Elementary Mechanics of Fluids

Viscosity
Some Simple Flows

- Flow between a fixed and a moving plate

Fluid in contact with the plate has the same velocity as the plate

\[ u = x\text{-direction component of velocity} \]

\[ u(y) = \frac{V}{B} y \]

\[ u = V \]

\[ u = 0 \]
Some Simple Flows

• Flow through a long, straight pipe
  Fluid in contact with the pipe wall has the same velocity as the wall
  \( u = x \)-direction component of velocity
Fluid Deformation

- Flow between a fixed and a moving plate
- Force causes plate to move with velocity $V$ and the fluid deforms continuously.
Fluid Deformation

Shear stress on the plate is proportional to deformation rate of the fluid

\[ \tau \propto \frac{\delta \alpha}{\delta t} \]

\[ \delta \alpha = \frac{\delta L}{\delta y} \quad \delta t = \frac{\delta L}{\delta V} \quad \frac{\delta \alpha}{\delta t} = \frac{\delta V}{\delta y} \quad \tau \propto \frac{\delta V}{\delta y} \]
Shear in Different Fluids

- Shear-stress relations for different types of fluids
- Newtonian fluids: linear relationship
- Slope of line (coefficient of proportionality) is “viscosity”

\[
\tau \propto \frac{dV}{dy}
\]

\[
\tau = \mu \frac{dV}{dy}
\]
Viscosity

- Newton’s Law of Viscosity \( \tau = \mu \frac{dV}{dy} \)

- Viscosity \( \mu = \frac{\tau}{dV/dy} \)

- Units \( \frac{N/m^2}{m/s/m} = \frac{N\cdot s}{m^2} \)

- Water (@ 20°C)
  - \( \mu = 1 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \)

- Air (@ 20°C)
  - \( \mu = 1.8 \times 10^{-5} \text{ N}\cdot\text{s/m}^2 \)

- Kinematic viscosity \( \nu = \frac{\mu}{\rho} \)
Flow between 2 plates

Force is same on top and bottom

\[ F_1 = \tau_1 A_1 = \tau_2 A_2 = F_2 \]
\[ A_1 = A_2 \]
\[ \tau_1 = \tau_2 \]

Thus, slope of velocity profile is constant and velocity profile is a straight line

\[ \tau_1 = \mu \frac{du}{dy} \bigg|_1 = \mu \frac{du}{dy} \bigg|_2 = \tau_2 \]
Flow between 2 plates

Shear stress anywhere between plates

\[ \tau = \mu \frac{du}{dy} = \mu \frac{V}{B} \]

\[ \mu = 0.1 \text{ N} \cdot \text{s} / \text{m}^2 \text{ (SAE 30 @ 38° C)} \]
\[ V = 3 \text{ m/s} \]
\[ B = 0.02 \text{ m} \]

\[ \tau = (0.1 \text{ N} \cdot \text{s} / \text{m}^2) \left( \frac{3 \text{ m/s}}{0.02 \text{ m}} \right) = 15 \text{ N} / \text{m}^2 \]
Flow between 2 plates

- 2 different coordinate systems

\[ u(r) = V \left[ 1 - \left( \frac{r}{B} \right)^2 \right] \]

\[ u(y) = C[y(B - y)] \]
Example: Journal Bearing

• Given
  – Rotation rate, $\omega = 1500$ rpm
  – $d = 6$ cm
  – $l = 40$ cm
  – $D = 6.02$ cm
  – $SG_{oil} = 0.88$
  – $\nu_{oil} = 0.003$ m$^2$/s

• Find: Torque and Power required to turn the bearing at the indicated speed.
Example: cont.

• Assume: Linear velocity profile in oil film

Shear Stress \[ \tau = \mu \frac{dV}{dy} = \mu \frac{\omega (d/2)}{(D-d)/2} \]

\[ = (0.88 \times 998 \times 0.003) \frac{(2\pi \times 1500)(0.06/2)}{(0.0002)/2} = 124 \text{ kN} / \text{m}^2 \]

Torque \[ M = (2\pi \tau \frac{d}{2} \frac{d}{2}) \]

\[ = (2\pi \times 124,000 \times 0.06 \times 0.4 \times 0.06) = 281 \text{ N} \cdot \text{m} \]

Power \[ P = M\omega = 281 \times 157.1 = 44,100 \text{ N} \cdot \text{m} / \text{s} = 44.1 \text{ kW} \]
Example: Rotating Disk

- Assume linear velocity profile: \( \frac{dV}{dy} = \frac{V}{y} = \frac{\omega r}{y} \)
- Find shear stress