Problem 1. What gets printed?

```c
main( )
{
    int i = 0, m = 0;
    int f(int j);

    m = f(i);
    printf("m = %i \n", m);
}

int f(int i)
{
    while (i <= 5)
    {
        i = i + 1;
        printf("i = %i \n", i);
    }
    i = i + 1;
    return(i);
}
```

Problem 2.

<table>
<thead>
<tr>
<th>Question</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.  A C program must have a function called “main”.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>b.  A calling function must pass arguments to the called function.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>c.  Every function returns a value to its calling function.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>d.  Every function must be defined in its own separate file.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>e.  Interpolation is used to estimate the relations between noisy data.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>f.  Newton Divided Difference and Lagrange functions are two forms of interpolation functions.</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
(17 pts.) Problem 3. Starting from an initial guess \( x_0 = 2.5 \), take 2 iterations of Newton’s method to find the minimum value of the function: (not minimizing: -3; wrong method: -10)

\[
f(x) = 2\sin x - \frac{x^2}{10}
\]

To find min/max, take the derivative and set it equal to zero:

\[
g(x) = f'(x) = 2\cos x - \frac{x}{5} = 0 \quad \text{and} \quad g'(x) = -2\sin x - \frac{1}{5}
\]

Now apply Newton’s method to find the root of this function

\[
x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} = x_i - \frac{2\cos x_i - \frac{x_i}{5}}{-2\sin x_i - \frac{1}{5}}
\]

where (as Dustin Wiggins pointed out it is a “Maximum” not a “Minimum”!)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x(i)</td>
<td>x(i+1)</td>
<td>g(x)</td>
<td>g'(x)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>0.995082</td>
<td>-2.10229</td>
<td>-1.39694</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.996082</td>
<td>1.469011</td>
<td>0.899863</td>
<td>-1.87761</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.469011</td>
<td>1.427642</td>
<td>-0.09058</td>
<td>-2.18965</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.427642</td>
<td>1.427552</td>
<td>-0.0002</td>
<td>-2.17954</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.427552</td>
<td>1.427552</td>
<td>-1.2E-09</td>
<td>-2.17952</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1.427552</td>
<td>1.427552</td>
<td>0</td>
<td>-2.17952</td>
<td></td>
</tr>
</tbody>
</table>

To find the minimum, graph the function and approximate the value.
(24 pts.) Problem 4. Start from an initial guess of \( x_1^0 = x_2^0 = x_3^0 = 0.0 \), show the first 2 iterations of the Gauss Seidel method for the solution of this system of equations. Compute the error after the second iteration. (Gauss Elimination = -15)

\[
\begin{align*}
3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\
0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\
0.3x_1 - 0.2x_2 + 10x_3 &= 71.4
\end{align*}
\]

Gauss Seidel method gives:

\[
\begin{align*}
x_1^{k+1} &= \left[ 7.85 + 0.1x_2^k + 0.2x_3^k \right] / 3 \\
x_2^{k+1} &= \left[ -19.3 - 0.1x_1^{k+1} + 0.3x_3^k \right] / 7 \\
x_3^{k+1} &= \left[ 71.4 - 0.3x_1^{k+1} + 0.2x_2^{k+1} \right] / 10
\end{align*}
\]

First iteration

\[
\begin{align*}
x_1^1 &= \left[ 7.85 \right] / 3 = 2.62 \\
x_2^1 &= \left[ -19.3 - 0.1(2.62) \right] / 7 = -2.79 \\
x_3^1 &= \left[ 71.4 - 0.3(2.62) + 0.2(-2.79) \right] / 10 = 7.01
\end{align*}
\]

Second iteration

\[
\begin{align*}
x_1^2 &= \left[ 7.85 + 0.1(-2.79) + 0.2(7.01) \right] / 3 = 2.99 \\
x_2^2 &= \left[ -19.3 - 0.1(2.99) + 0.3(7.01) \right] / 7 = -2.50 \\
x_3^2 &= \left[ 71.4 - 0.3(2.99) + 0.2(-2.50) \right] / 10 = 7.00
\end{align*}
\]

Error: See Table
(22 pts.) Problem 5. Consider the following integral:
(a) Evaluate the following integral analytically.
(b) Evaluate the integral numerically using Trapezoid rule, with \( n = 3 \) panels (that is, 4 points).
(c) Compute the error between your analytical and approximate results.

\[
I = \int_0^{3\pi/2} \sin(5x + 1) \, dx
\]

(a) Analytical

\[
I = \int_0^{3\pi/2} \sin(5x + 1) \, dx = \left. -\frac{1}{5} \cos(5x + 1) \right|_0^{3\pi/2}
\]
\[
= \left[ \cos\left(\frac{15\pi}{2} + 1\right) - \cos(1) \right] / 5
\]
\[
= \left[ 0.84 - 0.54 \right] / 5
\]
\[
= -0.06
\]

(b) Trapezoid Rule

\[
I \approx \frac{\Delta x}{2} \left\{ f(x_0) + 2[f(x_2) + f(x_3)] + f(x_4) \right\}
\]
\[
= \left. \frac{\pi}{2} \left[ \sin(1) + 2[\sin(\frac{5\pi}{2} + 1) + \sin(5\pi + 1)] + \sin(\frac{15\pi}{2} + 1) \right] \right|_{\frac{\pi}{2}}
\]
\[
= 1.27 \left\{ 0.84 + 2[0.54 + (-0.84)] + -0.54 \right\}
\]
\[
= -0.236
\]

(c) Error

\[
\varepsilon = \frac{|I - \hat{I}|}{I} \times 100 = \frac{-0.06 + 0.236}{-0.06} \times 100 = 293\%
\]
(17 pts.) Problem 6. Use the Euler method and a step size of $\Delta t = 0.5$, solve the initial value problem on the interval $t = [0, 1]$

$$\frac{dx}{dt} = (1 + t)\sqrt{x} \quad \text{where } x(0) = 1.$$

Euler

$$x_{i+1} = x_i + \Delta(1 + t_i)\sqrt{x_i}$$

so

$$x_{0.5} = 1 + 0.5\left[(1 + 0)\sqrt{1}\right] = 1.5$$
$$x_1 = 1.5 + 0.5\left[(1 + 0.5)\sqrt{1.5}\right] = 2.42$$

<table>
<thead>
<tr>
<th>t</th>
<th>x-0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
</tr>
<tr>
<td>0.5</td>
<td>1.50</td>
</tr>
<tr>
<td>1</td>
<td>2.42</td>
</tr>
</tbody>
</table>
PS. The analytical solution

\[
\frac{dx}{dt} = (1 + t)\sqrt{x} \\
\frac{dx}{\sqrt{x}} = (1 + t)dt \\
\int x^{-1/2}dx = \int (1 + t)dt + C \\
2x^{1/2} = t + \frac{t^2}{2} + C \\
x = 1 \text{ at } t = 0, \text{ so} \\
2(1) = 0 + 0 + C \Rightarrow C = 2 \\
2x^{1/2} = t + \frac{t^2}{2} + 2 \\
x(t) = \left( \frac{t + \frac{t^2}{2}}{4} + 1 \right)^2
\]
1. What is an array in C? How does an array differ from an ordinary variable?

   Several variables of the same type with the same name, addressed through an index.

2. Describe the array that is defined in each of the following C statements:

   a. `char name[30];`
   
   1-D array of characters with 30 elements

   b. `float c[4] = {2., 5., 3., -4.};`
   
   1-D array of floats with 4 elements
   
   c. `int param[5][5];`
   
   2-D array of integers with 25 elements

3. Write an appropriate array declaration for the following situation:

   Define a one-dimensional, 12-element integer array named c.

   ```c
   int c[12];
   ```

4. What is the purpose of the `while` statement? What is the minimum number of times that a `while` loop can be executed?

   The `while` statement allows looping in a program. The minimum times through the loop is zero, since the logical test is done at the beginning of the loop.
5. A C program contains the following declarations and initial assignments:

```c
int i = 8, j = 5;
float x = 0.005, y = -0.01;
```

Determine the value of each of the following expressions. Use the values initially assigned to the variables for each expression

- $(i > 0) \land (j < 5)$
- $(8 > 0) \land (5 < 5)$
- $T \land F$
- $F$
- $(x > y) \land (i > 0) \lor (j < 5)$
- $(0.005 > -0.01) \land (8 > 0) \lor (5 < 5)$
- $T \land (T) \lor (F)$
- $T$
- $\text{fabs}(x + y)$
- $\text{fabs}(0.005 + -0.01)$
- $0.005$
- $\text{pow}(x - y, 3.0)$
- $\text{pow}(0.005 + 0.01, 3.0)$
- $3.375e-6$

6. Describe the output generated by the following program.

```c
#include <stdio.h>
main()
{
    int a, b = 0;
    int c[10] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 0};
    for ( a = 0; a < 10; a = a+1)
    {
        b = b+c[a];
    }
    printf("%d \n", b);
}
```

45
7. Write a loop that will calculate the sum of every 3-rd integer, beginning with i=2 (i.e., calculate the sum 2+5+8+11+…) for all values of i that are less than 100. Write the loop 3 different ways.
   a. Using a while loop

   ```
   sum = 0;
   i=2;
   while(i<100)
   {
       sum=sum+i;
       i=i+3;
   }
   ```

   b. Using a do-while loop

   ```
   sum = 0;
   i=2;
   do
   {
       sum=sum+i;
       i=i+3;
   } while(i<100)
   ```

   c. Using a for loop

   ```
   sum = 0;
   for(i=2;i<100;i=i+3)
   {
       sum=sum+i;
   }
   ```
8. Describe the output that will be generated by the following C program

```c
#include <stdio.h>

int funct(int b);

main()
{
    int a, b;
    for (b = 1; b <= 5; b = b+1)
    {
        a = funct(b);
        printf("%d 
", a);
    }
}

int funct(int b)
{
    int c;
    c = b*b;
    return c;
}
```

1
4
9
16
25

9. Newton’s method, \( x_{i+1} = x_i - \frac{f(x_i)}{f''(x_i)} \), applied to the function \( f(x) = x^3 - 4x + 2 \) with

\[ x_1 = 1.0 \] gives \( x_2 \) as

a) 0.0 ___X__;  b) 2.0 _____;  c) 1.0 _____

\[
x_{i+1} = x_i - \frac{f(x_i)}{f''(x_i)} = x_i - \frac{x_i^3 - 4x_i + 2}{3x_i^2 - 4} = 1 - \frac{1 - 4 + 2}{3 - 4} = 0
\]
10. The values of $x$ and the corresponding values of $f(x) = x^3 - 4x + 2$ in the bisection method are

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.125</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.578125</td>
</tr>
</tbody>
</table>

The next value of $x$ is
a) 0.625 ___X___;  b) 0.875 ______;  c) 1.250 ______

11. What is meant by ill conditioning of a set of linear equations?

The slopes of the equations are so similar that the computing method can not distinguish between the solutions.

12. Is the identity matrix a matrix all of whose elements are equal to one? Yes ___X__, No _____

13. if $A = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then $C = \begin{bmatrix} 17 & 16 \\ 7 & 6 \end{bmatrix}$ denotes the

a) ___X___ product of A and B;  b) _____ sum of A and B;  c) _____ difference of A and B

14. Solve the following system of equations using the Gauss Elimination method. Show your work.

\[
\begin{align*}
x_1 + x_2 + x_3 &= 6 \\
3x_1 + 2x_2 + x_3 &= 10 \\
-2x_1 + 3x_2 - 2x_3 &= -2
\end{align*}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
3 & 2 & 1 \\
-2 & 3 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
6 \\
10 \\
-2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & -1 & -2 \\
0 & 3 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
6 \\
-8 \\
10
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & -1 & -2 \\
0 & 0 & -10 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{bmatrix}
=
\begin{bmatrix}
6 \\
-8 \\
-30 \\
\end{bmatrix}
\]

\[
x_1 = 1 \\
x_2 = 2 \\
x_3 = 3
\]
Problem 1. (10 Points)

a. What is the difference in meaning between the C variables \( x_3 \) and \( x[3] \)?

\( x_3 \) is a single variable and
\( x[3] \) is an array with three elements

b. Given the C declaration

```
char grades[5];
```

How many array elements are allocated for the data storage? \( ____5____ \)

What type of data can be stored there? \( \text{characters} \)

How does one refer to the initial array element? \( \text{grades}[0] \)

How does one refer to the final array element? \( \text{grades}[4] \)

c. Declare a C array for storing the values of the square roots of integers from 0 through 10 and a second array for storing the cubes of the same integers.

```
double roots[11];
int cubes[11]; or double cubes[11];
```

d. Show the contents of the array \( A[] \) after the execution of the following code fragment

```
int A[11], i;
for(i = 0; i < 11; i = i+1)
{
    A[i] = pow(i,2);
}
```

![Image showing command line output]

```
0 1 4 9 16 25 36 49 64 81 100
Press any key to continue
```
Problem 2. (5 Points)

INDICATE WHETHER THE FOLLOWING STATEMENTS ARE TRUE OR FALSE.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A C program must have a function called <strong>main</strong>.</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>b. An calling function must pass arguments to the called function.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>c. Every function returns a value to its calling function.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>d. Parameters are declared in a function's header.</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>e. Every function must be defined in its own separate file.</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
Problem 3. (30 points)

Consider the following integral:

\[
I = \int_0^{\frac{3\pi}{2}} \sin(5x + 1) \, dx
\]

(a) Evaluate the integral analytically.

\[
I = \frac{\cos(5x + 1)}{5} \bigg|_0^{\frac{3\pi}{2}}
= \frac{1}{5} \{ \cos[5(\frac{3\pi}{2}) + 1] - \cos[5(0) + 1] \}
= \frac{1}{5} \{ 0.8414709 - 0.5403023 \}
= -0.06023373
\]

(b) Evaluate the integral numerically using Simpson’s rule, with \( n = 4 \) panels (that is, 5 points).

![Graph showing the function \( \sin(5x + 1) \) from 0 to \( \frac{3\pi}{2} \) with numerical approximations at various points.]
\[
\int_a^b f(x)\,dx = \frac{3\pi/2/4}{3}\{f(0)+4[f(3\pi/8)+f(9\pi/8)]+2f(6\pi/8)+f(3\pi/2)\}
= 0.3927\{0.84147+4[0.57065 - 0.17716]+2 \cdot (0.21296) - 0.540302\}
= 0.90362
\]

(c) Compute the error between your analytical and approximate results.

\[
error = \frac{0.060234 - 0.90362}{-0.060234} = 1600\%
\]

```c
#include <stdio.h>
#include <math.h>
int main(void)
{
    int panels=10000;
    double a=0.0, b=4.712388, I;
    double integrate(double a, double b, int n);
    I = integrate(a, b, panels);
    printf("Using %d panels, the integral = %lf\n", panels, I);
    return 0;
}

double integrate(double a, double b, int n){
    double sum_odd = 0.0, sum_even = 0.0, x, dx, I;
    double f(double x);
    int k;
    dx = (b-a)/n;
    for(k=1; k<n; k+=2){
        x = a + dx*(k);
        sum_odd = sum_odd + f(x);
    }
    for(k=2; k<n; k+=2){
        x = a + dx*(k);
        sum_even = sum_even + f(x);
    }
    I = dx/3.0*(f(a) + 4*sum_odd+2*sum_even + f(b));
    return I;
}

double f(double x)
{
    double y;
    y=sin(5*x+1);
    return y;
}
```
Problem 4. (10 points)

What gets printed?

```c
#include <stdio.h>
int z;
void main ()
{
    void f(int x);
    z = 5;
    f(z);
    printf("z = %d \n", z);
}
void f(int x)
{
    x = 3;
    z += x;
}
```
Problem 5. (20 points)

The density of sodium at three temperatures is given in the following table:

<table>
<thead>
<tr>
<th>Temperature $T_i$ ($^\circ$C)</th>
<th>Density $\rho_i$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>929</td>
</tr>
<tr>
<td>205</td>
<td>902</td>
</tr>
<tr>
<td>371</td>
<td>860</td>
</tr>
</tbody>
</table>

Find the density for $T = 251 ^\circ$C by using a second-order Lagrange interpolation formula.

\[
f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)
\]

\[
\]

\[
= \frac{(46)(-120)}{-111(-277)} \cdot 929 + \frac{(157)(-120)}{(111)(-166)} \cdot 902 + \frac{(157)(46)}{(277)(166)} \cdot 860
\]

\[
= -0.1795(929) + 1.02247(902) + 0.15706(860)
\]

\[
= -166.783 + 922.266 + 135.073
\]

\[
= 890.556
\]
Problem 6. (25 points)

A flat plate of mass $m$ falling freely in air with a velocity $V$ is subject to a downward gravitational force and an upward frictional drag force due to air. The drag force $F_D$ is given by the expression

$$F_D = \frac{0.3V^2}{500 + (\ln V)^3} - 0.02V$$

Terminal velocity is reached when the drag force equals the gravitational force

$$F = F_D - mg = 0$$

Find the terminal velocity using the Bisection Method if $m = 1$ kg and $g = 9.8$ m/s$^2$. Use an initial interval of $V = 0$ to 200 m/s. Show your work for computing the first 2 iterations of the Bisection Method.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
a & b & c & F(a) & F(b) & F(c) \\
\hline
0.0001 & 100 & 200 & -9.81 & -6.79046 & 4.68753 \\
100 & 150 & 200 & -6.79046 & -2.0238 & 4.68753 \\
150 & 175 & 200 & -2.0238 & 1.09565 & 4.68753 \\
\hline
\end{tabular}
\end{table}

Bisection Method
#include <stdio.h>
#include <math.h>

void main ()
{
}

double g(double x)
{
}