT

Numerous models of travel timing have been calibrated in the literature. Some treat time as a discrete variable using familiar discrete choice methods, while others have treated time in a continuous fashion. Both approaches offer distinct advantages. Here, a continuous logit model of work tour departure time choice is estimated, which offers the advantage of continuous-time response using a random utility maximization structure, thus capitalizing on the key advantages of both main approaches to travel timing modeling. Bayesian techniques are used to estimate model parameters, and estimation results suggest a variety of predictive densities for departure times across different individuals. In addition, ordinary least squares (OLS) regression models are used to estimate travel times and their variance across times of day for the auto and transit modes. These network variables are used to inform estimation of the continuous logit model of departure time. The results are meaningful for multiple applications, and the continuous logit can readily be extended to a two-dimensional choice construct, such that the departure and return times can be modeled simultaneously. In addition, Bayesian estimation techniques allow for the utility function to take any number of forms, which may offer greater predictive ability.

1. INTRODUCTION

While activity scheduling is a fundamental aspect of any trip or tour, it is often greatly simplified in travel demand model specifications, particularly in comparison to other choice dimensions, like mode and destination. However, as transportation policies have become more focused on congestion and demand management (see, e.g., *1*), trip scheduling has become increasingly important in policy decisions. With improvements in dynamic traffic assignment (DTA) techniques and applications (see, e.g., *2, 3, and 4*), better models of travel timing are needed, compatible with the relatively fine time resolution used in DTA methods. In the face of such interest and need, the major weakness of most large-scale travel demand models remains time-of-day (TOD) modeling (*5, 6*).

Some of the earliest models of TOD choice used disaggregate methods over a limited temporal spectrum (e.g., 1-hour or the AM peak period). For instance, using the multinomial logit (MNL), Abkowitz (*7*) and Small (*8*) modeled TOD choice within a 1-hour AM peak period, while Hendrickson and Plank (*9*) examined seven 10-minute alternatives within the AM peak. To allow for likely correlations across the ordered alternatives, Chin (*10*) turned to the nested logit and Small (*11*) developed the ordered generalized extreme value (OGEV) model, both for limited temporal choice contexts within the AM peak period. While each of these models considers alternatives that are small (5 to 15 minute intervals), none considers the entire day's choice context; each is limited to the relatively narrow AM peak period.

Others have modeled TOD choice for the entire day. For instance, Bhat (*12*) used the OGEV model, Steed and Bhat (*13*) investigated both the MNL and OGEV models, and Okola (*14*) examined the MNL, each considering non-work trips and using broad alternatives (at least 3 hours each). Some of the most recent applications of discrete choice activity scheduling models have focused on the tour as the unit of measurement; and, of course, a tour has (at least) two timing components (departure or arrival time and activity duration or return time). Vovsha and Bradley (*15*), Abou Zeid et al. (*16*), and Popuri et al. (*17*) all modeled the two-dimensional choice of tour timing in a joint MNL context using 30-min or 1-hour time intervals. In some contrast, Ettema and Timmermans (*18*), Ettema et al. (*19*), and Ashiru et al. (*20*) developed utility equations to reflect the number of sources from which utility is derived, including home-stay duration, stop duration, and timing of activities.

Other research has treated departure time as a continuous response variable, with the usual approach involving a hazard function. Wang (*21*) used a parametric proportional hazard rate specification to model activity start times, while Bhat and Steed (*22*) and Komma and Srinivasan (*23*) developed non-parametric hazard models (for shopping trips and commute trips, respectively), both using a gamma mixing distribution to account for unobserved heterogeneity in departure time choices. The non-parametric hazard approach offers an advantage for departure time data, since reported data often suffer from 5-, 10-, 15-, and even 30-minute rounding errors: non-parametric hazards can allow consistency for grouped data of this nature (*24*). In contrast to the classical estimation approach of prior applications, Gadda et al. (*25*) used Bayesian estimation techniques and an accelerated hazard approach to model departure time.

While there are many advantages to the discrete and continuous response models described above, all suffer from some limitations. Bhat and Steed (*22*) indicate the following weaknesses of discrete choice methods: (1) Discrete choice models require that interval boundaries be set, which is

usually done in an arbitrary manner. (If interval boundaries are changed, different model results emerge.) (2) Two points very close in time may be classified in different intervals in discrete models, when, in reality, these points may be perceived as very similar options. Allowing for correlation across alternatives in discrete models can alleviate this issue to some extent, but continuous treatment of departure time seems preferable. (3) Discrete treatment of time also results in a loss of temporal resolution, and, as DTA methods become more common, the value of such models is diminished.

Of course, there are also disadvantages of existing continuous response models. The main limitation of such models is that they are not based in random utility theory. Random utility theory is a mainstay of choice modeling that not only provides a solid foundation for estimation of traveler welfare, but also can provide meaningful relationships between departure times and other traveler choices (e.g., destination and mode).

This paper offers a seemingly new method for modeling departure time, exploiting the advantages of continuous models while retaining the property of random utility maximization. The model formulation is actually not new. First proposed by McFadden (26) and extended by Ben-Akiva and Watanatada (27) and Ben-Akiva et al. (28), the continuous logit model provides a random utility framework in a continuous setting. However, the model has largely gone unused since that time, and has never been applied in departure time choice or duration analysis settings. (Ben-Akiva and Watanatada's [27] and Ben-Akiva et al.'s [28] applications were all for location choice). The model is estimated on work tour data from the 2000 San Francisco Bay Area Travel Survey (BATS) using Bayesian estimation techniques. Such methods avoid reliance on sample size asymptotics (needed with typical maximum likelihood estimation) and offer richer distributional inference of estimated parameters and associated outputs of interest (e.g., the multivariate distributions of all parameters are estimated rather than point estimates of means and covariance). Of course, assessing convergence with Bayesian methods can be problematic, since parameter draws converge to a distribution rather than a point. While a number of tools for convergence assessment exist (see, Gamerman and Lopes [29] for a thorough review), this remains an area of weakness in Bayesian statistics.

In order to include transportation (LOS) variables in the departure time model, continuously varying variables had to be thoughtfully imputed. For the auto mode, ordinary least squares (OLS) regressions (a la 16, 17, 23, and 30) were estimated to obtain time-varying travel times and time-varying reliability estimates. For the transit mode, similar regressions were estimated to obtain time-varying (by time-of-day) reliability estimates (with mean travel times, by time-of-day, coming from transportation network data).

In the next section, methods for and results of estimating time-dependent variables are discussed. Section 3 presents the continuous logit model's formulation, Section 4 presents results of the Bayesian model estimation procedure, and Section 5 offers some concluding remarks.

2. TRANSPORTATION LEVEL-OF-SERVICE MODELS

While the BATS data contain transportation LOS data for five TOD periods (early morning [before 6 am], AM peak [6 to 8:59 am], midday [9 am to 3:29 pm], PM peak [3:30 to 6:29 pm], and evening [6:30 pm to midnight]), it is useful to have this data vary continuously over time since the continuous logit model for departure time choices allows it. Four models were calibrated for these

purposes. The first is a model of travel speeds by the automobile mode, the second is a model of travel time variability by the automobile mode, and the third and fourth are models of travel time variability by transit modes. Since observations in the data represent travel tours, and LOS data were available based only on tour origin and primary destination, any tour leg involving stops was removed for this analysis. Moreover, only those tours with relevant LOS data were considered. This resulted in sample sizes of 86,358, 3,297, and 4,981 for automobile, drive-to-transit, and walk-to-transit tour modes, respectively.

2.1 Auto Mode Speed Regressions

To obtain time-varying average travel times as a continuous function over time, Popuri et al.'s (17) methods were followed very closely. In Popuri et al. (17), however, the response variable was the natural logarithm of the ratio of reported speed to free-flow speed (for all reported trip end pairs); and here free-flow speeds were not available. Instead, speeds in the early morning (EM) TOD period were used, as follows:

$$\ln \left(\frac{[\text{Reported Speed}]_{iq}}{[\text{EM Speed}]_q} \right) = X_{iq}' \varphi_1 + (\eta_1 + \sum_{j=1}^4 [\sum_{k=1}^3 \omega_{1jk} (h_j(t))^k]) * (\text{Delay})_q + \varepsilon_{iq} \quad (1)$$

Here, X_{iq} is a vector of covariates for trip i and origin-destination pair q , which includes a constant, the natural logarithm of trip distance, area type indicator variables for the origin and destination zones (with rural area type as the base), and day-of-week indicator variables (with Monday as the base). One can expect trip distance to affect speeds in a positive way, since longer distance trips would likely enjoy use of higher speed highways. Area type indicator variables (with rural as the base type) were expected to have a negative effect on speed, since, all else being equal, one can expect trips between rural areas to enjoy little congestion and higher speeds. The variable $(\text{Delay})_q$, is very important since it defines the normal level of congestion. For instance, if a particular origin-destination pair sees little or no congestion across the day, the $(\text{Delay})_q$ variable takes a value close to zero, and the predicted speeds will exhibit very little variation across different times-of-day. Similar to Popuri et al. (17), the delay variable was defined as follows:

$$(\text{Delay})_q = 1 - \frac{(\text{peak speed})_q}{(\text{EM speed})_q} \quad (2)$$

Here, the peak speed is taken as the lowest of five TOD speeds (typically from the PM peak period). The main reason for this variable (rather than simply a constant) is to account for differences in speed profiles across different origin-destination (OD) pairs. For instance, network links connecting some OD pair may enjoy little or no delays across the entire day, while links connecting another OD pair may exhibit extreme congestion during peak periods. The $(\text{Delay})_q$ variable accounts for these differences in a systematic way. As shown in equation 1, the coefficient on this delay variable is made up of a number of terms (13 in total), and describes how speeds vary over time t . The functions $h_1(t)$, $h_2(t)$, $h_3(t)$, and $h_4(t)$ are cyclic functions, as in Popuri et al. (17), as follows:

$$h_1(t) = \exp \left[\sin \left(\frac{2\pi t}{24} \right) \right], h_2(t) = \exp \left[\cos \left(\frac{2\pi t}{24} \right) \right],$$

$$h_3(t) = \exp \left[\sin \left(\frac{4\pi t}{24} \right) \right], h_4(t) = \exp \left[\cos \left(\frac{4\pi t}{24} \right) \right] \quad (3)$$

In order to alleviate issues with misreporting of travel times in the data, records were removed if the ratio of reported to free-flow speed exceeded 2.0 or if the ratio of reported to peak speed was less than 0.25 (totaling 17% of records). Of course, there is no fail-safe way of determining the accuracy of reported travel times. And, even if travel times seem very reasonable, they may not be accurate. These cutoffs were chosen to eliminate outliers while allowing for a reasonable range of potentially accurate reported travel times. The model estimates from equation (1) are shown in Table 1. While not all of the parameters are statistically significant, all have expected signs. Of the parameters not appearing in the delay coefficient, the most practically significant are indicators for regional core and central business district (CBD) zones (both origin and destination) and the natural logarithm of distance, not surprisingly. In addition, the shape of the delay coefficient, as it varies over departure time choice, appears reasonable, as shown in Figure 1a. The delay coefficient is computed from Table 1's parameter estimates. Also as shown in Figure 1a, speeds are predicted to be lowest (and travel times highest) during typical AM peak times (approximately 6 to 8 am) and PM peak times (approximately 3 to 6 pm). While the slowest speeds are expected during these peak times-of-day, it is worth noting that the location of speed profile peaks and valleys are assumed to be the same across all OD pairs (i.e., the slowest speeds will occur around 5 pm for all OD pairs).

Table 1 goes here

Figure 1 goes here

2.2 Auto Mode Travel Time Variability Regressions

Since the data contain reported travel times, and predicted travel times can be obtained from the speed regression (Equation 1), a natural measure of travel time variability is the difference between reported and predicted travel times. It should be noted that since some reported travel times suffer from rounding error and misreporting, some of the differences between reported and predicted travel times may not be attributable to natural fluctuations in travel times.

The covariates for auto travel time variability are very similar to those used in the average auto speed models, detailed in Section 3.1. The only difference in the set of covariates is that the number of terms in the delay coefficient was reduced from 13 to 5 (i.e., terms containing $h_j(t)$ to the powers of 2 and 3 were removed), because variations in travel time variance (over time-of-day) were found to be more consistent with expectations under this formulation. The model is formulated as follows:

$$\ln \left([TT_{resp} - TT_{pred}]_{iq}^2 \right) = X_{iq}' \varphi_2 + (\eta_2 + \sum_{j=1}^4 [\omega_{2j} h_j(t)]) * (\text{Delay})_q + \varepsilon_{iq} \quad (4)$$

Parameter estimates of the model are shown in Table 1. The most practically significant parameter is that associated with the natural logarithm of travel distance, which captures the effect of longer trips having higher travel time variance. As with the speed model, not all parameter estimates for the travel time variance regression are statistically significant. In addition, several signs are not so intuitive. For instance, a trip originating in a regional core zone is predicted to exhibit greater travel

time variability than one originating in a rural zone (all else equal), while trips originating in urban and suburban zones are predicted to enjoy lower travel time variance than those originating in rural zones (all else equal). Nonetheless, the results do not seem unreasonable, and more importantly, the fluctuations associated with the delay coefficient for this model (over times of day) appear as one would expect, as shown in Figure 1b: travel time variances highest during the normal AM peak period and the normal PM peak.

One key attribute of the models presented in Sections 3.1 and 3.2 is that travel time and variance profiles of different trips will share the same underlying shape (as predicted by the delay coefficient). While the heights of the profiles' peaks and valleys will differ (depending on the value of the $(\text{Delay})_q$ variable), the location and relative shape of the profiles will be the same. For instance, under the variance regression, all trips will share the feature that the maximum variance occurs around 3 to 4 pm. Of course, the models' search for a set of single parameters neglects the fact that different trips (with different free-flow speeds, travel distances, OD attributes, etc.) may have wholly different travel time and variance profiles – due to specific link characteristics and demand profiles over times of day. Nonetheless, without actual network data, it would be difficult to quantify such differences in any meaningful way. Such link-specific delay and travel time information, by time of day, may emerge through better instrumentation of highways, detailed dynamic traffic assignment estimates, controlled route-choice experiments, and the like.

2.3 Transit Mode Travel Time Variability Regressions

The final two regression equations used here are for transit travel time variability. Instead of estimating transit LOS characteristics as a continuous function of departure time, transit LOS attributes are assumed constant over five TOD periods. Since transit travel time estimates were provided for each of these five periods, those were used to represent average transit travel times. However, transit travel time variability was modeled similar to automobile variability in that the response variable is the squared difference between reported (or actual) and skimmed travel times.

Two models were estimated here, one for the drive-to-transit tour mode and one for the walk-to-transit tour mode. Empirical evidence suggested that the drive-to-transit mode would be best modeled as in equation (5), while the walk-to-transit mode would be best modeled as shown in equation (6). Essentially, the response variable of travel time variance is normalized by the average *motorized* travel time. The main reason for the difference between the two modes is that the majority of travel time variability for the walk-to-transit mode should be in vehicle, since access and egress times should be fairly stable by the walk mode.

$$\ln \left(\frac{(\text{Rep TT} - \text{Skim Total TT})^2}{(\text{Skim Total TT})^2} \right) \quad (5)$$

$$\ln \left(\frac{(\text{Rep TT} - \text{Skim Total TT})^2}{(\text{Skim In-Vehicle TT})^2} \right) \quad (6)$$

The two models were estimated using OLS regression techniques using specific covariates described earlier. To reduce the effect of travel time misreporting, records were removed if the ratio of reported to average travel times was less than 0.5 or greater than 2.0. Model estimation results are shown in Table 2, where it is clear that the smallest predicted travel time variances tend

to occur during the early morning for the drive-to-transit mode, and during the early morning and evening periods for the walk-to-transit mode. (Note that no evening period indicator was estimated for the *walk-to-transit* mode, since there were very few early morning observations for this mode. In other words, the two periods were merged. The *drive-to-transit* mode considers these periods separately.) For both models, travel time variances are predicted to be highest during the PM peak period, which seems very reasonable.

Table 2 goes here

3. CONTINUOUS LOGIT MODEL

3.1 Continuous Logit Formulation

The continuous logit model represents a generalization of the MNL model for a continuous response variable (26, 27, and 28). The model can be derived directly from the random utility assumption. For instance, assume that the utility of departure time choice, t , for an individual, i , is given by the following:

$$U_i(t) = V_i(t) + \varepsilon_i(t) \quad (7)$$

If the random utility component, $\varepsilon_i(t)$, is independent and identically distributed (iid) extreme value type I, then its cumulative distribution function at departure time choice t is given by the following (28):

$$F_{\varepsilon_i(t)}(\varepsilon) = \exp[-\exp(-\varepsilon)] \quad (8)$$

Ben-Akiva et al. (28) defined an opportunity density function, $\gamma_i(t)$, which describes the density of choice alternatives at a particular point in space. The authors use this function in the context of location choice, since some locations have more opportunities than others (e.g., a body of water will have no household locating opportunities while a densely populated urban area may have many). In the context of departure time choice, one may view this opportunity density function as a zero-one indicator. If an individual has already scheduled an activity during some particular time interval, the opportunity density will be zero over that interval, and the opportunity density will be one elsewhere. Ben-Akiva et al. (28) then derived the probability density for any choice alternative by the following:

$$f_i(t) = \frac{\gamma_i(t) \exp[V_i(t)]}{\int_a^b \gamma_i(z) \exp[V_i(z)] dz} \quad (9)$$

Here, a and b define the bounds of the choice space. If it is assumed that the opportunity density is constant over all alternatives (e.g., all TODs are feasible), then the probability density reduces to the following:

$$f_i(t) = \frac{\exp[V_i(t)]}{\int_a^b \exp[V_i(z)] dz} \quad (10)$$

Thus, the continuous logit model represents a generalization of the MNL model. Such a model of tour timing was described by Ettema and Timmermans (18), except that the denominator of the

density function was discretized, and the model was estimated as a MNL. Moreover, since time-varying travel time and cost were not available to Ettema and Timmermans, such time-dependent covariates were not included in their tour scheduling model.

Since utility is arguably (smoothly) continuous in departure time choice, it deserves special attention here. Similar to the typical MNL model, covariates that do not vary over time alternatives (e.g., an individual's gender or age) cannot be introduced in the normal way. There is only a single utility function for each individual, and the systematic utility function is specified using a collection of cyclical functions of departure time choice interacted with time-invariant covariates (similar to the specifications of Abou Zeid et al. [16] and Popuri et al. [17]). Time-varying covariates (i.e., average travel time, travel time variance, and travel cost) are introduced in the normal way. Systematic utility is formally specified as follows:

$$V_i(t) = X_i \beta s(t) + [-\exp(\alpha')] g_i(t) \quad (11)$$

$$s(t) = \left[\sin\left(\frac{2\pi t}{24}\right), \sin\left(\frac{4\pi t}{24}\right), \dots, \sin\left(\frac{2Q\pi t}{24}\right), \cos\left(\frac{2\pi t}{24}\right), \cos\left(\frac{4\pi t}{24}\right), \dots, \cos\left(\frac{2Q\pi t}{24}\right) \right]' \quad (12)$$

where $g_i(t)$ is the vector of time-varying covariates, α is the vector of parameters relating to those time-varying covariates, X_i is the vector of time-constant covariates for individual i , β is a vector of parameters to be estimated, and $s(t)$ is a vector of cyclical functions. Note that the parameter vector, α , is transformed in Eq. 11's utility function. This ensures that the parameter related to each time-varying covariate is negative, since one expects travel time, unreliability, and cost to reduce utility. There are a total of Q sine functions and Q cosine functions interacted with X_i , for a total of $2Q$ interactions. (Some covariates may be interacted with fewer than $2Q$ cyclical functions by restricting the applicable elements of β to be zero.) This formulation allows utility to take on a variety of shapes, including multimodal ones. In addition (and as indicated by Abou Zeid et al. [16] and Popuri et al. [17]), 24 hours is a multiple of the period of each cyclical function, which offers day-to-day consistency in the utility function.

3.2 Continuous Logit Data and Estimation

The departure time analysis undertaken here uses only home-based tours made on weekdays and for which the primary activity purpose was work¹. Further, if multiple home-based work tours exist for any individual in the original data set, only the first tour is used here². Thus, each record in the sample represents the first weekday work tour made over the 2-day survey period for all individuals. The sample was further restricted to observations where relevant transportation LOS data was available: tours with origins or destinations outside the region did not have LOS data and so could not be used. After restricting attention to such tours, 17,820 tour records remained.

¹ Work activities were selected for evaluation, thanks to their relatively predictable timing and to extensive existing literature on this type of trip-making. Of course, the methodology applies for any activity purpose and any other continuous response.

² It is assumed here that the first work tour occurring in the survey period is scheduled first. To avoid correlations across records of tours made by a single individual, only one work tour per individual is used.

In order to estimate this model's parameters, Bayesian methods were employed. For ease in notation, let X denote the set of independent variables in the model, let Y denote the set of response variables, and let θ denote the set of all parameters. In any Bayesian analysis, one can write the posterior distribution of the model parameters as proportional to the product of the likelihood and prior distribution (31):

$$p(\theta|Y,X) \propto p(Y|\theta,X)p(\theta) \quad (13)$$

Here, $p(Y|\theta,X)$ represents the likelihood function and $p(\theta)$ is the prior distribution of the model parameters. The prior distribution represents the analyst's prior beliefs about parameter values. In this work, the prior for each parameter (i.e., β and α) was chosen to be independent and normally distributed with vague or non-informative (i.e., large variance) prior parameters. Thus, the prior should play little role in the posterior distribution.

The posterior distribution of the parameters is obtained by constructing a Markov Chain with stationary distribution equal to the posterior distribution, and iteratively sampling the parameters using that Markov Chain. A Metropolis-Hastings (MH) algorithm (31) was employed for these purposes³. The MH algorithm requires a proposal density, $q(\theta|\theta_i)$, where θ_i is the current value of the parameters. The MH algorithm works as follows.

1. Let the current set of parameters be θ_i , and draw a new set of parameters θ_j , from the proposal density $q(\theta|\theta_i)$.
2. Compute $\alpha(\theta_i, \theta_j) = \min\{1, \frac{p(Y|\theta_j,X)p(\theta_j)q(\theta_i|\theta_j)}{p(Y|\theta_i,X)p(\theta_i)q(\theta_j|\theta_i)}\}$.
3. With probability α , let the new value of θ be θ_j . With probability $1 - \alpha$, let the new value of θ be θ_i .
4. Repeat.

Here, $q(\theta|\theta_i)$ is chosen to be a multivariate normal with mean θ_i . The covariance matrix is initially set to zero on the off-diagonal elements, and something very small on the diagonal elements. After 500 draws were obtained using the MH algorithm (and for each draw thereafter), the covariance matrix of the proposal density was updated (i.e., set to equal the estimated covariance matrix from the previous draws, not including the current draw). After 5,000 draws were obtained, and thereafter, the covariance matrix was estimated from the previous 5,000 draws (not including the current draw). Since the adaptive MH algorithm does not include information from the current value of the parameters (other than the mean of the proposal density), the algorithm will converge to the proper posterior distribution (32). Moreover, since the proposal density was defined to be symmetric about θ_i , $q(\theta_i|\theta_j) = q(\theta_j|\theta_i)$ and the acceptance probability reduces to the following:

$$\alpha(\theta_i, \theta_j) = \min\{1, \frac{p(Y|\theta_j,X)p(\theta_j)}{p(Y|\theta_i,X)p(\theta_i)}\} \quad (14)$$

³ The familiar Gibbs sampling is not needed here since all parameters are drawn simultaneously within the MH algorithm.

The algorithm was run to achieve some 250,000 draws from the posterior distribution. Posterior convergence was reached after about 200,000 draws. The remaining 50,000 draws were obtained for inference. Furthermore, only every 50th draw from those last 50,000 draws was used. This reduces the correlation in adjacent draws (since the algorithm will repeat identical draws). Posterior convergence was checked using Geweke's (33) diagnostic, and results were found to be acceptable for all parameters. Results of model estimation are presented in the next section.

4. EMPIRICAL RESULTS

The sample used in model estimation represents a 75% random sample of all usable departure time records. Each mode of the utility function utilizes eight individual-specific variables plus a constant. These variables include an indicator for males, age of the individual, an indicator for part-time workers, an indicator for high-income households (over \$75,000 per year), household size, the number of tours undertaken by the individual over the entire day (excluding the modeled tour), free-flow distance to the destination, and a variable indicating whether the destination zone is coded as part of San Francisco's central business district (CBD). The travel cost variable only varies (slightly) by TOD period (not continuously over time) and is taken to be \$0.15/mile (the assumed operating cost of automobiles) times the travel distance plus any tolls divided by the number of vehicle occupants for the auto modes, the transit fare for transit modes, and zero for the bike and walk modes.

Table 3 presents the estimation results. Mean parameter estimates relating to average travel time, travel time variance, and travel cost all have negative signs, by construction. The mean value of travel time implied by the parameter estimates is just \$2.20 per hour, with 95% interval bounds of \$0.30 and \$6.23. The mean estimate of the value of travel time variance is \$0.031 per squared minute (or \$10.51 per hour in terms of travel time's standard deviation). 95% interval bounds on the value of reliability are \$0.018 and \$0.054 per squared minute (or \$7.95 and \$13.93 per hour). Thus, it appears the marginal effect of travel time's variability exceeds that of average travel time here. Of course, the mean estimate of the value of travel time is quite low. It could be that unreliability is much more important for departure time decisions than average travel time, since workers are generally rather constrained in work start times. Alternatively, since imputed average travel times do not vary a great deal, maybe the model simply cannot distinguish the effect of average travel time in a meaningful way.

For parameters on interaction terms with time-invariant covariates, parameter estimates are difficult to interpret on their own. To understand the affects of time-invariant covariates, predictions for an individual having sample-average covariates can be examined, with covariate values varied one at a time. Figure 2 shows density profiles to illustrate the effect each covariate has on predictive densities. Note that the mean estimates for each parameter were used here, and time-varying variable effects were omitted.

Table 3 goes here
Figure 2 goes here

As shown in Figure 2, the predictive densities (of systematic utility for work-tour departure time choice) are greatest around the AM peak period, as expected, since most individuals depart for work in the morning. Practically speaking, the effects of gender, age, household income, household size, and CBD appear rather small (Figures 2a, 2b, 2d, 2e, and 2h). However, predictions appear in line

with expectations. For instance, males, older individuals, and those from households with larger number of individuals are predicted to depart earlier, all else equal (Komma and Srinivasan [23] found similar results for age and household size). The household size effect can, in some sense, be viewed as a proxy for the number of children, and those with children may have several other obligations en route to work, such as dropping off children at school or daycare. (Note that a number-of-children variable was not available in the San Francisco data set.) Individuals from high-income households may have more flexibility in work start times, which may explain why such individuals tend to depart later. (Similar results are noted by Komma and Srinivasan [23] and Popuri et al. [17].) Not surprisingly, individuals with longer travel distances depart earlier, since it takes longer for such individuals to arrive at work. Part-time worker status and additional tours undertaken during the day both have the effect of making later departure times more desirable. These results seem reasonable since part-time workers often do not work full 8-hour days and those with additional engagements are less likely to be working typical hours.

5. POLICY SIMULATIONS

This section provides three simple policy simulations, and details the predicted consumer surplus (CS) change and predicted departure time distribution changes under each. As noted by Ben-Akiva and Watanatada (27), CS under the continuous logit is computed as follows:

$$CS_i = \ln \left(\int_a^b \exp[V_i(t)] dt \right) \quad (15)$$

By dividing by the cost coefficient estimate, CS can be evaluated on a monetary scale. Three tolling policies are investigated here. In the first, it is assumed that \$0.15/mile tolls are assessed on all roads during the peak periods, resulting in peak period travel time delay reductions of 25%. In the second, the same tolls apply but peak period travel time delay reductions are just 5%. In the final simulation, \$0.30/mile tolls during peak periods are applied and assumed to reduce peak period travel time delay by 25%. The simulations are performed using the 25% out-of-sample data for those traveling by auto mode only, with 100 random draws from the collection of 1,000 posterior draws (using the same 100 random in each simulation). Transit users and those walking and biking are not included since they would not be expected to pay tolls. For those traveling by carpool, the toll is divided by the vehicle occupancy to reflect that those individuals pay less per vehicle occupant.

Figure 3a shows the CS results under the three policy scenarios. Average CS reductions per person are about \$1.05, \$1.11, and \$2.01 for scenarios 1, 2, and 3, respectively. Scenarios 1 and 2 (with the only difference being the assumed reduction in peak travel time delay) have very similar CS change distributions, suggesting that the effect of travel time and its variance on departure time choices are not practically significant. Since pre-toll out-of-pocket costs are assumed to be \$0.15/mile⁴, imposing \$0.15/mile tolls during the peak periods (where the AM peak period is highly preferred) essentially results in a doubling of total costs for most individuals. Thus, it is not surprising to see the large drop in CS, relative to the status quo scenario. Also not surprisingly, scenario 3's drop in CS is about twice that of scenarios 1 and 2, on average.

⁴ Some roadways in the San Francisco Bay Area are already tolled, so the \$0.15/mile toll represents less than a doubling of costs for travelers on those links.

Figure 3 goes here

Figure 3b shows the estimated densities of departure time choices under the three scenarios, along with the status quo (or actual) scenario, for the 364,300 data points (3643 individuals' times 100 draws for each). Given the similar CS results for scenarios 1 and 2, it is not surprising that their predictive departure time densities are quite similar. According to the predictive densities, tolls during peak periods will push travelers to other departure time alternatives. For instance, under the status quo scenario about 71% of travelers choose the AM peak period (6 am to 9 am), while 10% choose departure times before the AM peak and 19% depart after the AM peak. Under both scenarios 1 and 2, only 67% of travelers choose the AM peak, with 12% and 21% before and after, respectively. And under scenario 3, 63% choose the AM peak, 14% the before peak, and 23% the after peak period. Thus, the tolls imposed in the three scenarios have the expected effect on departure time choices. Simulated departure times also allow for analysis of total welfare (after accounting for revenues). Average total welfare changes are $-\$0.03$, $-\$0.09$, and $-\$0.25$ per traveler for the three scenarios, suggesting that each tolling scenario results in negative welfare changes. Of course, the scenarios here represent highly idealized settings and the tolls' effects on travel time and its variance may not be accurate; dynamic simulations of network performance under shifting departure times are needed to draw more definitive conclusions.

6. CONCLUSIONS

While the overall effect of travel time mean and variance variables on departure time profiles for work tours appears to be rather small, the value of the methods presented in this paper are clear. Using relatively straightforward methods and familiar datasets, regression equations were developed to obtain travel speeds as they vary across the day. Using similar regression equations, a new method for estimating variance in travel times (by TOD) was presented. The method relies on reported travel times and estimated travel times based on other regression models (for auto mode) and transportation network skims (for transit modes).

In addition, a new application of the continuous logit model was presented in the context of home-based work tour departure times. The continuous logit model offers the advantage of treating departure time in a continuous context, while retaining the theoretical and other advantages of a random utility framework. While model estimation does require numerical integration techniques in order to obtain likelihood estimates, computing power is not an issue.

The Bayesian methodology pursued here offers several advantages over traditional estimation techniques. Model estimation generates the multivariate posterior distribution of parameters, rather than single point estimates. And, with non-linear utility functions, such as those used here, the likelihood often becomes multimodal, leaving the possibility for traditional maximum likelihood methods to converge to local maxima. Since Bayesian methods do not search for parameter values that attain the maximum likelihood (draws from the posterior distribution are taken instead), this is not an issue (34). Lastly, the Bayesian method provides great flexibility in model specification, and other (possibly more appropriate) specifications can be examined with relative ease.

Such work offers numerous opportunities for extensions. For example, most recent models of tour timing consider not only the tour's departure time, but also the activity duration or return time. In theory, the continuous logit model can easily be extended to a second dimension of travel

timing by simply adding a second dimension of integration. In fact, Ben-Akiva et al.'s (28) original application was in the field of location choice (in two-dimensional space). Of course, adding this additional dimension of integration slows likelihood calculations, and thus, estimation. In addition, one can imagine a number of ways to formulate the utility function used in the continuous logit. The one presented here was chosen for its familiarity and allowance of bi-modality in departure time choice. However, other functional forms may allow better and richer models to emerge.

As activity-based travel demand modeling and DTA techniques advance rapidly, TOD modeling remains a key weakness of travel demand modeling. Moving toward continuous-time models (such as the continuous logit) and away from discrete choice methods will enhance the temporal resolution of such models. Almost no continuous models have a solid behavioral basis, while the continuous logit provides a direct measure of utility, which is important for project evaluation, policy analysis, and welfare calculations, and links the model to other travel choice dimensions (such as destination and mode).

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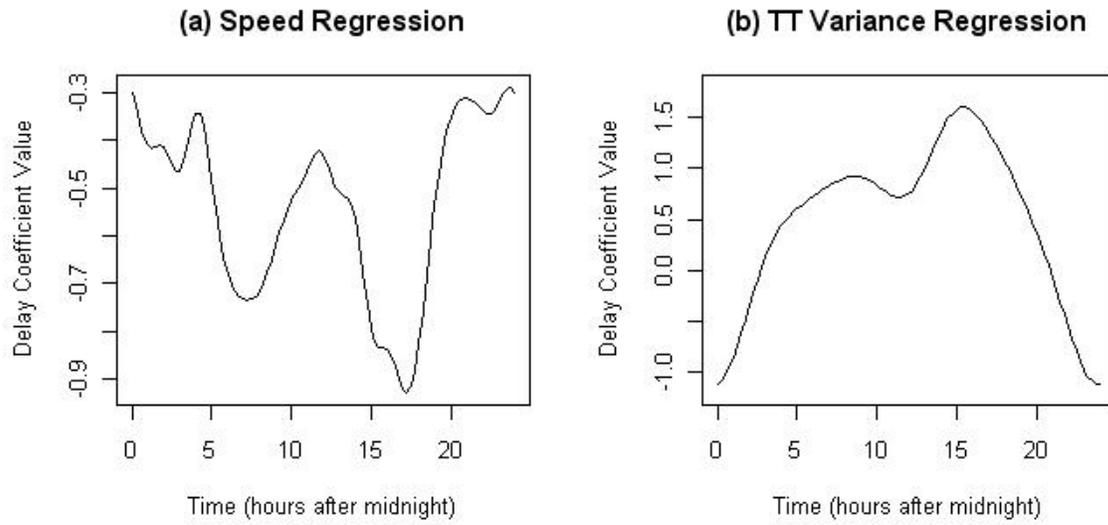


FIGURE 1: Delay Coefficient Variation in Auto Mode Speed and Travel Time Variance Regressions across Departure Times

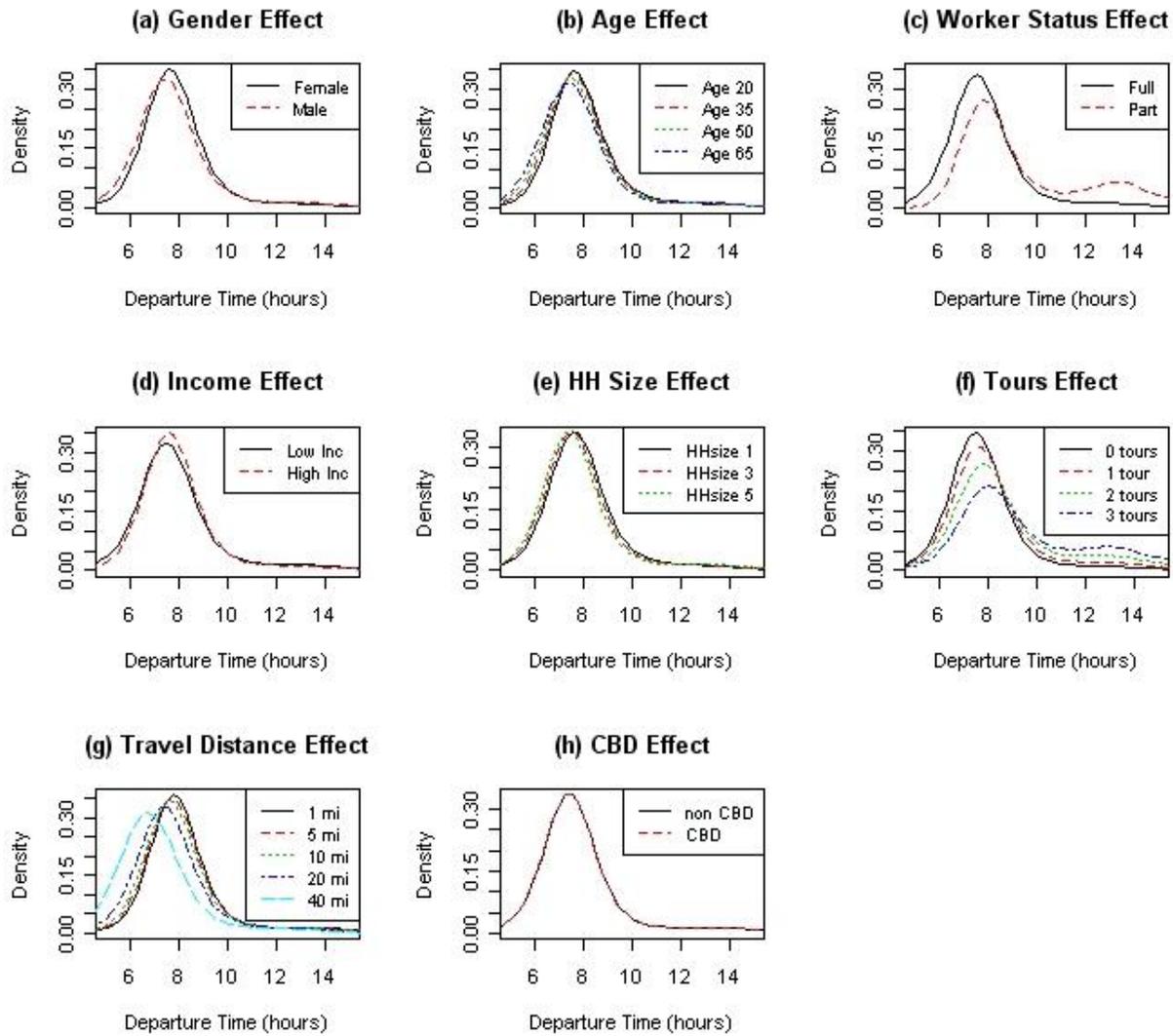


FIGURE 2: Predictive Density Profiles of Departure Time for Individuals with Different Attributes

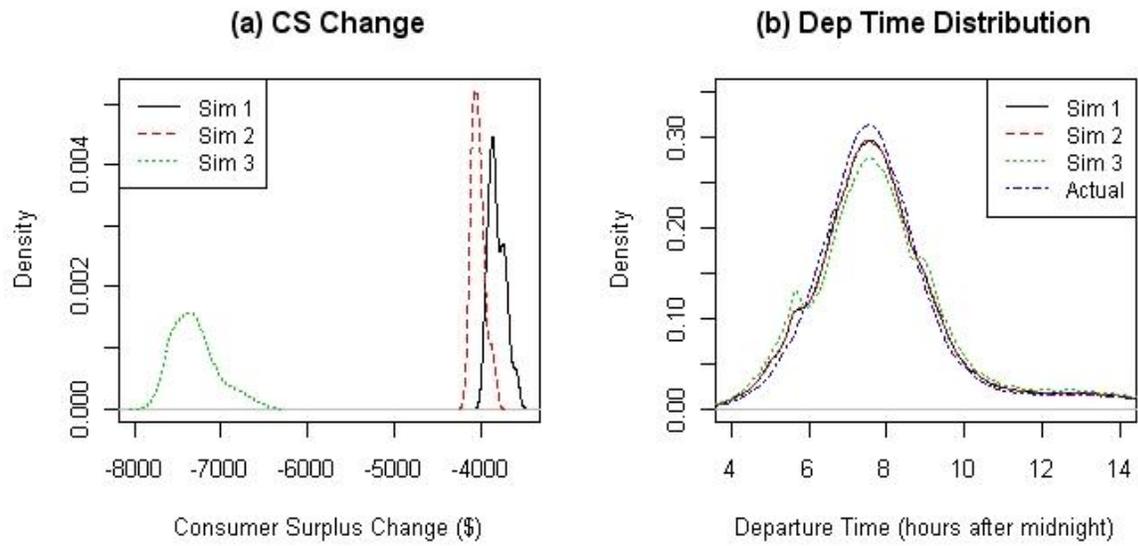


FIGURE 3: Simulation Exercise Consumer Surplus Change and Departure Time Choice Differences

TABLE 1: Auto Mode Speed and Travel Time Variance Regression Estimation Results

| Variable | Auto Speed Regression | | Auto Travel Time Variance Regression | |
|--|-----------------------|--------|--------------------------------------|--------|
| | Coeff. | t-stat | Coeff. | t-stat |
| Constant | -0.3205 | -30.08 | 1.4239 | 25.64 |
| Ln(distance) | 0.2008 | 94.02 | 0.7743 | 69.67 |
| Indicator (Tuesday) | -0.0052 | -1.10 | 0.0036 | 0.15 |
| Indicator (Wednesday) | -0.0143 | -2.96 | -0.0175 | -0.69 |
| Indicator (Thursday) | -0.0154 | -3.08 | 0.0067 | 0.26 |
| Indicator (Friday) | -0.0069 | -1.33 | 0.0161 | 0.60 |
| Indicator (Regional Core Origin) | -0.2015 | -10.48 | 0.5173 | 5.17 |
| Indicator (CBD Origin) | -0.1974 | -16.66 | 0.1732 | 2.81 |
| Indicator (Urban Business Origin) | -0.1608 | -16.71 | 0.0078 | 0.16 |
| Indicator (Urban Origin) | -0.1382 | -16.32 | -0.0670 | -1.52 |
| Indicator (Suburban Origin) | -0.0603 | -7.75 | -0.0795 | -1.96 |
| Indicator (Regional Core Destination) | -0.1998 | -11.18 | 0.4950 | 5.32 |
| Indicator (CBD Destination) | -0.2032 | -17.63 | 0.0619 | 1.03 |
| Indicator (Urban Business Destination) | -0.1529 | -15.99 | -0.1760 | -3.53 |
| Indicator (Urban Destination) | -0.1363 | -16.01 | -0.1424 | -3.21 |
| Indicator (Suburban Destination) | -0.0643 | -8.16 | -0.1540 | -3.75 |
| Indicator (Shared Ride 2) | -0.0361 | -9.45 | 0.1900 | 9.54 |
| Indicator (Shared Ride 3+) | -0.0429 | -10.71 | 0.1617 | 7.75 |
| Delay | -3.5696 | -7.46 | 1.8612 | 13.97 |
| Delay* h_1g1 | -0.2308 | -0.30 | -0.1509 | -4.57 |
| Delay* h_2g2 | 6.0955 | 6.64 | -0.7945 | -14.17 |
| Delay* h_3g3 | -1.9896 | -7.50 | 0.1838 | 4.53 |
| Delay* h_4g4 | 2.0779 | 5.17 | -0.3127 | -8.73 |
| Delay*($h_1^2g1^2$) | 0.0906 | 0.18 | | |
| Delay*($h_2^2g2^2$) | -3.5927 | -5.73 | | |
| Delay*($h_3^2g3^2$) | 1.2948 | 7.25 | | |
| Delay*($h_4^2g4^2$) | -0.8263 | -3.75 | | |
| Delay*($h_1^3g1^3$) | 0.0015 | 0.02 | | |
| Delay*($h_2^3g2^3$) | 0.5920 | 4.90 | | |
| Delay*($h_3^3g3^3$) | -0.2434 | -6.95 | | |
| Delay*($h_4^3g4^3$) | 0.1438 | 3.51 | | |
| Observations | 86,358 | | 86,358 | |
| R-Squared | 0.137 | | 0.147 | |
| Adj. R-Squared | 0.137 | | 0.147 | |

TABLE 2: Transit Travel Time Variance Regression Estimation Results

| Variable | Drive-to-Transit Model | | Walk-to-Transit Model | |
|--|------------------------|------------|-----------------------|------------|
| | Coeff | t-stat | Coeff | t-stat |
| Constant | -0.1656 | -0.14 | 1.8097 | 10.71 |
| Indicator (Regional Core Origin) | -0.8284 | -5.56 | -0.6344 | -4.74 |
| Indicator (CBD Origin) | -0.8248 | -4.64 | -0.7108 | -5.56 |
| Indicator (Urban Business Origin) | 0.2899 | 1.68 | -0.6192 | -4.90 |
| Indicator (Urban Origin) | 0.2034 | 1.59 | -0.5489 | -4.92 |
| Indicator (Regional Core Destination) | -0.7507 | -4.99 | -0.5245 | -4.04 |
| Indicator (CBD Destination) | -0.5704 | -3.32 | -0.4949 | -3.91 |
| Indicator (Urban Business Destination) | -0.0319 | -0.17 | -0.3951 | -3.10 |
| Indicator (Urban Destination) | 0.1764 | 1.31 | -0.1457 | -1.27 |
| Indicator (AM Peak Period) | 0.3246 | 0.28 | 0.3531 | 2.51 |
| Indicator (Midday Period) | 0.5217 | 0.44 | 0.1685 | 1.20 |
| Indicator (PM Peak Period) | 0.7685 | 0.65 | 0.3630 | 2.75 |
| Indicator (Evening Period) | 0.5286 | 0.45 | | |
| Total Travel Time (from skim) | -0.0493 | - 23.46 | | |
| In-Vehicle Travel Time (from skim) | | | -0.0877 | - 35.23 |
| Observations | 3,297 | | 4,981 | |
| R-Squared | 0.185 | | 0.204 | |
| Adj. R-Squared | 0.182 | | 0.202 | |

TABLE 3: Continuous Logit Model Estimation Results

| Variable | Mean Estimate | 95% Interval | Variable | Mean Estimate | 95% Interval |
|----------------------------------|---------------|---------------------|---------------------------------|---------------|---------------------|
| Departure Time Functions | | | Age Interactions | | |
| Sin(2*pi*t/24) | 1.1982 | (0.7754 ,1.6867) | Sin(2*pi*t/24) | 0.0179 | (0.0077 ,0.0275) |
| Sin(4*pi*t/24) | -0.8244 | (-1.3688 , -0.2132) | Sin(4*pi*t/24) | 0.0176 | (0.0034 ,0.0306) |
| Sin(6*pi*t/24) | 0.1447 | (-0.2358 ,0.5518) | Sin(6*pi*t/24) | 0.0092 | (-0.002 ,0.0198) |
| Sin(8*pi*t/24) | 0.1854 | (-0.0201 ,0.3906) | Sin(8*pi*t/24) | 0.0103 | (0.0051 ,0.016) |
| Cos(2*pi*t/24) | 0.8141 | (0.0573 ,1.5237) | Cos(2*pi*t/24) | -0.1269 | (-0.1482 , -0.1058) |
| Cos(4*pi*t/24) | 1.7545 | (1.2901 ,2.228) | Cos(4*pi*t/24) | -0.0981 | (-0.1143 , -0.0826) |
| Cos(6*pi*t/24) | 2.4718 | (2.1416 ,2.8026) | Cos(6*pi*t/24) | -0.0625 | (-0.073 , -0.0526) |
| Cos(8*pi*t/24) | 0.8101 | (0.6243 ,1.0058) | Cos(8*pi*t/24) | -0.0208 | (-0.0258 , -0.0161) |
| Part-Time Indicator Interactions | | | Free-flow Distance Interactions | | |
| Sin(2*pi*t/24) | -2.4095 | (-2.8176 , -2.0755) | Sin(2*pi*t/24) | 0.0453 | (0.0317 ,0.059) |
| Sin(4*pi*t/24) | -1.4741 | (-1.9063 , -1.1043) | Sin(4*pi*t/24) | 0.0292 | (0.0144 ,0.0451) |
| Sin(6*pi*t/24) | -0.8085 | (-1.0981 , -0.5442) | Sin(6*pi*t/24) | 0.0049 | (-0.0026 ,0.0131) |
| Cos(2*pi*t/24) | 0.0410 | (-0.231 ,0.2982) | Cos(2*pi*t/24) | -0.0267 | (-0.0522 , -0.0019) |
| Cos(4*pi*t/24) | 0.3921 | (0.1629 ,0.6296) | Cos(4*pi*t/24) | -0.0246 | (-0.0384 , -0.0103) |
| Cos(6*pi*t/24) | 0.0433 | (-0.1991 ,0.2635) | Cos(6*pi*t/24) | -0.0256 | (-0.0311 , -0.0198) |
| Male Indicator Interactions | | | No. of Other Tours Interactions | | |
| Sin(2*pi*t/24) | 0.3882 | (0.2386 ,0.542) | Sin(2*pi*t/24) | -0.6171 | (-0.6901 , -0.5439) |
| Sin(4*pi*t/24) | 0.4646 | (0.3479 ,0.5841) | Sin(4*pi*t/24) | -0.1454 | (-0.2107 , -0.0764) |
| Cos(2*pi*t/24) | 0.1693 | (-0.0816 ,0.4067) | Cos(2*pi*t/24) | 0.2875 | (0.119 ,0.46) |
| Cos(4*pi*t/24) | 0.2110 | (0.0835 ,0.3366) | Cos(4*pi*t/24) | 0.2373 | (0.1466 ,0.3341) |

| High Income HH Indicator Interactions | | | CBD Destination Zone Indicator Interactions | | |
|---------------------------------------|---------|--------------------|---|---------|--------------------|
| Sin(2*pi*t/24) | -0.0338 | (-0.1828 ,0.1174) | Sin(2*pi*t/24) | -0.0899 | (-0.2881 ,0.1121) |
| Sin(4*pi*t/24) | -0.2063 | (-0.3113 ,-0.094) | Sin(4*pi*t/24) | -0.0238 | (-0.1746 ,0.1276) |
| Cos(2*pi*t/24) | -0.5007 | (-0.7408 ,-0.2359) | Cos(2*pi*t/24) | -0.3114 | (-0.7025 ,0.0978) |
| Cos(4*pi*t/24) | -0.2813 | (-0.3994 ,-0.1489) | Cos(4*pi*t/24) | -0.1254 | (-0.3298 ,0.087) |
| HH Size Interactions | | | LOS Variables | | |
| Sin(2*pi*t/24) | 0.0291 | (-0.029 ,0.087) | Travel Time | -0.0047 | (-0.0126 ,-0.0007) |
| Sin(4*pi*t/24) | 0.1210 | (0.0761 ,0.1706) | Travel Time Variance | -0.0040 | (-0.0053 ,-0.0029) |
| Cos(2*pi*t/24) | -0.1169 | (-0.2207 ,-0.0116) | Cost | -0.1352 | (-0.2047 ,-0.0782) |
| Cos(4*pi*t/24) | -0.0972 | (-0.1507 ,-0.0452) | | | |
| | | | | | |
| Observations | 13,338 | | | | |