Elementary Mechanics of Fluids

Hydrostatic Forces on Plane Surfaces
Pressure on Plane Surface

Surfaces exposed to fluids experience a force due to the pressure distribution in the fluid.

\[ F = \int p dA \]
\[ = \int \gamma y \sin \alpha dA \]
\[ = \gamma \sin \alpha \int y dA \]

\[ F = (\bar{y} \gamma \sin \alpha) A \]
\[ F = \bar{p} A \]
Example

\[ F = \bar{p}A \]
\[ = (\gamma \bar{y} \sin \alpha) A \]
\[ = (23,600 \times 1.22 \times 1) \times (1.22 \times 2.44) \]
\[ F = 85.8 \text{kN} \]
Line of Action of Force

• Lies below centroid, since pressure increases with depth

\[ y_{cp} F = \int y dF \]
\[ = \int y (p dA) \quad \frac{A}{A} \]
\[ = \int y (\gamma y \sin \alpha) dA \quad \frac{A}{A} \]
\[ y_{cp} (\gamma y \sin \alpha A) = \gamma \sin \alpha I_0 \]
\[ y_{cp} (\bar{y} A) = \bar{I} + \bar{y}^2 A \]
\[ y_{cp} = \bar{y} + \frac{I}{\bar{y} A} \]
Example (3.59)

- $F \uparrow$ as $H \uparrow$?
- $y_{cp} - \bar{y} \downarrow$ as $H \uparrow$?
- $y_{cp} - \bar{y}$ is constant as $H \uparrow$?
- $T \uparrow$ as $H \uparrow$?
- $T$ is constant as $H \uparrow$?
\[
\begin{align*}
A &= \frac{bh}{2} \\
\bar{I}_{xx} &= \frac{bh^3}{36} \\
A &= \frac{\pi r^2}{2} \\
\bar{I}_{xx} &= 0.110r^4 \\
A &= \frac{\pi r^2}{4} \\
\bar{I}_{xx} &= \frac{\pi r^4}{4} \\
A &= 2.5981L^2 \\
\bar{I}_{x} &= 0.5127L^4 \\
A &= \pi ab \\
\bar{I}_{xx} &= \frac{\pi a^4b}{4}
\end{align*}
\]
Example (3.73)

**Find:** Force of block on gate

\[ F = \bar{p}A \]
\[ = (\bar{y}\sin \alpha)A \]
\[ = (9810 \times 10 \times 1) \times (4 \times 4) \]
\[ = 4 \times 4^3 / 12 \]
\[ = 0.133 \ m \]

\[ y_{cp} - \bar{y} = \frac{I}{\bar{y}A} \]
\[ \sum M = 0 \]
\[ = 0.133F_{w,g} - 2F_{b,g} \]
\[ F_{b,g} = \frac{0.133}{2} F_{w,g} \]
\[ = \frac{0.133}{2} 1569.6 \ kN \]
\[ F_{b,g} = 104.378 \ kN \]
Example (3.78)

\[ F = \bar{p}A = (\gamma \bar{y} \sin \alpha)A \]
\[ = 9810 \times (3 + 3 \cos 30) \times (4 \times 6) \]
\[ = 1,318,000 \text{ N} \]

\[ y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{4 \times 6^3 / 12}{(6.464 \times 24)} \]
\[ = 0.4641 \text{ m} \]

\[ \sum M = 0 \]
\[ = 6R_A - (3 - 0.4641)F \]

\[ R_A = \frac{3 - 0.4641}{6}F \]
\[ = (0.42265)1318 \text{ kN} \]
\[ R_A = 557.05 \text{ kN} \]
HW (3.92)

Water

$T = 20^\circ C$

View D-D

View E-E

$d$

$H$

$W$
Example

Given: Gate AB is 4 ft wide, hinged at A. Gage G reads -2.17 psi
Find: Horizontal force at B to hold gate.
Solution:

\[ F_{oil} = \bar{p}A \]
\[ = 0.75 \times 62.4 \times 3 \times (4 \times 6) \]
\[ = 3,370 \text{lbf} \]

\[ y_{cp} - \bar{y} = \frac{I}{yA} \]
\[ = \frac{4 \times 6^3}{12} / (3 \times 24) = 1 \text{ ft} \]

Convert negative pressure in tank to ft of water

\[ h = \frac{p}{\gamma} = \frac{-2.17 \times 144}{62.4} = -5.01 \text{ ft} \]
Example

\[ F_w = \bar{p}A \]
\[ = 62.4 \times (15 - 5.01) \times (4 \times 6) \]
\[ = 15,000 \text{ lbf} \]

\[ y_{cp} - \bar{y} = \frac{I}{\bar{y}A} \]
\[ = \frac{4 \times 6^3 / 12}{(15 - 5.01) \times 24} = 0.3 \text{ ft} \]

\[ \sum M_A = 0 \]
\[ = F_w \times 3.3 - F_{oil} \times 4 - F_B \times 6 \]
\[ = 15000 \times 3.3 - 3700 \times 4 - F_B \times 6 \]
\[ F_B = 6000 \text{ lbf} \]
HW (3.96)