(14 pts.) Problem 1. What gets printed?

```c
main( )
{
    int i = 0, m = 0;
    int f(int j);

    m = f(i);
    printf("m = %i \n", m);
}

int f(int i)
{
    while (i <= 5)
    {
        i = i + 1;
        printf("i = %i \n", i);
    }
    i = i + 1;
    return(i);
}
```

(6 pts. 1 pts ea.) Problem 2.

<table>
<thead>
<tr>
<th>Question</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A C program must have a function called “main”.</td>
<td>X</td>
<td></td>
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<tr>
<td>b. A calling function must pass arguments to the called function.</td>
<td></td>
<td>X</td>
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<tr>
<td>c. Every function returns a value to its calling function.</td>
<td></td>
<td>X</td>
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<tr>
<td>d. Every function must be defined in its own separate file.</td>
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<td>X</td>
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<tr>
<td>e. Interpolation is used to estimate the relations between noisy data.</td>
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<td>X</td>
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<tr>
<td>f. Newton Divided Difference and Lagrange functions are two forms of interpolation functions.</td>
<td>X</td>
<td></td>
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</tbody>
</table>
Problem 3. Starting from an initial guess $x_0 = 2.5$, take 2 iterations of Newton’s method to find the minimum value of the function: (not minimizing: -3; wrong method: -10)

$$f(x) = 2\sin x - \frac{x^2}{10}$$

To find min/max, take the derivative and set it equal to zero:

$$g(x) = f'(x) = 2\cos x - \frac{x}{5} = 0 \quad \text{and} \quad g'(x) = -2\sin x - \frac{1}{5}$$

Now apply Newton’s method to find the root of this function

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} = x_i - \frac{2\cos x_i - \frac{x_i}{5}}{-2\sin x_i - \frac{1}{5}}$$

where (as Dustin Wiggins pointed out it is a “Maximum” not a “Minimum”!)
(24 pts.) Problem 4. Start from an initial guess of $x_1^0 = x_2^0 = x_3^0 = 0.0$, show the first 2 iterations of the Gauss Seidel method for the solution of this system of equations. Compute the error after the second iteration. (Gauss Elimination = -15)

\[
\begin{align*}
3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\
0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\
0.3x_1 - 0.2x_2 + 10x_3 &= 71.4
\end{align*}
\]

Gauss Seidel method gives:

\[
\begin{align*}
x_1^{k+1} &= \left[\frac{7.85 + 0.1x_2^k + 0.2x_3^k}{3}\right] \\
x_2^{k+1} &= \left[\frac{-19.3 - 0.1x_1^{k+1} + 0.3x_3^k}{7}\right] \\
x_3^{k+1} &= \left[\frac{71.4 - 0.3x_1^{k+1} + 0.2x_2^{k+1}}{10}\right]
\end{align*}
\]

First iteration

\[
\begin{align*}
x_1^1 &= \left[\frac{7.85}{3}\right] = 2.62 \\
x_2^1 &= \left[\frac{-19.3}{7}\right] = -2.79 \\
x_3^1 &= \left[\frac{71.4}{10}\right] = 7.10
\end{align*}
\]

Second iteration

\[
\begin{align*}
x_1^2 &= \left[\frac{7.85 + 0.1(-2.79) + 0.2(7.01)}{3}\right] = 2.99 \\
x_2^2 &= \left[\frac{-19.3 - 0.1(2.99) + 0.3(7.01)}{7}\right] = -2.50 \\
x_3^2 &= \left[\frac{71.4 - 0.3(2.99) + 0.2(-2.50)}{10}\right] = 7.00
\end{align*}
\]

Error: See Table
Problem 5. Consider the following integral:
(a) Evaluate the following integral analytically.
(b) Evaluate the integral numerically using Trapezoid rule, with \( n = 3 \) panels (that is, 4 points).
(c) Compute the error between your analytical and approximate results.

\[
I = \int_{0}^{\frac{3\pi}{2}} \sin(5x + 1) \, dx
\]

(a) Analytical

\[
I = \left[ \frac{1}{5} \cos(5x + 1) \right]_{0}^{\frac{3\pi}{2}} = \frac{1}{5} \left[ \cos(15\pi/2 + 1) - \cos(1) \right] = \frac{1}{5} \left[ 0.84 - 0.54 \right] = -0.06
\]

(b) Trapezoid Rule

\[
I \approx \frac{\Delta x}{2} \left\{ f(x_0) + 2[f(x_2) + f(x_3)] + f(x_4) \right\}
\]

\[
= \frac{\pi/2}{2} \left\{ \sin(1) + 2[\sin(15\pi/2 + 1) + \sin(5\pi + 1)] + \sin(15\pi/2 + 1) \right\} = 1.27 \left\{ 0.84 + 2[0.54 + (-0.84)] + -0.54 \right\} = -0.236
\]

(c) Error

\[
\varepsilon = \left| \frac{I - \hat{I}}{I} \right| \times 100 = \left| \frac{-0.06 + 0.236}{-0.06} \right| \times 100 = 293\%
\]
**Problem 6.** Use the Euler method and a step size of $\Delta t = 0.5$, solve the initial value problem on the interval $t = [0, 1]$

\[
\frac{dx}{dt} = (1 + t)\sqrt{x} \quad \text{where } x(0) = 1.
\]

Euler

\[x_{i+1} = x_i + \Delta \left[(1 + t_i)\sqrt{x_i}\right]
\]

so

\[x_{0.5} = 1 + 0.5 \left[(1 + 0)\sqrt{1}\right] = 1.5
\]
\[x_1 = 1.5 + 0.5 \left[(1 + 0.5)\sqrt{1.5}\right] = 2.42
\]

<table>
<thead>
<tr>
<th>t</th>
<th>x-0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>0.5</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>2.42</td>
</tr>
</tbody>
</table>

PS. The analytical solution

\[ \frac{dx}{dt} = (1 + t)\sqrt{x} \]
\[ \frac{dx}{\sqrt{x}} = (1 + t)dt \]
\[ \int x^{-1/2} dx = \int (1 + t) dt + C \]
\[ 2x^{1/2} = t + \frac{t^2}{2} + C \]
\[ x = 1 \text{ at } t = 0, \text{ so } \]
\[ 2(1) = 0 + 0 + C \Rightarrow C = 2 \]
\[ 2x^{1/2} = t + \frac{t^2}{2} + 2 \]
\[ x(t) = \left( \frac{t + \frac{t^2}{4} + 1}{2} \right)^2 \]