Elementary Mechanics of Fluids

Description of Motion
Fluid Motion

- Two ways to describe fluid motion
  - Lagrangian
    - Follow particles around
  - Eularian
    - Watch fluid pass by a point or an entire region
  - Flow pattern
    - Streamlines – velocity is tangent to them

\[
V = \frac{dx}{dt} i + \frac{dy}{dt} j + \frac{dz}{dt} k
\]

\[
V = ui + vj + wk
\]
Flow Patterns

- Uniform flow \( \frac{\partial V}{\partial s} = 0 \)
- Non-uniform flow \( \frac{\partial V}{\partial s} \neq 0 \)
- Steady flow \( \frac{\partial V}{\partial t} = 0 \)
- Unsteady flow \( \frac{\partial V}{\partial t} \neq 0 \)
Example (4.1)

- Valve at C is opened slowly
- Classify the flow at B while valve is opened
- Classify the flow at A
HW (4.2)
Laminar vs Turbulent Flow

- Laminar

- Turbulent
Flow Rate

- **Volume rate of flow**
  - Constant velocity over cross-section
    \[ Q = VA \]
  - Variable velocity
    \[ Q = \int VdA \]

- **Mass flow rate**
  \[ \dot{m} = \int \rho VdA = \rho \int VdA = \rho Q \]
Examples

• Prob. 4.17
Discharge in a 2-cm pipe is 0.03 m³/s. What is the average velocity?

\[ Q = VA \]
\[ V = \frac{Q}{A} = \frac{\pi}{4} \left( \frac{d}{2} \right)^2 = \frac{0.03}{\pi (0.25)^2} = 0.611 \text{ m/s} \]

• Prob. 4.20
A pipe whose diameter is 8 cm transports air with a temp. of 20°C and pressure of 200 kPa abs. At 20 m/s. What is the mass flow rate?

\[ \rho = \frac{p}{RT} = \frac{200000}{287 \times 293} = 2.378 \text{ kg/m}^3 \]
\[ \dot{m} = \rho VA \]
\[ = 2.378 \times 20 \times \frac{\pi}{4} (0.08)^2 = 0.239 \text{ kg/s} \]
Flow Rate

- Only $x$-direction component of velocity ($u$) contributes to flow through cross-section

$$Q = \int VdA = \int udA = \int V \cos \theta dA$$

or

$$Q = \int V \cdot dA$$

or

$$Q = V \cdot A$$
Example (4.24)

- Find: \[ \frac{V}{V_o} \]

\[ Q = \int VdA = \int_0^R V_o (1 - r/R) 2\pi r dr \]

\[ = 2\pi V_o \left( \frac{r^3}{2} - \frac{r^3}{3R} \right) \bigg|_0^R = 2\pi V_o \left( \frac{R^2}{2} - \frac{R^2}{3} \right) \]

\[ = \frac{1}{3} \pi V_o R^2 \]

\[ \frac{V}{V_o} = \frac{Q}{A} = \frac{1}{3} \pi V_o R^2 \frac{1}{\pi R^2 V_o} = \frac{1}{3} \]
Example (4.28)

- Find: \( Q, \bar{V}, \dot{m} \)

\[
Q = 2 \int V dA = 2 \int_{0}^{0.5} 20 y dy
\]

\[
= 40 \left[ \frac{y^2}{2} \right]_0^{0.5} = 5 \text{ m}^3 / \text{s}
\]

\[
\bar{V} = \frac{Q}{A} = \frac{5}{1} = 5 \text{ m/s}
\]

\[
\dot{m} = \rho Q = 1.2 \times 5 = 6 \text{ kg/s}
\]
Vertical depth = 1 m

$u = y^{1/3}$ m/s

$30^\circ$
Acceleration

- Acceleration = rate of change of velocity
- Components:
  - Normal – changing direction
  - Tangential – changing speed

\[ \vec{V} = V(s, t) \hat{e}_t \]

\[ \ddot{a} = \frac{d\vec{V}}{dt} = \frac{dV}{dt} \hat{e}_t + V \frac{d\hat{e}_t}{dt} \]

\[ \frac{dV}{dt} = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \]

\[ \frac{d\hat{e}_t}{dt} = \frac{V}{r} \hat{e}_n \]

\[ \ddot{a} = (V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}) \hat{e}_t + \frac{V^2}{r} \hat{e}_n \]
Acceleration

- Cartesian coordinates
  \[ \vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \]
  \[ \vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \]

  \[ a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt} + \frac{\partial u}{\partial t} \]
  \[ a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x} \frac{dx}{dt} + \frac{\partial v}{\partial y} \frac{dy}{dt} + \frac{\partial v}{\partial z} \frac{dz}{dt} + \frac{\partial v}{\partial t} \]
  \[ a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} + \frac{\partial w}{\partial t} \]

  \[ \begin{align*}
  \text{Convective} & = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \\
  \text{Local} & = \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t}
  \end{align*} \]

- HW (4.43)
Example (4.49)

\[ Q = Q_o - Q_1 \frac{t}{t_o} = 0.985 - 0.5t \]

\[ \frac{\partial V}{\partial s} = 2 \text{ m/s} \]

\[ V = \frac{Q}{A} = \frac{Q_o - Q_1 \frac{t}{t_o}}{\frac{\pi}{4} d^2} = \frac{0.985 - 0.5(0.5)}{\frac{\pi}{4} (0.5)^2} = 3.4743 \text{ m/s} \]

\[ a_L = \frac{\partial V}{\partial t} = \frac{\partial (Q/A)}{\partial t} = -\frac{Q_1}{\frac{\pi}{4} d^2 t_o} = -\frac{0.5}{\frac{\pi}{4} (0.5)^2} = -2.55 \text{ m/s}^2 \]

\[ a_C = V \frac{\partial V}{\partial s} = 3.743 \times 2 = 7.49 \text{ m/s}^2 \]
HW (4.50 & 4.51)
Example

Given: \( \vec{V} = 3t\vec{i} + xz\vec{j} + ty^2\vec{k} \)

Find: Acceleration, \( \vec{a} \)

\( u = 3t; \quad v = xz; \quad w = ty^2 \)

\[
a_x = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t} = 0(3t) + 0(xz) + 0(ty^2) + 3 = 3
\]

\[
a_y = \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t} = z(3t) + 0(xz) + x(ty^2) + 0 = 3zt + xy^2 t
\]

\[
a_z = \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w + \frac{\partial w}{\partial t} = 0(3t) + 2ty(xz) + 0(ty^2) + y^2 = 2xyzt + y^2
\]

\[
\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} = 3\vec{i} + (3zt + txy^2)\vec{j} + (2xyzt + y^2)\vec{k}
\]