Problem 1. What gets printed?

```c
main( )
{
    int i = 1, m = 0;
    int f(int j);

    m = f(i);
    printf("m = %i \n", m);
}

int f(int i)
{
    while (i <= 5)
    {
        i = i + 1;
        printf("i = %i \n", i);
    }
    i = i + 1;
    return(i);
}
```

Problem 2.

<table>
<thead>
<tr>
<th>Question</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A C program must have a function called “main”.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>b. A calling function must pass arguments to the called function.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>c. Every function returns a value to its calling function.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>d. Every function must be defined in its own separate file.</td>
<td>X</td>
<td></td>
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<tr>
<td>e. Interpolation is used to estimate the relations between noisy data.</td>
<td>X</td>
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<tr>
<td>f. Newton Divided Difference and Lagrange functions are two forms of interpolation functions.</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
Problem 3. Starting from an initial guess \( x_0 = 2.5 \), take 2 iterations of Newton’s method to find the minimum value of the function: (not minimizing: -3; wrong method: -10)

\[
f(x) = 2 \sin x - \frac{x^2}{8}
\]

To find min/max, take the derivative and set it equal to zero:

\[
g(x) = f'(x) = 2 \cos x - \frac{x}{4} = 0 \quad \text{and} \quad g'(x) = -2 \sin x - \frac{1}{4}
\]

Now apply Newton’s method to find the root of this function

\[
x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} = x_i - \frac{2 \cos x_i - \frac{x_i}{4}}{-2 \sin x_i - \frac{1}{4}}
\]

where (as Dustin Wiggins pointed out it is a “Maximum” not a “Minimum”!)
(24 pts.) Problem 5. Start from an initial guess of \( x_1^0 = x_2^0 = x_3^0 = 1.0 \), show the first 2 iterations of the Gauss Seidel method for the solution of this system of equations. Compute the error after the second iteration. (Gauss Elimination = -15)

\[
\begin{align*}
3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\
0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\
0.3x_1 - 0.2x_2 + 10x_3 &= 71.4
\end{align*}
\]

Gauss Seidel method gives:

\[
\begin{align*}
x_1^{k+1} &= \frac{7.85 + 0.1x_2^k + 0.2x_3^k}{3} \\
x_2^{k+1} &= -\frac{19.3 - 0.1x_1^{k+1} + 0.3x_3^k}{7} \\
x_3^{k+1} &= \frac{71.4 - 0.3x_1^{k+1} + 0.2x_2^{k+1}}{10}
\end{align*}
\]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gauss-Seidel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>2</td>
<td>( x_1 )</td>
<td>( x_2 )</td>
<td>( x_3 )</td>
<td>( x_1^* )</td>
<td>( x_2^* )</td>
<td>( x_3^* )</td>
<td>( \text{err1} )</td>
<td>( \text{err2} )</td>
<td>( \text{err3} )</td>
</tr>
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<td>1.0000</td>
<td>1.0000</td>
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<td>-2.7531</td>
<td>7.0034</td>
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<td>136.3228%</td>
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<tr>
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<td>2.7167</td>
<td>-2.7531</td>
<td>7.0034</td>
<td>2.9913</td>
<td>-2.4997</td>
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<td>9.1968%</td>
<td>10.1355%</td>
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<td>2</td>
<td>2.9913</td>
<td>-2.4997</td>
<td>7.0003</td>
<td>3.0000</td>
<td>-2.5000</td>
<td>7.0000</td>
<td>0.2744%</td>
<td>0.0102%</td>
</tr>
<tr>
<td>6</td>
<td>3.0000</td>
<td>-2.5000</td>
<td>7.0000</td>
<td>3.0000</td>
<td>-2.5000</td>
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<td>0.0000%</td>
<td>0.0004%</td>
<td>0.0000%</td>
</tr>
<tr>
<td>7</td>
<td>3.0000</td>
<td>-2.5000</td>
<td>7.0000</td>
<td>3.0000</td>
<td>-2.5000</td>
<td>7.0000</td>
<td>0.0000%</td>
<td>0.0003%</td>
<td>0.0000%</td>
</tr>
</tbody>
</table>

First iteration

\[
\begin{align*}
x_1^{k+1} &= \frac{7.85 + 0.1(1) + 0.2(1)}{3} = 2.72 \\
x_2^{k+1} &= \frac{-19.3 - 0.1(2.72) + 0.3(1)}{7} = -2.75 \\
x_3^{k+1} &= \frac{71.4 - 0.3(2.72) + 0.2(-2.75)}{10} = 7.003
\end{align*}
\]

Second iteration

\[
\begin{align*}
x_1^{k+1} &= \frac{7.85 + 0.1(-2.75) + 0.2(7.003)}{3} = 2.99 \\
x_2^{k+1} &= \frac{-19.3 - 0.1(2.99) + 0.3(7.003)}{7} = -2.5 \\
x_3^{k+1} &= \frac{71.4 - 0.3(2.99) + 0.2(-2.5)}{10} = 7.003
\end{align*}
\]

Error: See Table
(22 pts.) Problem 6.
(a) Evaluate the following integral analytically.
(b) Evaluate the integral numerically using Trapezoid rule, with \( n = 3 \) panels (that is, 4 points).
(c) Compute the error between your analytical and approximate results.

\[
I = \int_{0}^{\frac{3\pi}{2}} \sin(4x + 2) \, dx
\]

(a) Analytical

\[
I = \int_{0}^{\frac{3\pi}{2}} \sin(4x + 2) \, dx = \left. \frac{1}{4} \cos(4x + 2) \right|_{0}^{\frac{3\pi}{2}}
\]

\[
= \frac{1}{4} [\cos(6\pi + 2) - \cos(2)]
\]

\[
= 0
\]

(b) Trapezoid Rule

\[
I \approx \frac{Ax}{2} \{ f(x_0) + 2[f(x_2) + f(x_3)] + f(x_4) \}
\]

\[
= \frac{\pi}{4} \{ \sin(2) + 2[\sin(2\pi + 2) + \sin(4\pi + 2)] + \sin(6\pi + 2) \}
\]

\[
= 1.27 \{ 0.91 + 2[0.91 + (0.91)] + 0.91 \}
\]

\[
= 4.62
\]

(c) Error

\[
\varepsilon = \left| I - \hat{I} \right| * 100 = 0 + 4.62 * 100 = 462\%
\]
(17 pts.) Problem 7. Use the Euler method and a step size of $\Delta t = 0.25$, solve the initial value problem on the interval $t = [0, 1]$

$$\frac{dx}{dt} = (1 + t)\sqrt{x} \quad \text{where} \ x(0) = 1.$$ 

Euler

$$x_{i+1} = x_i + \Delta t \left[(1 + t_i)\sqrt{x_i}\right]$$

so

$$x_{0.25} = 1 + 0.25 \left[(1 + 0)\sqrt{1}\right] = 1.25$$
$$x_{0.5} = 1.25 + 0.25 \left[(1 + 0.25)\sqrt{1.25}\right] = 1.6$$
$$x_{0.75} = 1.6 + 0.25 \left[(1 + 0.5)\sqrt{1.6}\right] = 2.07$$
$$x_1 = 2.07 + 0.25 \left[(1 + 0.75)\sqrt{2.07}\right] = 2.70$$

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x_{0.25}$</th>
<th>$x_{0.5}$</th>
<th>$x_{0.75}$</th>
<th>$x_1$</th>
</tr>
</thead>
<tbody>
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<td>1.00</td>
<td>1.25</td>
<td>1.60</td>
<td>2.07</td>
</tr>
<tr>
<td>0.25</td>
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<td>2.07</td>
<td>2.70</td>
</tr>
<tr>
<td>0.5</td>
<td>1.60</td>
<td>2.07</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>2.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.70</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing the solution using Euler's method with step sizes 0.25 and 0.025]
PS. The analytical solution

\[ \frac{dx}{dt} = (1 + t) \sqrt{x} \]
\[ \frac{dx}{\sqrt{x}} = (1 + t) dt \]
\[ \int x^{-1/2} dx = \int (1 + t) dt + C \]
\[ 2x^{1/2} = t + \frac{t^2}{2} + C \]
\[ x = 1 \text{ at } t = 0, \text{ so} \]
\[ 2(1) = 0 + 0 + C \Rightarrow C = 2 \]
\[ 2x^{1/2} = t + \frac{t^2}{2} + 2 \]
\[ x(t) = \left( \frac{t + \frac{t^2}{4} + 1}{2} \right)^2 \]