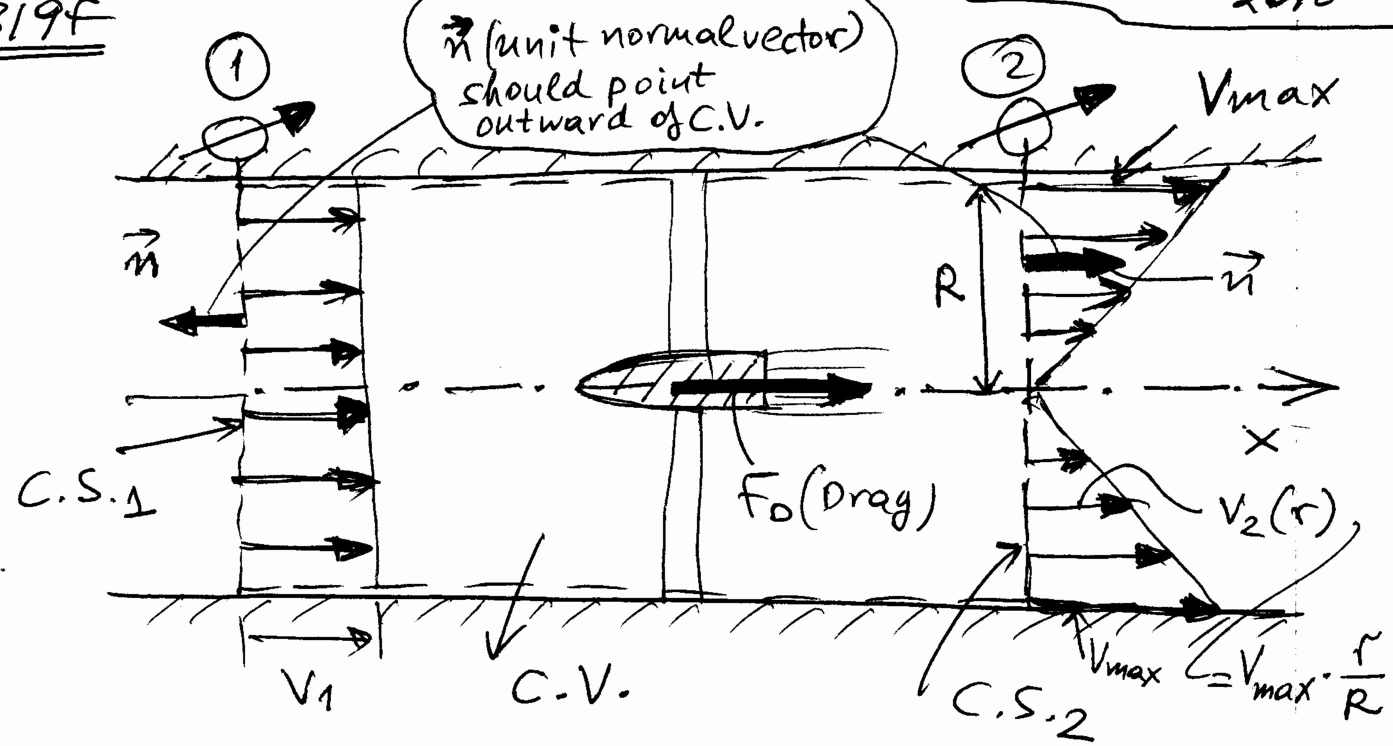
Supplement to Ex. 6.8 of textbook | ©S.A. Kinnas 2010

CE319F



(a)

$$\rho \int_{C.S._1} V_x (\vec{V} \cdot \vec{n})\, dA = \rho \int_{C.S._1} V_1 (-V_1)\, dA = -\rho V_1^2 \int_{C.S._1} dA = -\rho V_1^2 A_1$$

- in station (1) flow is uniform
- in station (1): $\vec{V} \cdot \vec{n} = \underbrace{|\vec{V}|}_{=V_1} \cdot \underbrace{|\vec{n}|}_{=1} \cos(180°) = V_1(-1) = -V_1$

(b)

$$\rho \int_{C.S._2} V_x \underbrace{(\vec{V} \cdot \vec{n})}_{= |\vec{V}| \cdot |\vec{n}| \cdot \cos(0°) = V_2(r)} dA = \rho \int_{C.S._2} V_2(r)(V_2(r))\, dA = \rho \int_{C.S._2} V_2^2\, dA$$

$$= \rho \int_0^R V_2^2(r)\, 2\pi r\, dr$$

Also continuity requires: $V_1 A_1 = \int_0^R V_2 (2\pi r)\, dr \Rightarrow$

$$\Rightarrow V_1 \pi R^2 = \int_0^R V_{max}\left(\frac{r}{R}\right)(2\pi r)\, dr = \frac{2\pi V_{max}}{R} \int_0^R r^2 dr = \frac{2\pi V_{max}}{R} \frac{R^3}{3}$$

$$\Rightarrow V_{max} = \frac{3}{2} V_1$$