Modeling the Hydrodynamics of Stratified Lakes

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ABSTRACT: Stratified lakes present challenges to numerical modeling not encountered in estuarine flows. The vertical mixing processes are small, both instantaneously and cumulatively, as evidenced by the small seasonal change in hypolimnion temperature found in monomictic temperate lakes. The small cumulative vertical flux which does exist is important for ecological processes, but the mixing is energized by a variety of sources and occurs predominantly within a benthic boundary layer at small scales relative to practical model grid resolution. The fundamental control for the cascade of energy from the wind into mixing and benthic boundary layer transport appears to be the basin-scale internal wave field, which can be modeled on a relatively coarse mesh. However, attention must be given to the processes in the surface layer that lead to the setup of a baroclinic tilt. Below the wind-mixed surface layer the advective motion is driven principally by the basin-scale internal waves, whose net transport is a function of nonlinear effects from topography and wave amplitude. A further modeling complication is the convoluted bathymetry often associated with reservoirs formed in drowned river valleys and many lakes formed naturally in geological rifts. Cold inflows associated with storm events may flow rapidly through the narrow bottom of reservoirs where model grid resolution is generally poor. In this paper we discuss some of the challenges in numerical modeling of stratified lakes and some of the techniques which we are using and developing to address these challenges.

1 INTRODUCTION

1.1 Why model lake hydrodynamics?

Lakes are a meter for gauging the success or failure of catchment management decisions. When nutrient loads exceed assimilative capacity and algae blooms become a public concern, it is clear that the meter is reading “failure”. However, gradations of “successful” catchment management are difficult to discern as assimilative capacity can be modified by load distribution, burial, resuspension and recycling - in short, by effects of transport on the biogeochemical cycle. Transport processes are inherently three-dimensional (3D), driven by interactions of wind, surface thermo-dynamics, topography and the seasonal thermal history of the lake. It is difficult to design simple box models which accurately reflect effects of 3D transport and spatial inhomogeneity on assimilative capacity, so water quality modeling in a stratified lake inherently requires transport modeling.

On present computers, one-dimensional models (e.g. DYRESM, Imberger and Patterson, 1981) that couple laterally-averaged lake physics and water quality modeling (e.g. CAEDYM, Herzfeld and Hamilton, 1997) are extremely efficient in CPU time and memory. Multiple computer runs forecasting evolution of lake stratification and water quality over 50 to 100-year intervals can be readily conducted to allow examination of long-term sensitivity to anthropogenic changes in the catchment or fluctuations in local climate. For the near future, this approach will remain the primary method for predicting changes in lake water quality over inter-annual and decadal time scales.

However, advances in computer power and numerical methods are bringing 3D modeling of lakes for seasonal to annual time scales within the reach of desktop workstations (e.g. Schwab and Bedford, 1995; Casulli, 1997; Umlauf et al., 1999; Hodges et al., 2000). Modeling in 3D provides better resolution of topographical effects, internal waves, mixing, and spatial gradients of environmental forcing. Unfortunately, practical 50-year simulations of 3D lake dynamics require increasing available computational speed and memory by at least two orders of magnitude. While this is perhaps a decade away, we can presently apply 3D models to shorter time periods to: (1) gain greater understanding of transport processes; (2) investigate the limitations of our present methods; and (3) develop approaches to "tune" 1D models to the physics of individual lakes.

1.2 A brief review of lake transport processes

In this paper, we examine the challenges to modeling transport processes in stratified lakes. The wind, surface thermodynamics and inflow dynamics provide the energy sources for transport, turbulent kinetic energy (TKE), internal waves and mixing. The framework of the transport processes associated with wind energy is shown schematically in Figure 1. A strong temperature gradient separates a lake upper layer (epilimnion) from the lower layer (hypolimnion), limiting the scales of transport processes across the thermocline (metalimnion). When the wind-mixed layer is sufficiently deep (relative to the depth of the thermocline), downwind transport of the surface water provides a barotropic tilt to the lake surface; in response, the thermocline takes on an opposing (baroclinic) tilt which initiates basin-scale internal waves. These form a set of rotational Kelvin and Poincare waves when Coriolis forces are significant (Gill, 1982).

Basin-scale internal motions may persist as free waves for several days after their initiation, dissipating energy in the benthic boundary layer and slowly transferring energy to groups of free, short-period internal waves which are then either dissipated locally or shoal on the sloping boundaries losing their energy through breaking (Antenucci et al., 2000). As they propagate about the lake, the basin-scale internal waves develop oscillating advective motion in the hypolimnetic waters and a deep benthic boundary layer (Imberger, 1998). Thorpe (1999) showed breaking internal waves along the sloping boundary could generate an alongslope current that is greater than the Lagrangian drift for certain conditions. A critical piece of our picture appears to be the nonlinear effects of topography and finite amplitudes of internal waves, leading to onshore and offshore transport (Lemckert et al., 1998).

The basin-scale internal wave response depends on the stratification, which is in a delicate balance between intensification by solar radiation penetrating the water column and destratification due to vertical mixing. The vertical mixing processes in the interior are themselves the result of a complicated series of energy cascades involving the direct introduction of TKE at the free surface, production of TKE by the shear of the basin-scale motions along with breaking and traumata of short-period internal waves. The latter is formed by the interaction of groups of waves with themselves, with the background mean shear flow (due to the basin-scale seiching) and at the boundary where the resulting turbulence sets up a benthic boundary

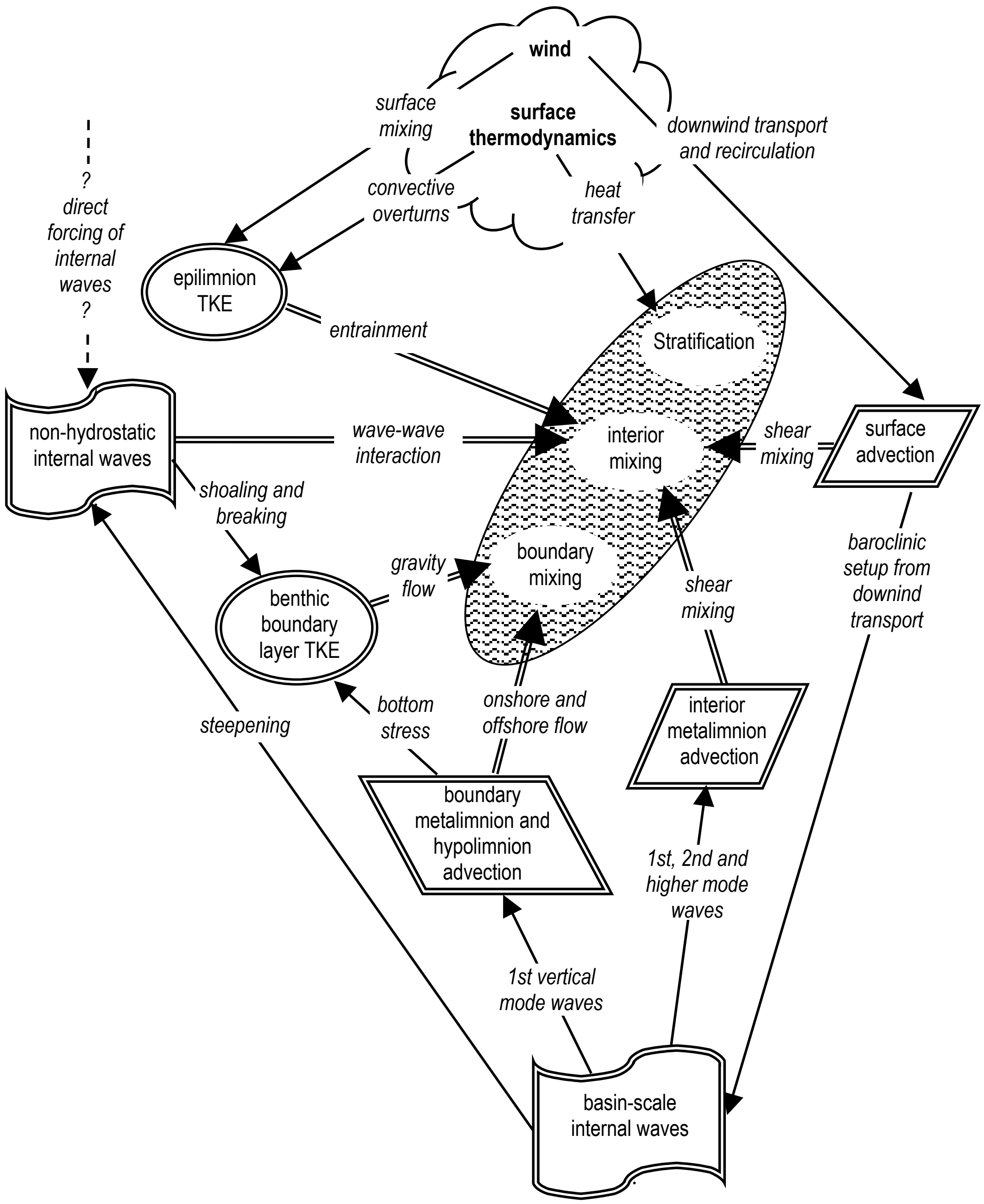

Figure 1: Transformation of wind and thermodynamic energy into stratification. The energy from the wind and thermodynamics directly influences the TKE and stratification in the upper layer (epilimnion) while indirectly influencing the mixing across the thermocline (metalimnion) through entrainment and generation of internal waves. Evidence of Antenucci and Imberger (2000) points to the existence of direct forcing of non-hydrostatic internal waves by a mechanism that is as yet unknown.

layer flux. The mixing resulting from these complicated sets of processes is quite small, as evidenced by the small seasonal change in temperature found in the hypolimnion of most temperate lakes. Furthermore, the non-linear mass transport associated with the basin-scale internal waves can lead to a vertical cycling of the water quality which is large enough to explain the observed rates of change of the hypolimnetic water.

Finally, inflow dynamics provide a further source of stratification and energy for advection and mixing. Storm events typically produce catchment runoff that is cooler than epilimnion temperatures, resulting in downslope flow and entrainment of epilimnetic water along a boundary. The inflow inserts itself into the lake temperature structure at neutral buoyancy. Similarly, coastal lakes may experience salinity underflows during storms or high tides. In extreme cases, such inflows may have a sharp front that propagates downslope and initiates large-scale internal waves as the front passes through the metalimnion.

The above can only be considered a simple gloss over the transport processes. More comprehensive reviews of transport and boundary effects can be found in Imberger (1998) and Thorpe (1998)

1.3 Modeling the transport processes

We have three principal research areas aimed at modeling transport processes on seasonal and longer time scales: (1) energy inputs that are the top-level forcing of a lake, including surface thermodynamics, wind forcing and inflow events; (2) evolution of the basin-scale internal waves that control the spatial and temporal distribution of energy; and (3) sub-basin scale processes which dissipate the basin-scale internal waves and affect mixing.

Our present work has demonstrated success in the first two areas. Indeed, it can be said that modeling the first-order setup and evolution of basin-scale internal waves for a lake is primarily a matter of providing sufficient computing resources and using algorithms which have been developed and proven for stratified lakes. However, much work remains to be done in the last area: “closure” models for small-scale processes whose cumulative effect is noticeable at seasonal time scales but may be smaller than numerical truncation error at the model time scale. In particular, most numerical models applied to stratified lakes at practical grid resolutions will experience slow numerical diffusion of the thermocline. The effect is minor for simulations of days or a few weeks; however, over monthly or seasonal time scales the accumulation of numerical diffusion changes the response of the lake to wind forcing and allows excessive transport from the hypolimnion to the metalimnion and epilimnion.

The cumulative effects of numerical diffusion, the importance of internal waves and the absence of strong inflow/outflow boundary forcing challenges stratified lake models in ways not seen in estuary models. In an estuary, the advective motions are driven primarily by interactions of tidal forcing and river inflows: energetic processes determined at the boundary of the system. If the exchange at the tidal boundary and the river inflows are correctly represented, an estuary model will produce reasonable results over long time scales without noticeable degradation by numerical diffusion. Furthermore, while internal waves occur in estuaries (Stephens and Imberger 1997), they play a secondary role in transport and mixing due to the dominance of tidal effects.

In section 2 of this paper, we discuss modeling the fundamental energy inputs into a stratified lake. In section 3, we examine problems and solutions to modeling the basin-scale motions. Finally, in section 4, we pose the future challenges associated with modeling the subgrid-scale processes. To keep this paper at a reasonable length, the following is necessarily focussed on the lake modeling being conducted at the Centre for Water Research at the University of Western Australia and undoubtedly neglects much that is going on in other organizations.

2 BASIN-SCALE ENERGY INPUT

2.1 Forcing by thermodynamics and wind stress at the surface

Implementation of 3D surface thermodynamics and wind forcing can, at first glance, be simply adapted from oceanic modeling and 1D lake modeling. Indeed, the fundamental bulk heat transfer models used successfully in 1D DYRESM can be applied separately for each discrete water column in a lake as the basis of surface thermodynamics for a 3D model. The primary difficulty with accurately modeling surface effects is usually a lack of accurate environmental data rather than any fundamental inadequacy of existing models.

In particular, the wind over the entire surface of most lakes is directly influenced by the surrounding topography such that local wind blocking may give rise to consistent spatial wind gradients which directly drive surface gyres. Rueda and Schladow (pers. comm., 1999) have demonstrated that a 3D model of Clear Lake, California, will produce topographically-driven circulation gyres in the wrong direction if a spatially-uniform wind field is used as environmental forcing. When spatial gradients of the wind field are included by interpolating from 14 wind stations around the lake, the correct directions for surface gyres are obtained using the Estuary and Lake Computer Model (ELCOM), described in Hodges et al. (2000).

As multiple wind stations cannot be practically maintained around most lakes, a major challenge to 3D hydrodynamic lake modeling is prediction of spatially-varying wind fields. Direct atmospheric modeling (Pan et al., 1998) and subgrid-scale interpolation from large-scale atmospheric data sets (Nadaoka et al., 2000) have been successful in *a posteriori* modeling of events, but add another layer of computational and research effort to 3D lake modeling (as well as being difficult to apply in a forecasting mode). An approach that may prove more flexible is under investigation by Rueda and Schladow (pers. comm., 1999) that involves modeling the effect of topography on the local wind pattern for prevailing wind directions.

2.2 Dense underflow events

Reservoirs formed from flooded river valleys or natural lakes in geographical rifts may experience dense inflows moving down the bottom channel for tens of kilometers before plunging through the metalimnion. Such flows may take several days from initiation in the catchment to insertion in the hypolimnion. Capturing the speed of propagation, entrainment rate and depth of the underflow are critical to modeling its correct insertion in the lake density structure. Modeling these inflows is not simply an academic exercise, but is critical to catchment management decisions. In 1998, a winter storm near Sydney, Australia flushed the catchment of Lake Burragorang with a pulse of cold water that traveled the 40 km length of the reservoir in one week. The storm runoff maintained its identity as a turbid underflowing cold water mass, transporting *Giardia* and *Cryptosporidium* spores from the catchment to the dam wall, where the offtake for the city of Sydney became contaminated (Antenucci and Imberger, 1998). The resulting broadcast warnings of possible contamination to the reservoir that provides 70% of Sydney's drinking water were both embarrassing and costly for the water authority.

The complex morphologies of reservoirs present a modeling challenge as a dense inflow will seek the lowest path, i.e. the old river bed. While a reservoir may have a width on the order of one to several kilometers, the old river bed is likely to be on the order of tens or a few hundred meters in width. If model bathymetry does not include the old river bed (through lack of data, insufficient grid resolution or boundary smoothing required for curvilinear coordinate generation), the inflow will be diluted and slowed as it spreads across a wide region. Furthermore, where the drowned river channel is known, it is likely to be sinuous and under-resolved at the grid scale.

Figure 2a shows the convoluted bathymetry of Lake Burragorang, Australia, while inset Figure 2b illustrates a Cartesian grid on a relatively straight section of the channel. Poor resolution in the channel bottom causes numerical dissipation of momentum as an underflow changes from 'x' to 'y' directions. After a storm in winter 1997, Lake Burragorang thermistor chain field data showed propagation of a cold water inflow from the arms of the reservoir to the dam wall in a week. The mean velocity of 11 cm s^{-1} could not be reproduced in a model using the finest practical grid (50 x 50 m) and the physical bathymetry. In order to capture the first-order dynamics of the underflow with available computer power, the bathymetry was "straightened" as shown in Figure 2c. This reduced the numerical dissipation of momentum, allowing the inflow propagation to be successfully modeled (Jha et al., 1999).

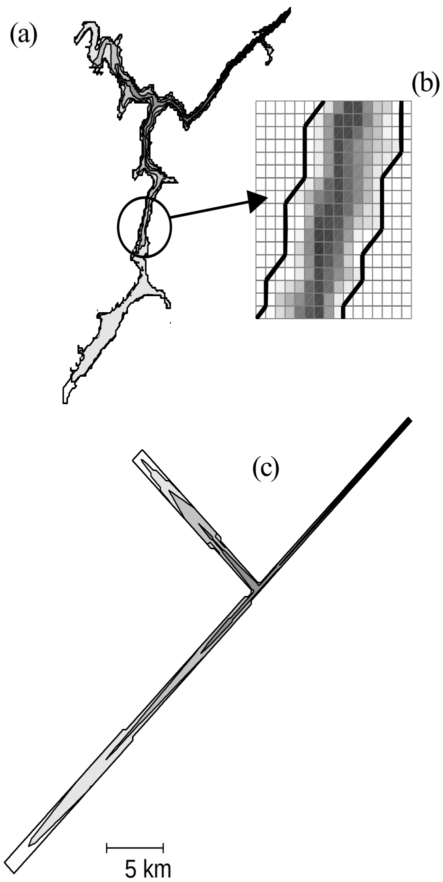

Figure 2: Lake Burragorang, NSW, Australia: (a) real bathymetry; (b) 50 x 50 meter grid resolution in a section; (c) straightened bathymetry.

The disadvantage of straightening the bathymetry is the loss of dynamic effects of flow curvature. To remedy this, we have developed a new "perturbation curvilinear" approach that can be applied when the radius of curvature of the bends is smaller than the channel width (Hodges and Imberger, 2000). Using this approach, the first-order curvature effects can be included in a straightened Cartesian grid model without the extra costs and complexity of full curvilinear methods (e.g. Meakin and Street, 1988).

3 BASIN-SCALE PROCESSES

3.1 Introduction

Given a model that can successfully compute the energy inputs at the basin scale, the next requirement is correct computation of the evolution of energy at large scales. In this we are concerned with capturing the oscillating transfer between potential energy and advective kinetic energy by basin-scale internal waves, along with their interactions and transport. This requires accurate representation of the background stratification that sets the period and amplitude of the waves generated by the environmental forcing. The basin-scale internal waves are the primary store of kinetic energy in a stratified lake, so accuracy in their modeling is critical to successfully representing transport processes.

3.2 The surface mixed layer and setup of internal waves

The depth of the wind-mixed surface layer is dependent on the TKE introduced by the wind, the heat transfer across the surface and the stratification in the upper layer. The physics of surface mixing is fundamentally an exchange of TKE for potential energy as the stratification is mixed upwards. The vertical transport processes in the diurnal wind-mixed layer are too small to directly simulate at practical grid and time scales in a lake model, so a turbulence closure scheme is required. There is a wide variety of approaches from which to choose: two-equation transport, Richardson number mixing, algebraic stress closures, mixed-layer models and large-eddy simulation (LES) methods. The review by Rodi (1987), while somewhat dated, still provides a good overview of methods and fundamentals.

The processes of mixing in a stratified flow are generally not well-represented in the classic high-order turbulence closure schemes. Indeed, it has been shown in 1D tests (McCormick and Meadows, 1988) that the popular Mellor-Yamada model under-predicts the surface-mixed-layer depth, which has important consequences, as discussed below. Recent LES work with fine vertical resolution (50 grid layers in the surface-mixed layer) provides a promising step toward modeling surface mixing dynamics in a quantitatively correct manner, but is presently limited to supercomputer applications in simple rectangular boxes (Skyllingstad et al., 1999). Richardson number approaches have had some success (Beletsky et al., 1997), but are generally formulated to parameterize mixing in terms of a vertical eddy-diffusivity, requiring inclusion of an ad hoc coefficient that is a function of the grid spacing. Mixed-layer models (e.g. Kraus and Turner, 1967) have a significant advantage at coarse grid resolutions as the vertical diffusion equation can be eliminated and physics of mixing directly parameterized in terms of discrete energetics

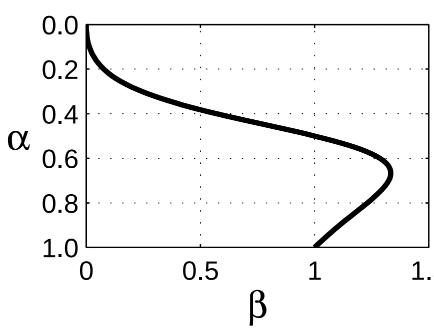

Figure 3: Effect of the ratio of wind-mixed-layer depth to epilimnion depth (α) on the scaling coefficient (β) for the equilibrium thermocline setup due to downwind transport (after Hodges, 2000).

of mixing across homogenous layers. This approach has been shown to produce a good representation of the mixed-layer depth in a strongly stratified lake (Hodges et al., 2000).

Our interest in correctly predicting the mixed-layer depth is due to its role in the setup of basin-scale internal waves. Under-prediction of the mixed-layer depth leads to under-prediction of the baroclinic tilt ($\partial h/\partial x$) that initiates the internal wave motion (Hodges, 2000). The baroclinic tilt is governed by wind shear velocity (u_*), depth (H), reduced gravity of stratification (g') by

$$\frac{\partial h}{\partial x} \approx \beta \frac{u_*^2}{g'H} \tag{1}$$

where β is related to the depth of the wind-mixed layer as shown in Figure 3. It has been demonstrated that recent modeling approaches can adequately address this problem, as shown in the comparison of field data and modeled internal waves for Lake Kinneret in Figure 4 (after Hodges et al., 2000) Thus, we consider this a challenge that has been met: given the correct wind forcing, we can model mixing

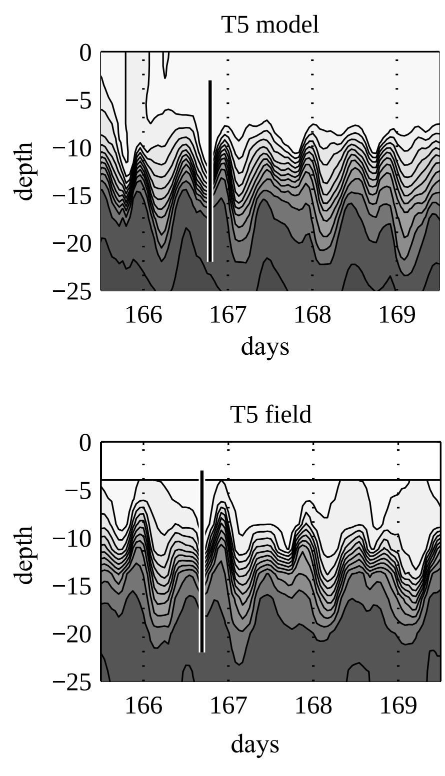

Figure 4: Field and model data: temperature isotherms at station T5 in Lake Kinneret from days 165 to 169 in 1997. Solid line shows trough of basin-scale Kelvin wave (after Hodges et al. 2000).

and transport using a coarse grid resolution in the surface layer and attain a reasonable representation of the mixed-layer depth, along with correct set-up of wind-forced internal waves.

3.3 Numerical diffusion

For a given wind forcing, the amplitudes and periods of basin-scale internal waves are set by the vertical density stratification. It follows that the evolution of the thermocline depth and thickness must be correctly predicted to model the physics of internal waves. Unfortunately, numerical models have a tendency to artificially diffuse sharp temperature gradients significantly faster than physical mixing processes (Griffies et al., 2000). Numerical diffusion mixes cool hypolimnetic water up into the warm epilimnetic water, accelerating the erosion of the thermocline. As the modeled thermocline diffuses, the amplitudes and periods of the basin-scale internal waves increase, which alters physical mixing as well as numerical diffusion. The effects of numerical diffusion accumulate with model run-time, slowly warming the hypolimnion.

Numerical diffusion is a function of the size of the computational grid relative to the sharpness of physical gradients in the flow and the type of numerical method applied. A significant amount of effort in the numerical community has gone into producing methods that minimize the numerical diffusion so as to allow propagation of sharp gradients (e.g. Leonard, 1991). Unfortunately, these methods still require relatively fine grid spacing to be effective. As sufficient refinement is generally impractical (especially over seasonal time scales), we have been seeking other approaches to countering numerical diffusion.

A promising method for reducing the effects of numerical diffusion involves the use of a sharpening filter applied vertically (Laval et al., 2000). By using a numerical scheme that calculates surface thermodynamics, turbulent mixing and advection separately, it is possible to reverse the effects of numerical diffusion. Since advection is an adiabatic process, the increase in the potential energy of the adiabatically relaxed density field (i.e. background potential energy, Winters et al., 1995) occurring during the advection calculation is directly attributable to numerical diffusion. Vertical application of a sharpening filter “undoes” the artificial smoothing of sharp gradients induced by numerical diffusion

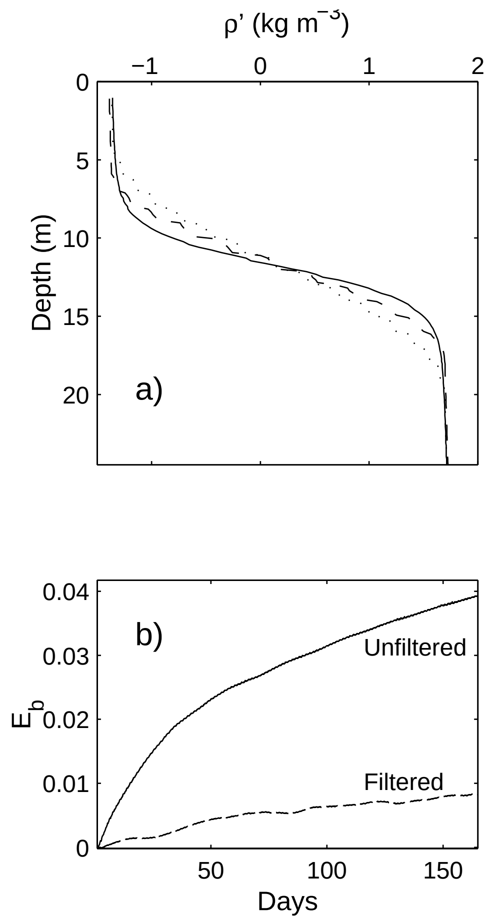

Figure 5: (a) Comparison of adiabatically relaxed density profiles for a modeled basin-scale internal seiche after 180 days of simulation using a 15 min time step: initial profile —— ; final profile with no filtering applied ⋯⋯ ; final profile with filtering applied - - - - . (b) Change in background potential energy (E_b) from initial state, normalized by potential energy change required to completely mix the stratification (after Laval et al., 2000).

during advection as shown in Figure 5a. The filtering technique has been tested through simulation of a forced basin-scale internal seiche. The turbulent mixing processes are identically zero so numerical diffusion is the only means of altering the adiabatically relaxed density profile from its initial state. Figure 5b demonstrates the effectiveness of the method at reducing the numerical diffusion that results in a cumulative increase in the background potential energy.

3.4 Net transport

The primary difficulty in predicting transport by internal waves is that net transport is an order of magnitude smaller than the wave-induced motions. For example, in Lake Kinneret, Israel, the boundary Kelvin waves cause a back-and-forth motion along the boundaries with velocities of 0.2 to 0.5 ms^{-1}, (Antenucci, 1999) while the net transport flux around the edge of the lake, due to nonlinear topographical and finite-amplitude effects, is modeled at 0.02 to 0.04 ms^{-1}. Comparisons of field and model transport data are encouraging as we appear to capture the correct magnitude and direction for the critical area of transport at the intersection of the hypolimnion and metalimnion. This is an area where more work is underway in order to quantify our ability to model the cumulative transport over seasonal time scales. It is likely that the accuracy of our results will be dependent on developing better closure schemes for unresolved effects.

4 UNRESOLVED EFFECTS

4.1 Introduction

Our past efforts have been directed at modeling the first-order energy cascade from the environment into the internal wave field and the resulting large-scale evolution of the transport field. The next level of refinement is developing models for the processes that are not resolved due to: (1) their temporal or spatial scale relative to the numerical grid and time step, (2) neglect of fundamental processes in the model equations, or (3) difficulties in numerically capturing the effect.

4.2 Non-hydrostatic evolution of internal waves

Analyses of internal wave frequency spectra for the metalimnion typically have peaks in the range of periods between around 2 to 4 minutes as shown in Figure 6. These waves mostly occur in groups as free, short-period internal waves and have typical length scales between about 70 and 100 meters. Their generation from nonlinear steepening (Horn et al., 2000) and shoaling (Thorpe, 1997) has been investigated, however there also seems to be a direct forcing by the wind through a mechanism that has yet to be identified (Antenucci and Imberger, 2000). Although the energy content of short-period waves is smaller than the basin-scale waves, they are presumed to be important as a trigger for internal turbulence and as a mechanism for

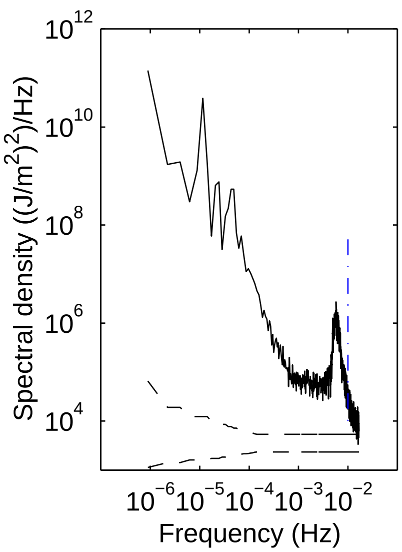

Figure 6: Potential energy displacement spectra for Lake Kinneret (Antenucci, 1998). Confidence interval - - - -; buoyancy frequency - · - · -; Spectra shows energy spike below the buoyancy frequency consistent with non-hydrostatic internal waves.

energy and momentum transport to the lake boundary. Two major challenges need to be addressed to adequately capture the short-period waves in practicable lake models: the non-hydrostatic behavior of their generation and their sub-grid length scales.

Lake models typically apply the hydrostatic assumption, which can be stated variously as a neglect of dynamic pressure, a neglect of vertical acceleration or that horizontal length scales are much larger than vertical length scales. This assumption becomes problematic when dealing with internal waves. The processes by which basin-scale internal waves evolve into trains of smaller-scale internal waves are fundamentally non-hydrostatic, depending on the balance between nonlinear steepening and non-hydrostatic dispersion. This has been demonstrated in recent laboratory work of Horn et al. (2000) which examines the scales at which linear waves evolved into trains of solitons (see Figure 7).

In a hydrostatic model, internal waves steepen until nonlinear steepening balances numerical dissipation and diffusion (which brings the interesting observation that a hydrostatic model *without* numerical diffusion will have internal waves that steepen until they break - introducing a form of numerical diffusion into a supposedly diffusion-free model). Modeling hydrostatic internal waves has several consequences: (1) numerical dissipation of momentum is accompanied by numerical diffusion of the density structure so the internal wave has a trail of numerical mixing; (2) the steepness of waves becomes a function of numerical dissipation and diffusion, rather than physics, and is therefore highly dependent on grid resolution; and (3) the amplitude of the modeled wave is damped faster than physical processes transform energy into smaller scales.

The non-hydrostatic equations can be formulated and solved numerically (e.g. Mahadevan et al., 1996), but there is some question as to whether we can actually resolve non-hydrostatic effects when the grid aspect ratio is often 100:1 or greater for practical lake models. Discretization on high aspect ratio grids implies the non-hydrostatic effects will be small and may be dominated by truncation error. Non-hydrostatic processes occur with horizontal and vertical length scales of similar orders of magnitude, requiring a grid scale that is similar in the horizontal and vertical directions to accurately resolve the effects. Indeed, the primary purpose of the non-hydrostatic mesoscale model of Mahadevan and Street (1996) was to solve problems in the ill-posedness of open boundary conditions in hydrostatic models rather than capturing the non-hydrostatic processes.

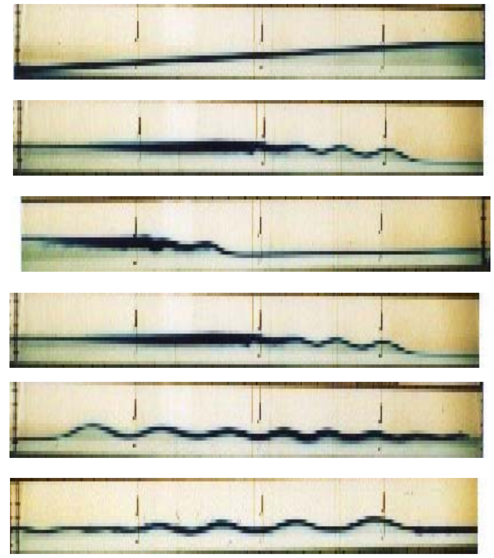

Figure 7: Time-series of the evolution of a 2-layer interface in a laboratory experiment showing degeneration of basin-scale internal waves into non-hydrostatic wave trains. Digitized photographs from experiments by Horn et al. (2000) with vertical scale stretched by 300% for visualization.

It follows that non-hydrostatic internal waves are a closure problem for geophysical numerical models (Horn et al., 1999). The processes themselves are not resolved by the governing equations and the effects are dependent on processes that are fundamentally sub-grid scale. This is a basic challenge that has not been addressed in any existing turbulence closure model. A method is needed to remove energy from resolved internal waves, then store, transport and dissipate this energy in a framework which accounts for the propagation of numerically unresolvable waves.

4.3 Phase interactions of internal waves

A series of scalar transport studies using ELCOM (Hodges et al., 2000) to model Lake Kinneret in 3D showed surprising differences for transport in the metalimnion depending on whether a scalar is introduced into the western or eastern boundary as shown in Figure 8. The transport patterns show characteristics of chaotic behavior, leading to a hypothesis that phase interactions between the various internal waves leads to a dynamically chaotic system. If this hypothesis is correct, then a new challenge is faced in numerical modeling: a chaotic system depends on the interactions of wave phases, which are arguably the most difficult aspect to quantitatively capture in any discrete model of the Navier-Stokes equations. Indeed, careful examination of Figure 4 shows the model of Lake Kinneret produced a slight phase lag in the Kelvin wave trough at station T5. However, the prospect of describing a system by chaos theory has its own benefits: if one can describe the evolution of the saddle points and attractors in a lake as a function of stratification, wind forcing and topography, then perhaps a simpler model of dynamical transport can be substituted for the deterministic transport equations (Boffetta et al., 2000). If this proves possible, then the evolution of net transport over seasonal to decadal time scales may be posed as a problem in understanding the evolution of the nodal points that describe chaos.

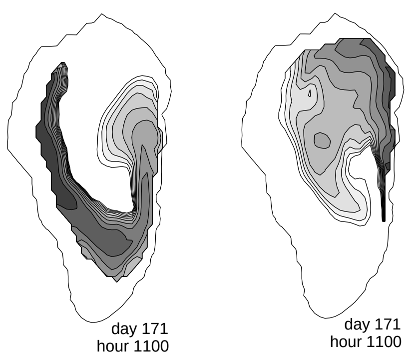

Figure 8: Plan view of modeled scalar transport by internal waves in Lake Kinneret. Contours represent meta-limnion concentrations of passive scalars along the 21 ºC isosurface of temperature after 7 days of simulation with a 7.5 minute time step: Left: tracer introduced on western boundary advected alongshore by Kelvin wave. Right: tracer introduced along eastern boundary advected cross-shore by Poincaré wave.

4.4 Hypolimnetic benthic boundary layer

The basin-scale seiching of the metalimnion sets up an oscillating benthic boundary layer in the hypolimnion. From the perspective of the hydrodynamic model, the boundary layer provides drag that slowly dissipates the large-scale internal wave motions. However, the majority of momentum dissipation occurs in the shallow regions affected by the sloshing metalimnion, so accurate resolution of the drag in the hypolimnion is of secondary importance in modeling basin-scale internal wave dynamics. Unfortunately, being of secondary importance in the physics of large-scale motions does not mean secondary importance in transport processes for biogeochemical state variables. Indeed, the interaction of the benthic boundary layer with the mass transport due to the Kelvin waves may be a critical component in the transport into the metalimnion of nutrients deposited in the benthos of the hypolimnion.

There are two challenges faced in modeling the hypolimnion benthic boundary layer: (1) computing the effective resuspension rate of sediments; and (2) computing the net transport along the boundary. A resuspension model should take into account the inaccuracies of the modeled large-scale velocity field in the hypolimnion, characterized by low velocities, large grid scales, and velocity reversals on the time scale of the first mode internal waves period.

The net transport in the benthic boundary layer is probably a greater challenge as it is a near-boundary effect resulting from the mismatch in the transport on opposite oscillations of the boundary layer as it is seiches with the basin-scale internal waves. We have little faith in a net flux computed from the resolved small velocities computed in the hypolimnion as they are near the numerical truncation error of the larger velocities in the surface layer and metalimnion. Arguably, the resolved net flux may be a function of the net truncation error with directional bias caused by numerical algorithm implementation. What is needed is a method that uses information on the basin-scale internal wave motions in the water column (i.e. the forcing at the boundary due to the slope of the metalimnion and the periodicity of the motion) along with the boundary slope, composition and roughness length, to compute a sub-grid scale flux along the boundary. It appears that significantly more field, laboratory and numerical research is required to obtain the robust model formulation that is necessary. The critical point is that basin-scale hydrodynamics can be modeled adequately without a sophisticated benthic boundary layer model, but the biogeochemical transport in the near-boundary region may be incorrect.

4.5 Metalimnetic boundary mixing

Along the sloping boundaries of a lake, the metalimnion seiching induces sloshing in a manner similar to, but slower than, surface waves on a beach. Laboratory experiments (e.g. Ivey et al., 1999) have shown that significant mixing can occur as a result of these motions; effects which are amplified by shorter wavelength (i.e. sub-grid scale) internal waves which readily break on a sloping boundary. Field data suggests that this mixing plays a role in the nutrient fluxes into the metalimnion (Ostrovsky et al., 1996).

It appears that mixing effects along the boundaries may dominate the mass flux between the hypolimnion and metalimnion. The processes at the juncture of the metalimnion and the benthos are more energetic than those in the hypolimnion boundary layer, but numerical modeling is not any easier. Because of the slope of the boundary, the vertical velocity near the boundary becomes important. In a Cartesian mesh, the slope has a "stair-step" representation which dominates the resolved near-boundary flow. While boundary-conforming grids (i.e. the 'σ-method') eliminate this problem, they have a tendency to distort baroclinic processes - adding a different type of error to the boundary flow field. Whichever grid method is used, the fundamental problem remains the excessive computational cost of a grid fine enough to resolve the small-scale processes along a sloping boundary. Again, what is needed is a subgrid-scale model applying large-scale velocities, stratification and internal wave propagation at the boundary to predict the correct sub-grid scale mixing.

5 CONCLUSIONS

Numerical modeling of stratified lakes in 3D is advancing rapidly and has exposed the inadequacies of numerical methods designed for estuaries. Using the numerical techniques and computational power that are presently available, the fundamental basin-scale motions can be modeled with reasonable accuracy over weekly to monthly time scales. Unfortunately, this does not mean that longer-term simulations can be conducted by simply applying more computer power. Accurate simulation of seasonal evolution of the thermal structure rests on resolution of two numerical issues: (1) control of numerical diffusion, and (2) modeling of the unresolved turbulent processes that lead to mixing. The former issue we are approaching by a filtering technique that reverses numerical diffusion. The latter area still requires a significant amount of integrated field, laboratory and numerical work to parameterize the interactions of non-hydrostatic internal waves, basin-scale internal waves and the turbulent benthic boundary layer where mixing primarily occurs.

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REFERENCES

Antenucci, J., *Lake Kinneret Data Report, Experiment 98177,* Centre for Water Research Technical Report WP1473, The University of Western Australia, 1998.

Antenucci, J. and J. Imberger, *Limnological Modeling of Lake Burragorang: Final Report* Centre for Water Research Technical Report WP11462JA, The University of Western Australia, 1998.

Antenucci, J. and J. Imberger, "On long internal waves near the high frequency limit in an enclosed basin," Centre for Water Research Manuscript ED1427JA, The University of Western Australia, 2000.

Antenucci, J., J. Imberger and A. Saggio, "Seasonal evolution of the basin-scale internal wave field in a large stratified lake," *Limno. Oceanogr.,* (in press), 2000.

Beletsky, D., W.P. O'Connor, D.J. Schwab, and D.E. Dietrich, "Numerical simulation of internal Kelvin waves and coastal upwelling fronts," *J. Phys. Oceanogr.* **27**, pp. 1197-1215, 1997.

Boffetta, G., M. Cencini, S. Espa, and G. Querzoli, "Chaotic advection and relative dispersion in an experimental convective flow," (draft manuscript), Università di Torino, Italy, 2000.

Casulli, V., "Numerical simulation of three-dimensional free surface flow in isopycnal co-ordinates," *J. Num. Meth. Fluids*, **25**, pp. 645-658, 1997.

Gill, A.E. Atmosphere-Ocean Dynamics, Academic Press, 662 pp., 1982.

Griffies, S.M., R.C. Pacanowski, and R.W. Hallberg, "Spurious diapycnal mixing associated with advection in a z-coordinate ocean model," *Mon. Weather. Rev*. (in press), 2000.

Herzfeld, M. and D.P. Hamilton, "A computational aquatic ecosystem model of the Swan River," in *Proceedings, International conference on Modelling and Simulation 8-11 December 1997*, Eds: A.D. McDonald and M. McAleer, University of Tasmania, Hobart, Vol. II, pp. 663-668, 1997.

Hodges, B.R. "Recirculation and equilibrium displacement of the thermocline in a wind-driven stratified lake," *Fifth International Symposium on Stratified Flows*, Vancouver, Canada, July 2000.

Hodges, B.R., and J. Imberger, "A curvilinear perturbation form of the Navier-Stokes equations," Centre for Water Research manuscript ED1120BH, University of Western Australia, 2000.

Hodges, B.R., J. Imberger, A. Saggio, and K.B. Winters, "Modeling basin-scale internal waves in a stratified lake," *Limno. Oceanogr*., (in press), 2000.

Horn, D.A., J. Imberger, and G.N. Ivey, "Internal solitary waves in lakes - a closure problem for hydrostatic models," Proceedings of the 11th 'Aha Haliko'a Hawaiian Winter Workshop, Jan 19-22, University of Hawaii, Manoa, 1999.

Horn, D.A., J. Imberger, and G.N. Ivey, "The degeneration of large-scale interfacial waves in lakes" *submitted to J. Fluid Mech.,* 2000.

Imberger, J., "Flux paths in a stratified lake: A review," in J. Imberger, editor, *Physical Processes in Lakes and Oceans,* Coastal and Estuarine Vol. 54, American Geophysical Union, pp. 1-18, 1998.

Imberger, J. and J.C. Patterson, "A dynamic reservoir simulation model - DYRESM 5." in H. Fischer, editor, *Transport Models for Inland and Coastal Waters*, Academic Press, pp. 310-361, 1981.

Ivey, G., K. Winters, and I. DeSilva, "Turbulent mixing in an internal wave energized benthic boundary layer," *submitted to J. Fluid Mech.*, 1999.

Jha, A., J. Romero, and J. Imberger, *Limnological study of Lake Burragorang, Final Report*, Centre for Water Research Technical Report WP1374AJ, The University of Western Australia, 1999.

Kraus, E.B., and J.S. Turner, "A one-dimensional model of the seasonal thermocline: II. The general theory and its consequences," Tellus, **19**, pp. 98-106, 1967.

Laval, B., B.R. Hodges and J. Imberger, "Vertical transport in large stratified lakes," *Fifth International Symposium on Stratified Flows*, Vancouver, Canada, July 2000.

Lemckert, C.J. and J. Imberger, "Turbulence in the benthic boundary layer," in J. Imberger, editor, *Physical Processes in Lakes and Oceans*, Coastal and Estuarine Studies Volume 54, 503-516., American Geophysical Union (1998).

Leonard, B.P., "The ULTIMATE conservative difference scheme applied to unsteady one-dimensional advection," *Comp. Meth. Appl. Mech. Eng*., **88**, pp. 17-74, 1991.

McCormick, M.J. and G.A. Meadows, "An intercomparison of four mixed layer models in a shallow inland sea," *J. Geo. Res.*, **93**, pp. 6774-6788, 1988.

Mahadevan, A., J. Oliger, R.L. Street, "A nonhydrostatic mesoscale ocean model. Part I: well-posedness and scaling," *J. Phys. Ocean.*, **26**, pp. 1868-1880, 1996.

Meakin, R.L., and R.L. Street, "Simulation of environmental flow problems in geometrically complex domains. Part I: A general coordinate transformation," *Comp. Meth. in Appl. Mech. and Eng.*, **68,** pp. 151-175, 1988.

Nadaoka, K., T. Yoshino and Y. Nihei, "Oil spill simulation with improved coastal ocean and wind models," Int. Conf. Hydro Dynamics 2000, Japan, Sept. 2000.

Ostrovsky, I., Y.Z. Yacobi, P. Walline, and I. Kalikhman, "Seiche-induced mixing: Its impact on lake productivity," *Limno. Oceanogr.*, **41**, pp. 323-332, 1996.

Pan, H., R. Avissar, and D.B. Haidvogel, "Three-dimensional numerical modeling of summer water motions and temperature structure in Lake Kinneret," *Eos Transactions,* **79**(1) Ocean Sciences Meeting Supplement, OS120-7, American Geophysical Union, 1998.

Rodi, W. "Examples of calculation methods for flow and mixing in stratified flow," *J. Geo. Res.*, **92**, pp. 5305-5328, (1987).

Schwab, D.J. and K.W. Bedford, "Operational Three Dimensional Circulation Modeling in the Great Lakes," in *Computer Modelling of Seas and Coastal Regions,* Comp. Mech. Pubs., Boston, pp. 387-395. 1995.

Skyllingstad, E.D., W.D. Smyth, J.N. Moum, and H. Wijesekera, "Upper-ocean turbulence during a westerly wind burst: A comparison of large-eddy simulation results and microstructure measurements," J. Phys, Ocean., **29**, pp. 5-28, 1999.

Stephens, R. and J. Imberger, "Intertidal motions within the deep basin of the Swan River Estuary. *J. Hydr. Eng.*, ASCE **123,** pp. 863-873, 1997.

Thorpe, S.A., "Some dynamical effects of internal waves and the sloping sides of lakes," in J. Imberger, editor, *Physical Processes in Lakes and Oceans,* Coastal and Estuarine Vol. 54, American Geophysical Union, pp. 1-18, 1998.

Thorpe, S.A., "On the interactions of internal waves reflecting from slopes," *J. Phys. Ocean.* 27, pp. 2072-2078, 1997.

Umlauf, L., Y. Wang, and K. Hutter, "Comparing two topography-following primitive equation models for lake circulation," *J. Comp. Phys.*, **153**, pp. 638-659, 1999.

Winters, K.B., P.N. Lombard, J.J. Riley, and E.A. D'Asaro, "Available potential energy and mixing in density-stratified fluids," *J. Fluid Mech*., **298**, pp. 115-128, 1995.