AN ANALYSIS OF MONTHLY WATER DEMAND  
IN PHOENIX, ARIZONA  
by  
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1 INTRODUCTION

1.1 PURPOSE

In 1990 the Water and Wastewater Department and the Water Rates Advisory Committee of the City of Phoenix, Arizona, developed a new water rate structure simpler than the one previously in use. This new water rate structure encourages water conservation. Preliminary calculations done by city staff confirmed the advantages of the new billing system (City of Phoenix, 1990). However, a specific water forecast model is required for a more precise estimation of the influence of the new rate structure on water consumption and on revenue collection. Such a model will allow testing of different rate structures and different conditions affecting water use and therefore revenue (water price, conservation programs, weather conditions).

The water forecasting system is very powerful decision support tool. The precise estimation of the future water consumption is essential for determining the water management policy including the efficient water use and for the water purchase planning. The revenue projections are necessary for budget preparation. The knowledge of the future water production is indispensable for utility planning and management.

1.2 SCOPE

The scope of this work is determined by the local requirements of the Water and Wastewater Department in Phoenix, Arizona.

Taking into consideration specific environment of the City of Phoenix, Arizona, the forecasting model distinguishes between three water user categories: two residential categories--single-family and multi-family--and a single non-residential user category. The non-residential category includes: commercial, industrial and institutional consumers. Some of the factors supporting such differentiation between the water consumers are variations in water consumption per account, seasonal
variability, distribution of meter sizes, sensitivity to the weather conditions, price elasticity, and response to the conservation programs.

The model also takes into account the spatial division of the City service area which results from different rights to the water stored and developed by the Salt River Project (more information about Salt River Project is presented in Section 1.3).

Some factors influencing water consumption such as: weather conditions, water conservation programs, or water price changes, are not utilized in the present model as they demand more detailed study of additional data which are unavailable at the moment. Such a study was performed at Southern Illinois University in parallel with this study.

Based on the water use forecast and the prediction of the number of accounts the model presented here estimates monthly revenues generated by the volume and base charges in accordance with the water rate structure. Since some of the customers do not pay their bills, the bad debt is also incorporated into revenue projections.

The Water and Wastewater Department also needs the predictions of the revenue from the "Water Resource Acquisition Fees" and the "Development and Occupational Fees" which are designed to cover some of the costs of providing necessary water resources to a new development.

One of the applications of the model is in the process of budget preparations for next fiscal year and corrections of the budget within the current fiscal year. Therefore, the model must have a procedure which updates the fiscal year predictions, utilizing the values observed in elapsed months of the current fiscal year.

The Phoenix Department of Water and Wastewater estimates future water use by the application of a forecasting model named IWR-MAIN (Institute for Water Resources - Municipal And Industrial Needs). The IWR-MAIN model predicts the water consumption in 5 or 10 year time steps taking into consideration the factors which determine water demand, for instance: population, number of users, market value and type of housing units in the residential sector, employment in various service and manufacturing industries, water and wastewater rates, weather conditions, personal income, and conservation programs (City of Phoenix, 1989, Dziegielewski and Boland 1989). The present study is intended to supplement IWR-
MAIN by making a more detailed forecast on monthly time steps within the next 5-10 years.

The essence of the model is based on the methodology of water use projections published in the professional journals, however, two new procedures have been developed in this research. One of them is a method of updating the projections of annual water use, the other is a method for the estimation of the unaccounted-for-water from time-shifted data of production and consumption.

1.3 OUTLINE OF THE THESIS

The following Section 1.4 introduces the spatial division of the water and wastewater service area. Section 1.5 describes the features of the water rate structure recently proposed by the Water and Wastewater Department. Section 2 reviews selected literature on the forecasting of water use. Sections 3.1 to 3.7 are devoted to a detailed mathematical description of the model components. Sections 4.1 to 4.6 give the real estimation of the parameters of the model described mathematically in Section 3. Section 5 presents summary, general conclusions and recommendations for further study.

1.4 SPATIAL DIVISION OF THE SERVICE AREA

The Salt River Project (SRP) is an organization originating from the Salt River Valley Water Users' Association (SRVWUA) formed in 1903. In 1904 the members of the Association negotiated a contract promising to repay the federal government the cost of building Roosevelt Dam. They gave their lands as security for a loan to finance the dam construction (Luckingham, 1989). Although the debt was paid back 40 years ago, the owners of the lands within the boundaries of the original SRP are still entitled to the water from the dams and reservoirs built at the turn of the century. These regions are called the "on-project" water service area.
There are some sectors within on-project area which are referred to as "non-member" service area. The owners of this territories declined the original SRVWUA membership for two reasons: their property had little agricultural value or it received enough water from the Salt and Verde Rivers (City of Phoenix 1989). In 1910, the Kent Decree confirmed a preestablished system of water rights based on the principle of prior appropriation--the SRVWUA (and later the SRP) was to supervise the distribution of project water (Luckingham 1989).

The City of Phoenix provides water to all users within the City who are not served by SRP, and it also serves the non-member areas within the SRP jurisdiction. In this thesis, "off-project" means any area not supplied with SRP water. This term labels the regions which lie outside the original SRP water rights boundaries. The members of both non-member and off-project service areas are not entitled to the water of the SRP. Therefore, these two spatial categories are grouped together and considered as a single off-project category.

1.5 BILLING SYSTEM IN PHOENIX, ARIZONA

Since the model developed in this work answers the requirements of the newly introduced water rate structure, it is pertinent to describe this water billing system. The current Section makes this presentation, based on (City of Phoenix, 1990). The old rate structure was a complex system: each consumer group had different price of water and the charges during the summer season were related to the amount of water consumed during winter months.

The new water rate structure assumes the same price of water for all the consumers: single-family users, multi-family users and nonresidential consumers. It defines a certain monthly amount of water called a Lifeline--10 ccf (ccf is one hundred cubic feet of water) for summer months and 6 ccf for the rest of the year. The cost of the water consumed within the Lifeline is included into the base charges. This provides an inexpensive, stable monthly water bill for low water use consumers. The amount of water above the Lifeline is sold at a constant unit price.
The new water rate structure introduces different charges per unit of water for three seasons, which are the same for all the consumer groups. The lowest price of water is in "Low" months, December to March. The highest water rates are in "High" months, June to September. The remaining months of the year: April, May, October, and November have "Medium" rates.

There are additional fees which are assessed for new meters in service area. The Water Resource Acquisition Fee is designated to assist in paying capital costs of new projects. It is different for the three areas: on-project, off-project South of Jomax Road and off-project North of Jomax Road. The fee varies by meter size for single-family and non-residential categories within each area. For multi-family dwellings or apartments the fee depends on the number of the units in the structure.

The development and occupation fees are designated to offset the cost of new development. They are composed of the water fee and the sewer fee, and it is constant for the whole service area. Each unit from the single-family category and each unit from the multifamily category have a specified fee ($/unit) which do not depend on the meter size. The non-residential fee varies with the meter size.
The models of water use which are discussed in professional journals are based on the cross-sectional data, time-series data or pooled cross-sectional and time-series data. The cross-sectional regression models do not really examine water use patterns over time, and therefore their application for forecasting purposes is questionable. Cross-sectional-time-series models are complex models. They are usually focused on the modelling of the relationship between water use and variables which are able to explain the variations in water consumption. For example, average and marginal price of water, personal or household income, evapotranspiration, rainfall depth (Morgan and Smolen, 1976, Agthe and Billings, 1980, Carver and Boland, 1980), temperature, lot value, household size (Danielson, 1979) are variables which are customarily considered as primary determinants of water use. Some authors attempt to explain variations in water use by such factors as: daylight hours (Hansen and Narayanan, 1981), number of weekend days in month, or average elevation of the census tract (Cassuto and Ryan, 1979). Some studies have more than 15 variables which are regarded as influential factors on water consumption (Billing, 1987).

Time-series models seem to be more applicable for the complex Phoenix forecasting system. The general time-series model was introduced in 1972 by Salas-LaCruz and Yevjevich. They separated three components of the water use: trend (linear or quadratic function of time), seasonality (approximated by Fourier series), and stochastic part (modeled by a autoregressive stochastic model). In modeling the trend and periodicity, they used both the monthly means and the standard deviations. They also studied cross-correlation functions of water use, air temperature, and precipitation. They found that trend accounts for 10-69 percent of the variance in water use, whereas seasonality accounts for 24-79 percent of the variance.

Oh and Yamauchi (1974) estimated the trend and the cyclical component of monthly water consumption in Honolulu, Hawaii, by the application of the twelve-month centered moving average. They expressed the seasonal and irregular components of analyzed time series by the ratio of the observed water use to the moving average. The seasonal component was separated from the irregular part by
the averaging ratios for corresponding months of the year. Oh and Yamauchi attempted to explain some of the seasonal variations by the rainfall.

Maidment and Parzen (1984a) give a comprehensive description of modeling water consumption by the "cascade model". The model is composed of four components:

- long-term trend and cyclical changes resulting from city development, increasing service area, and changes in socioeconomic factors.
- seasonal variations resulting from the natural variations in climatic factors and related human behavior including commercial/industrial activity.
- irregular components which are resulting from the past events (autocorrelation).
- irregular components related to the factors affecting water use e.g. weather conditions.

This methodology was applied to model the water use in six Texas cities (Maidment and Parzen, 1984b). The regression of observed annual water consumption against population, number of water connections, household income, and water price determined the trend in mean annual water use. Household income and the water price were found not significant for all six cities. Since the population and the number of connections are highly correlated, in each city only one of them were found significant. The seasonal component was estimated by fitting a Fourier series to the monthly means of the detrended time series. After having removed the trend and the seasonal components from the observed water consumption, the residuals were filtered by the auto-regression method. Such "prewhitened" data were regressed against the detrended, deseasonalized, and autoregressive filtered time series of precipitation, evaporation, and maximum air temperature. The cascade models explained 80-87% variance of the water use. Each city exhibited different distribution of the percentage of the variation in water consumption explained by particular model component: trend explained 0-63% of variation, seasonality 11-76%, autocorrelation 2-23%, and climate correlation 1-11%. On average (for six Texas cities) the trend and seasonality accounted for 70% of total variability, the autocorrelation and climate correlation 15%, and residual error 15%.

Franklin and Maidment (1986) applied the cascade modelling to the weekly and monthly municipal water use in Deerfield Beech, Florida. They used a polynomial
function of population to model the trend. To increase the applicability of the model for risk analysis, they added a rainfall component, so that the distribution of the possible future water consumption could be determined based on the distribution of historical rainfall. They found that the inclusion of the auto-correlation term considerably improves the accuracy of the forecasts made on the weekly basis but it does not improve the precision of the monthly projections.

The cascade modelling methodology is very flexible. It allows one to increase the precision of the model by relatively simple model extension (addition of required components). Therefore the model can be developed by the "step by step" method--a very convenient procedure for the development of the complex water forecasting system for Phoenix, Arizona. The base of this system should be constructed from the regression lines since the regression is considered in literature as a superior to the autoregressive-moving average methods and to the exponential smoothing method for fitting actual consumption and forecasting future consumption (Weber, 1989).

The wide application of the Phoenix forecasting system, the required precision of predictions, and the character of the available data requires development of new procedures, which constitute the author’s contribution to water demand modelling. These methods are:

- application of Bayesian methods for updating predicted annual water consumption, revenue and water production from new data.
- disaggregated modelling of water consumption, water production, and city revenue divided by two areas, three user types, and seven meter diameter categories.
- estimation of the unaccounted for water from two shifted in time data sets: the production data and the consumption data.
3 MATHEMATICAL DESCRIPTION OF THE FORECASTING SYSTEM

In this section a detailed mathematical description of the model components is presented. Selected model elements are presented in Figure 3.1.

FIGURE 3.1 Selected components of the Phoenix forecasting system.
In Figure 3.2, the relationship between different categories, which are used in the description of the forecasting system, is graphically introduced:

FIGURE 3.2 Categories of water demand used in the modelling process.
The following Section 3.1 presents the method of modelling water use per account. Section 3.2 describes the regression models of the number of accounts. In the Section 3.3 the model of water consumption per account and the model of number of accounts are used to estimate the future total water consumption. Section 3.4 presents the method of estimation of the amount of water which is consumed on each account below specified amount--Lifeline. Section 3.5 introduces revenue estimation method. Section 3.6 presents the technique of updating the predicted water consumption within the current fiscal year. The method of the estimation of the unaccounted-for-water and the projection of the water production are discussed in the Section 3.7. Based on the discussed here methodology the model parameters are estimated in Section 4.

3.1 PROJECTING WATER USE PER ACCOUNT

The issues discussed in this section include: monthly water consumption per account and its seasonal variations; removal of the seasonal component from the water use per account time series; prediction of the normal weather water consumption per account; and application of the demand adjustment factors

3.1.1 Monthly water consumption

The monthly water consumption per account is determined by the division of total water use recorded in a given month by the respective number of accounts:

\[ V(i,m,y) = \frac{W(i,m,y)}{A(i,m,y)} \]  

(3.1)

where:

\[ V(i,m,y) \] = water use per account observed in the spatial area and type of use category i in the month m of the fiscal year y [ccf/acct/month]
\( W(i,m,y) \) = recorded water use in the spatial area and type of use category \( i \) in the month \( m \) of the fiscal year \( y \) [ccf/month]

\( A(i,m,y) \) = observed number of accounts in the spatial area and type of use category \( i \) in the month \( m \) of the fiscal year \( y \) [#acct].

\( i \) = index of planning area/type of use category

= 1 means on-project planning area, single-family residential
= 2 means on-project planning area, multi-family residential
= 3 means on-project planning area, non-residential
= 4 means off-project planning area, single-family residential
= 5 means off-project planning area, multi-family residential
= 6 means off-project planning area, non-residential.

3.1.2 **Seasonal variations**

The water consumption has a very strong seasonal pattern. The modelling of the seasonal variations of the water use per account is performed by the application of seasonal indices. The seasonal indices are calculated under the assumption that the observed value of the time series equals a trend factor multiplied by seasonal and irregular factors. This multiplicative decomposition model, which is widely used to describe seasonal changes, is described by the equation (all variables are for the month \( m \) of the fiscal year \( y \)):

\[
V = T \times S \times R
\]

(3.2)

where:

\( V = V(m,y) \) = observed value of the water use per account

\( T \) = trend component

\( S \) = seasonal component

\( R \) = irregular component.

The following method of the estimation the decomposition model parameters is carried out by using Statgraphics' "seasonal decomposition" procedure
(Statgraphics is the name of a statistical software package, which will be described later). The trend (T) is estimated by a centered twelve-month moving average $V_{ma}(m,y)$. The twelve-month moving average is calculated by adding the observations from the 12 successive months and dividing the resulting sum by 12--number of time periods included in the average (see: Fig. 3.3) The term "moving" means that each successive average is determined by dropping the first month observation included in the previous average and by adding the observation from the next month. Since twelve is an even number, each moving average corresponds to a point of time situated midway between the sixth and the seventh month of the 12-month period.

To obtain an average corresponding to original months (not to the points of time between months) a centered moving average must be calculated. The centralization process is a two-month moving average of the previously computed 12-month moving averages.

For example, the average calculated for months 1/86-12/86 corresponds to the midpoint between 6/86 and 7/86 (this average is equal 99.7 in Figure 3.3). The next moving average is calculated for months 2/86 to 1/87 and it corresponds to the midpoint between 7/86 and 8/86 (value 101.2 in Figure 3.3). Once the moving averages are calculated, they are centered. The first two-month moving average is determined for the "midpoints": 6/86-7/86 and 7/86-8/86 and it corresponds to July, 1986 (100.4 in Figure 3.3). The second one is calculated for the "midpoints" 7/86-8/86 and 8/86-9/86 and it corresponds to August, 1986 (101.8 in Figure 3.3).
### FIGURE 3.3 Calculation of the centered twelve-month moving average.

The ratio of the observed monthly water consumption to the centered twelve-month moving average in each individual fiscal year is an estimate of the seasonal and irregular components (SR). The seasonal part is isolated from the SR elements by averaging the calculated ratios. This process can be represented by the equation:
\[
S(i,m) = \frac{1}{Y} \sum_{y=1}^{Y} \frac{V(i,m,y)}{V_{ma}(i,m,y)}
\] (3.3)

where:

\(S(i,m)\) = seasonal factor for month \(m\)

\(V(i,m,y)\) = observed water use per account in month \(m\) of the fiscal year \(y\) [ccf/acct/month]

\(V_{ma}(i,m,y)\) = twelve-month moving average of the water consumption per account centered in the month \(m\) of the fiscal year \(y\) [ccf/acct/month]

\(Y\) = number of historical years of record

\(i\) = index of planning area/type of use category.

Figure 3.4 presents an example of the seasonal factors.

**FIGURE 3.4** Example of seasonal factors (single-family user category, total service area).

Finally the seasonal factors are normalized to ensure that their sum equals twelve (number of months in year). The results of Eq. (3.3) do not add up to twelve because different time periods are used for the calculation of the centered moving average and the seasonal factors. For example, if the monthly observations are available for the time period 7/86 to 12/90, the twelve-month centered moving average calculated from these data corresponds to the period of 1/87 to 6/90.
Therefore, the seasonal factors can be estimated only from the 1/87-6/90 data, but not from the full available data set used for the calculation of the moving average (7/86-12/90). Since the sum of seasonal factors is very close to twelve, the normalization process is not critical.

The normalization can be described by the formula:

\[ I_s(i,m) = \frac{S(i,m)}{\sum_{m=1}^{12} S(i,m)} \]  

(3.4)

where:

- \( I_s(i,m) \) = seasonal index for month \( m \)
- \( S(m) \) = seasonal factor (Eq. 3.3).

These seasonal indices are estimated from the historical data, separately for each of the six core models, and then they are combined whenever possible because the differences between some planning area/user category indices are not significant.

### 3.1.3 Average deseasonalized monthly water use

To estimate trend from year to year data, or to examine price or conservation effects, the seasonal variations need to be removed from the data. In some studies, there are sufficient annual data to determine trends in water use directly from annual water consumption values. In this study, however, only four years of annual data are available for the six basic use categories, thus the trend must be calculated from monthly values with the regular seasonal cycle in water use removed--deseasonalized data.

The seasonal component is removed from the raw data by dividing the recorded values of water use per account by the seasonal indices:

\[ V_{t(i,m,y)} = \frac{V(i,m,y)}{I_s(i,m)} \]  

(3.5)

where:
\(V_{ti}(i,m,y) = \) trend and irregular components of water consumption per account [ccf/month/acct]

\(V(i,m,y) = \) water consumption per account [ccf/month/acct]

\(I_s(i,m) = \) seasonal index.

Figure 3.5 shows an example of the two time series: one of them is the recorded water consumption per account, the other is the same water consumption per account where the seasonal component has already been removed.

**FIGURE 3.5** Example of the observed and the deseasonalized water use per account (Off-project/single-family category).

Deseasonalization decreases the variability in water consumption, what improves the fitting of the regression estimate of the trend line. This deseasonalization of data before the fitting of the regression line is especially useful when data do not cover complete twelve-month periods. For instance, a decreasing trend may be estimated by the least squares method if the data set applied into analysis begins in summer (high water consumption) and terminates in winter (low water consumption).

To isolate the month to month random effect of weather from systematic trends in climate, the departures of weather variables from their normal values
(monthly averages through the period of data applied in regression analysis) are taken and used in the water use regressions. Rainfall and maximum air temperature data from the Arizona State University weather station at Tempe have been studied and no trends or jumps in the annual and seasonal weather observations from period 1953-1989 was found. Consequently the assumption has been made that there is no trend in the monthly weather data during the time period analyzed by the regression employed in this study.

The general form of the model of the deseasonalized water consumption per account has additive form ($V_t = \text{constant} + \text{trend} + \text{weather related components}$) and can be represented by equation:

$$V_t(i,m,y,T,P) = a_1(i) + a_2(i) m_y + a_3(i) \Delta T_{\text{max}}(m,y) + a_4(i) \Delta P_d(m,y) + a_5(i) \Delta P_{0.01}(m,y)$$  \hspace{1cm} (3.6)

where:

$V_t(i,m,y) =$ deseasonalized water consumption per account as a function of planning area, consumer type, temperature and precipitation [ccf/month/acct]

$m_y =$ dummy variable representing monthly time step ($m_y = 1$ for January 1986, $m_y = 2$ for February 1986, etc.)

$\Delta T_{\text{max}} =$ departure of the observed monthly mean maximum temperature from its normal value (normal value means here an average over the time period applied into regression analysis)

$\Delta P_d =$ departure of the observed monthly depth of precipitation from its normal value

$\Delta P_{0.01} =$ departure of the observed number of days with the rainfall greater than 0.01 inch from the normal value in month $m$

$a_1(i)$ .. $a_5(i) =$ coefficients determined by the least squares method.

Stepwise regression is used to determine which variables are significant for each of the six core models. Deviations of temperature, precipitation and number of rainy days are set to the expected values which equal zero. The weather variables are in Eq. (3.6) only to adjust the average water use for abnormal weather effects.
3.1.4 Normal weather predictions

According to the methodology presented in previous section, six values, determined for two planning areas and for three categories of consumption, represent the normal weather average water consumption per account without seasonal variations. These values are determined from the relationship:

\[ V_{ad}(i) = a_1(i) \]  

(3.7)

where \( V_{ad}(i) \) is the normal weather average water consumption per account without seasonal component, \( a_1(i) \) is a constant from the equation (3.6) and \( i \) indicates planning area/type of use category, \( i=1, 2, ..., 6. \)

The normal weather water use per account is described by the following formula:

\[ V_n(i,m) = V_{ad}(i) I_s(i,m) \]  

(3.8)

where:

- \( V_n(i,m) \) = expected water use per account in the given planning area/type of use category \( i \) (\( i=1..6 \)) in the month \( m \) (\( m = 1..12 \) [ccf/acct/month])
- \( V_{ad}(i) \) = normal weather average water use per account without seasonal component (Eq. 3.7) [ccf/acct/month]
- \( I_s(i,m) \) = seasonal index (Eq. 3.4) [dimensionless].

3.1.5 Demand adjustment factors

In any month the observed demand will rarely be exactly equal to the expected normal weather demand computed by Eq. (3.8). The demand adjustment factors are ratios of the observed water use per account to the normal weather water consumption:
\[ f_a(i,m,y) = \frac{V(i,m,y)}{V_n(i,m,y)} \]  

where:

- \( f_a(i,m,y) \) = demand adjustment factor
- \( V(i,m,y) \) = water use per account observed in the spatial area and type of use category \( i \) in the month \( m \) of the fiscal year \( y \) \([\text{ccf/acct/month}]\)
- \( V_n(i,m,y) \) = normal weather water use per account for the planning area/user category \( i \) and in the month \( m \) of the fiscal year \( y \) \([\text{ccf/acct/month}]\).

Figure 3.6 presents an example of the demand adjustment factors:

**FIGURE 3.6** Example of the demand adjustment factors (Off-project/single-family category).

The adjustment factors defined above reflect the historical conditions influencing the water consumption. For instance the adjustment factors \( f_a(i,m=\text{July-June}, y=1987) \) calculated for all the months of the fiscal year 1987 reflect the deviations in water use from the normal values as influenced by the conditions (weather, price changes, conservation programs, etc.) specific for the fiscal year in question. The projection of water use per account for the month \( m \) of some future fiscal year \( y_p \) with the assumed conditions of some past historical year \( y_h \) can be estimated from the relationship:
\[ V(i,m,y_p | y_h) = V_n(i,m,y_p) \cdot f_a(i,m,y_h) \]  

(3.10)

where:

- \( V(i,m,y_p | y_h) \) = projection of water use in the month \( m \) of the fiscal year \( y_p \) under the conditions affecting water use from the fiscal year \( y_h \) [ccf/month/acct]
- \( V_n(i,m,y_p) \) = projection of the normal weather water use per account in the month \( m \) of the fiscal year \( y_p \) [ccf/month/acct]
- \( f_a(i,m,y_h) \) = demand adjustment factors determined for the month \( m \) of the historical year \( y_h \) (Eq. 3.9) [dimensionless]
- \( i \) = identifier of the planning area/type of use category.

These adjustment factors allow for the reconstruction of demand in any future year as if the conditions affecting water use of some past year repeated themselves, without attempting to explain all of the many factors which go into the relationship between water demand and weather, price changes, and water conservation programs.

Because the expected water use per account modeled by the equation (3.8) depends only on the month of the year and it does not depend on the year (there is no trend), the equation (3.10) just copies the water consumption per account from the fiscal year \( y_h \) to the fiscal year \( y_p \).

To simulate conditions which did not occur in the past, but which may occur in the future, the demand adjustment factors as a function of these conditions may be formulated:

\[ f_a(i,X) = f(X) + \text{error} \]  

(3.11)

where \( f_a \) is an adjustment factor, \( f \) indicates a function, \( X \) symbolize variables which explain the variability in adjustment factors (weather conditions, socioeconomic conditions, water rate, conservation programs, etc.), and "error" contains an unexplained variability.

The weather coefficients from Eq. (3.6) can be used as a part of the function \( f \) but other factors such as price changes and conservation impacts may also affect the results, and their study has yet to be done. It is important to note that any model
simulating demand variability will always have less variation than the true demand, if the error term in Eq. (3.11) is set to zero, which usually occurs, as deterministic models do not contain a stochastic components. The demand adjustment factors in their raw form, however, do capture the full demand variation.

3.2 PROJECTING NUMBER OF ACCOUNTS

To make the prediction of the total water consumption, the projection of the number of accounts is required for all the planning area/user categories of demand. Additionally, the projection of the number of accounts in each meter size category is needed so that the revenue derived from base charges, from the water resource acquisition fee, and from the development and occupation fee can be calculated.

3.2.1 Average monthly number of accounts

It is assumed that the number of accounts is not influenced by weather conditions. Therefore, temperature, precipitation, and evaporation are not considered in modelling the number of accounts. Six core account models are developed to describe the relationship between the dummy variable \( m_y \) which indicates the monthly time step and the monthly number of accounts. These six models correspond to the two administrative areas and three types of demand defined previously.

The city officials consider the number of building permits issued to be an indicator of the growth status of the city at any particular time. The building permits are not used as an explanatory variable in modelling changes of the number of accounts. Yet, if suitable data would be available, the trend estimated for number of accounts could be corrected according to the relationship determined by additional analysis between the monthly number of accounts and the number of building permits issued.

The trend in number of accounts for each planning area/type of water use is described by the equation:
\[ A(i,m_y) = a(i) + b(i) m_y \] (3.12)

where:

\( A(i,m_y) \) = estimated average monthly number of accounts for a given combination of spatial area with the type of use \( i \), and for the month specified by dummy variable \( m_y \) [#acct/month]

\( m_y \) = dummy variable indicating monthly time step

\( = 1 \) for Jan. 1986, \( m_y = 2 \) for Feb. 86 . . . \( m_y = 60 \) for Dec. 1990, etc.

\( a(i), b(i) \) = coefficients determined by regression analysis.

3.2.2 Meter size fractions

In each class of accounts water is delivered through meters of several sizes (5/8", 1", 1&1/2", 2", 3", 4", 6"). Each account pays a base service charge based on the size of the meter, so the projection of the total number of accounts in each category must be broken down by the meter size. It is done by calculating *meter size fractions*--the ratio of the number of meters of a specified size to the total number of meters in this category. The meter size fractions are calculated from the formula:

\[ r(t,ms) = \frac{A(t,ms)}{A(t)} \] (3.13)

where:

\( r(t,ms) \) = average fraction of the number of meters (accounts) from meter size category \( ms \) for the time \( t \) [dimensionless]

\( A(t,ms) \) = observed average number of accounts falling into meter size category \( ms \) in the time \( t \) [#acct]

\( A(t) \) = observed average number of accounts in the time \( t \) [#acct]

\( ms \) = index which indicates the meter size category

\( t \) = represents time period (here half of the year).

If a trend in t-month average meter size fractions exists, the best constant estimator of these fractions is calculated from the most recent historical record--in
our case the last six months (July-December) of the 1990. Figure 3.7 presents an example of the meter size fractions:

![Graph showing meter size fractions from 1986 to 1990](image)

**FIGURE 3.7** Example of the meter size fractions (Off-project/single-family category, meter size: 5/8”).

The number of accounts in a given month can be predicted by using the equation (3.12). The subdivision of the number of accounts according to the meter size category in a month of the fiscal year can be made by the multiplication of predicted value by the appropriate meter-size fraction:

\[
A(i,m_y,ms) = A(i,m_y) r(ms)
\]  

(3.14)

where:

- \(A(i,m_y,ms)\) = number of accounts projected for the combination of the planning area and the type of use \(i\) and for the month indicated by dummy variable \(m_y\), which falls into the meter size category \(ms\) [\#acct]
- \(A(i,m_y)\) = number of accounts projected for the combination of the spatial area and the type of use \(i\) and for the month \(m_y\) (3.12) [\#acct]
- \(r(ms)\) = average meter-size fraction (3.13).
- \(m_y = f(m,y)\) = dummy variable indicating monthly time step.
3.3 PROJECTING TOTAL WATER CONSUMPTION

The prediction of the normal weather water consumption for a given month of the fiscal year $y$, indicated by the monthly time step variable $m_y$, can be made by the multiplication of the predicted water use per account (Eq. 3.8) and the predicted number of accounts (Eq. 3.12):

$$ W_n(i,m,y) = V_n(i,m) A(i,m,y) \quad (3.15) $$

where:

$ W_n(i,m,y) $ = expected water use in the given combination of spatial area and the type of use $i$ under normal weather conditions [ccf/month]

$ V_n(i,m) $ = expected water use per account in the given combination of spatial area and the type of use $i$ under normal weather conditions (Eq. 3.8) [ccf/acct/month]

$ A(i,m,y) $ = expected number of accounts in the month $m$ of the fiscal year $y$ ($m$ and $y$ are specified by the dummy time step variable $m_y$ used in Eq. 3.12) [#acct].

Water consumption predictions under specific conditions which affect water use (weather conditions, socioeconomic conditions, water rate, conservation programs, etc.) are made by the application of the demand adjustment factors discussed in Section 3.1.4:

$$ W(i,m,y) = W_n(i,m,y) f_a(i,X) \quad (3.16) $$

where:

$ W_n(i,m,y) $ = expected normal weather water use (Eq. 3.15) [ccf/month]

$ f_a $ = demand adjustment factor as a function of planning area/type of use category $i$ and variables $X$ representing conditions influencing water use [dimensionless].

The prediction of the fiscal year water consumption is calculated as a sum of the monthly projections:
\[ W(i,y) = \sum_{m=Jul}^{Jun} W(i,m,y) , \quad (3.17) \]

and the projection of the total water consumption for the city in a given fiscal year is determined by summing the water use predicted separately for six planning area/type of use categories:

\[ W(y) = \sum_{i=1}^{6} W(i,y) . \quad (3.18) \]

The total fiscal year water consumption for the normal weather conditions can be calculated from the equations similar to the Eq. (3.17) and Eq. (3.18).

### 3.4 CONSUMPTION ABOVE THE LIFELINE

In each demand category, an amount of water is supplied to each account within a part of the base charge for having a water account. This amount is called the Lifeline, and it varies seasonally: from October through May the Lifeline is equal 6 \([\text{ccf}/\text{acct/month}]\), while in June through September it is 10 \([\text{ccf}/\text{acct/month}]\). To calculate the revenue generated from the volume charges, the proportion of water sold above the Lifeline must be known, and these proportions can be estimated by:

\[ h(i,m,y) = 1 - \frac{W_{Lb}(i,m,y)}{W(i,m,y)} \quad (3.19) \]

where:

- \( h(i,m,y) \) = proportion of water consumption above the Lifeline for the month \( m \) of the fiscal year \( y \) [dimensionless]
- \( W_{Lb}(i,m,y) \) = observed water consumption in month \( m \) of the fiscal year \( y \), below the specified Lifeline [ccf/month]
\[ W(i,m,y) = \text{observed total water use in month } m \text{ of the fiscal year } y \text{ [ccf/month]} \]
\[ i = \text{indicator of the planning area/type of use category.} \]

The projection of the water consumption above the Lifeline is calculated for each category and for a given month from the relationship:

\[ W_{La}(i,m,y) = W(i,m,y) h(i,m) \quad (3.20) \]

where \( W_{La} \) is the water consumption above the Lifeline, \( W(i,m,y) \) is the projection of water use (Eq. 3.16), and the \( h(i,m) \) is the average proportion of water consumption above the Lifeline.

### 3.5 PROJECTING REVENUE

The new block rate structure introduced by the City of Phoenix Water and Wastewater Department in *Description and Evaluation of the Current and Proposed Water Rate Structure*, Feb. 13, 1990, makes possible the calculation of the total revenue to be divided into two independent parts. One of them is the revenue from base charges, while the other is the revenue from volume charges.

#### 3.5.1 Revenue from volume charge

The prediction of water consumption in each month of the fiscal year is needed for the purpose of projection of revenue from the volume charge. The proposed block rate structure indicates that the predicted monthly water use must be split into two ranges determined by the Lifeline of 6 ccf from October through May and 10 ccf from June through September. The amount of water which belongs to the 0-6 ccf range in the low and medium seasons and 0-10 ccf range in the high season
does not produce revenue from the volume charge, and therefore it does not need to be considered in modelling the revenue from volume charge.

Although the proposed rate structure sets the same volume charge for all the customers, the City of Phoenix Water Conservation and Resources Division needs a separate forecast--for each planning area: on-project and off-project and--for each type of use: single-family, multi-family and non-residential.

The prediction of monthly revenue made from the volume charges is calculated in three steps:

- Prediction of water consumption in the two spatial areas and for the three categories of water use in the specified month of the fiscal year (Eq. 3.15 or Eq. 3.16);
- Determining the total amount of water which is used above the Lifeline (Eq. 3.20); and
- Calculation of revenue from the volume charge for each month of the given fiscal year and for each service area and each type of use.

The monthly revenue for the fiscal year $y$, for the particular type of use within the given planning area, and for the "normal" weather conditions is determined by relationship:

$$ R_{vn}(i,m,y) = h(m, y) \, W_n(i,m,y) \, N_v(m) $$

(3.21)

where:

$ R_{vn}(i,m,y) $ = revenue from the volume charge produced in month $m$ of the fiscal year $y$, for the given spatial area/type of use category $i$, and for normal weather conditions [$$/month$] 

$ W_n(i,m,y) $ = normal weather monthly water use in the given combination of spatial area and the type of use $i$ determined from equation (Eq. 3.15) [ccf/month] 

$ h(m, y) $ = projected proportion of water consumption above Lifeline, for the months $m$ of the fiscal year $y$

$ N_v(m) $ = volume charge [$$/ccf].
The volume-charge per ccf is the same for all consumers; but differs by season, from *Low Months* (Dec, Jan, Feb, Mar), through *Medium Months* (Apr, May, Oct, Nov), to *High Months* (Jun, Jul, Aug, Sep).

Total normal weather fiscal year revenue produced by the volume charges is calculated by adding monthly revenues:

\[
R_{vn}(y) = \sum_{m=1}^{12} R_{vn}(m,y)
\]  

(3.22)

where:

\(R_{vn}(y)\) = total service area revenue from the volume charge produced in the normal weather month m of the fiscal year y [$/year], which is found by summing the revenue from the six core models:

\[
R_{vn}(m,y) = \sum_{i=1}^{6} R_{vn}(i,m,n)
\]

\(R_{vn}(i,m,y)\) = revenue from the volume charge produced in month m of the fiscal year y, for the given spatial area/type of use category i, and for normal weather conditions [$/month]

### 3.5.2 Deviations of revenue projections from normal values

The deviations of revenue from normal values are simulated indirectly by the application of the demand adjustment factors \(f_a\) in the prediction of the water consumption per account (Eq. 3.16), whereas the normal weather values are calculated under assumption that \(f_a = 1\). For example, the prediction of monthly revenue generated in year y from volume charges under specific conditions for year \(y_h\) which affect water use is calculated from the relationship:

\[
R_v(i,m,y|y_h) = h(m,y) \: W_n(i,m,y) \: N_v(m) \: f_a(i,m,y_h)
\]  

(3.23)

where:
\( R_v(i,m,y|h) = \) revenue from the volume charge produced in month \( m \) of the fiscal year \( y \), for the given spatial area/type of use category \( i \), under the conditions affecting water use from the fiscal year \( y_h \) [$/month]

\( W_n(i,m,y) = \) prediction of normal weather water use (Eq. 3.15) [ccf/month]

\( h(m, y) = \) projected proportion of water consumption above Lifeline, for the months \( m \) of the fiscal year \( y \)

\( N_v(m) = \) volume charge [$/ccf]

\( f_d(i,m,y_h) = \) demand adjustment factors determined for the month \( m \) of the historical year \( y_h \) (Eq. 3.9) [dimensionless]

\( i = \) identifier of the planning area/type of use category.

### 3.5.3 Revenue from base charges

The description of the block rate structure (Phoenix, 1990) indicates that the base charges depend only on the size of the meter. The user must pay for the meter readings and connection maintenance, regardless of the amount of water he/she consumes. Any amount of water smaller than the "lifeline" is supplied without a volume charging. Therefore, the revenue due to base charges can be calculated by the multiplication of the number of meters falling into the particular size category by the appropriate base charge. The revenue produced by the base charge is calculated in three steps:

- prediction of the average number of accounts for each of the two spatial areas subdivided into three categories of water use in the specified fiscal year (Eq. 3.12)

- subdivision of predicted number of accounts into particular meter size categories (Eq. 3.14)

- calculation of revenue from base charge for each month of the given fiscal year, for each service area and each type of use, and for each meter size.

The revenue for the given month \( m \) of the fiscal year \( y \) and for the particular type of use within the given spatial area is calculated from the formula:
\[ R_{b}(i,m,y) = \sum_{ms=1}^{7} A(i,m,y,ms) \cdot N_{b}(ms) \quad (3.24) \]

where:

- \( R_{b}(i,m,y) \) = revenue from the base charge produced in the month \( m \) of the fiscal year \( y \), for the given spatial area/type of use category \( i \) [$/month].
- \( A(i,m,y,ms) \) = number of accounts projected for the month \( m \) of the fiscal year \( y \), and for the spatial area/type of use category \( i \), which falls into meter size class \( ms \) (Eq. 3.14) [#acct/month].
- \( N_{b}(ms) \) = base charge specific for meter size \( ms \) [$/acct].

The total revenue produced by the base charges in the fiscal year \( y \) can be calculated from summation of the monthly results:

\[ R_{b}(y) = \sum_{i=1}^{6} \sum_{m=July}^{June} R_{b}(i,m,y) \quad (3.25) \]

where:

- \( R_{b}(y) \) = expected revenue from the base charge in the fiscal year \( y \) [$/year].
- \( R_{b}(i,m,y) \) = expected revenue from the base charge in the month \( m \) of the fiscal year \( y \), for the given spatial area/type of use category \( i \) (Eq. 3.24) [$/month].

### 3.6 FORECASTING WATER CONSUMPTION WITHIN THE CURRENT YEAR

The correction of the predictions within the current fiscal year is important for the updating the budget, which usually is made after the fourth and the seventh month of the fiscal year. Forecasting water consumption and water revenues within the current fiscal year presents a particular challenge because a part of the forecast consumption has already taken place. Thus, if four months of the current fiscal year
have already elapsed, then the requirement for an annual forecast is really reduced to forecasting the remaining eight months of consumption. Likewise, if seven months of consumption are already known, only five months consumption need to be forecast.

The methods outlined so far do not take into account the measured information from the current year—they all assume that the projection is to be made for all the months by only projection of trends from the past years. In this section a method is developed to balance the trend forecast computed in the manner described in previous sections of this thesis with another estimate of annual consumption inferred from consumption already recorded during the current year. A method of determining the standard error of the estimate of this forecast is also presented. Corresponding estimates of revenue from water sales in the current fiscal year can thus be constructed.

3.6.1 Updating projections of annual water consumption

The current fiscal year water consumption can be estimated by two different methods. One of them is the direct calculation of the water use from the core models described in Section 3.3. These models give the amount of the water consumption for each planning area subdivided into three categories of the type of use for every month m of the fiscal year. The detailed formula which describes the core models of the monthly water consumption is:

\[
W(i,m,y) = V_{ad}(i) \cdot I_s(i,m) \cdot A(i,m,y) \cdot f_a(i,m,X) \quad (3.26)
\]

where:

- \(W(i,m,y)\) = expected water use in month m of the fiscal year y [ccf/month]
- \(V_{ad}(i)\) = normal weather average water use per account without seasonal component (Eq. 3.7) [ccf/month/acct]
- \(I_s(i,m)\) = seasonal index (Eq. 3.4) [dimensionless]
- \(A(i,m,y)\) = expected number of accounts in the month m of the fiscal year y (m and y are specified by the dummy time step variable \(m_y\) used in Eq. 3.12) [#acct].
\( f_d(i,m,X) \) = demand adjustment factor as a function of planning area/type of use category \( i \), month \( m \), and variables \( X \) representing conditions influencing water use (Eq. 3.11) [dimensionless]

\( i = \) index of the planning area/user category.

The total fiscal year consumption is the sum of the monthly predictions:

\[
W(i,y) = \sum_{m=1}^{12} W(i,m,y) . \quad (3.27)
\]

The second prediction of the water use can be made after \( k \) months of the fiscal year elapsed by dividing the accumulated consumption over these \( k \) months by the fraction of total consumption that would normally have been accumulated in these \( k \) months. Thus, if July has 0.15, August 0.14 and September 0.11 of the total annual consumption, the accumulated consumption for these three months is 0.40 of the whole year. Dividing the recorded consumption for July to September by 0.40 would give an estimate of annual consumption for the whole fiscal year. This procedure is written formally as:

\[
W_d(i,y,k) = \frac{\sum_{m=1}^{k} W(i,m,y)}{\sum_{m=1}^{k} d_w(m)} . \quad (3.28)
\]

where:

\( W_d(i,y,k) \) = expected water consumption in the spatial area and the type of use category \( i \) and in the fiscal year \( y \) after \( k \) months passed [ccf/year]

\( W(i,m,y) \) = observed water use in month \( m \) of the fiscal year \( y \) [ccf/month]

\( d_w(m) \) = monthly water consumption fraction for the month \( m \) (Eq. 3.30)

\( k \) = number of elapsed months in the current fiscal year.

The monthly water use fractions are usually defined as an average ratio of the observed monthly water consumption to the annual consumption:
\[
d_{W}(m) = \frac{1}{Y} \sum_{y=1}^{Y} \frac{W(m,y)}{W(y)} \quad (3.29)
\]

where:
- \( d_{W}(m) \) = monthly water consumption fraction for the month \( m \)
- \( W(m,y) \) = observed water use in month \( m \) of the fiscal year \( y \) [ccf/month]
- \( W(y) \) = observed water consumption in the fiscal year \( y \) [ccf/year]
- \( Y \) = number of historical years of record

In this case, the monthly water use fractions can be estimated from the modeled water consumption:

\[
d_{W}(m) = \frac{W(i,m,y)}{W(i,y)} \quad (3.30)
\]

where:
- \( d_{W}(m) \) = monthly water consumption fraction for the month \( m \)
- \( W(i,m,y) \) = projection of the water use for month \( m \) of the fiscal year \( y \) (Eq. 3.26) [ccf/month]
- \( W(i,y) \) = projection of the water consumption in the fiscal year \( y \) (Eq. 3.27) [ccf/year].

Monthly fractions calculated from equation (3.30) preserve the distribution of the monthly water consumption which is assumed for the core models. If the effects of the price changes or conservation programs are modelled by a specified demand adjustment factors \( f_{a}(i,m,X) \), the monthly water use fractions \( d_{W}(i,m) \) also reflect these effects.

The updated fiscal year water consumption is a weighted average of the two available fiscal year forecasts:

\[
W_{C}(i,y,k) = [1-\beta(k)] \cdot W(i,y) + \beta(k) \cdot W_{d}(i,y,k) \quad (3.31)
\]

where:
\[ W_C(i,y,k) = \text{updated prediction of water use in the current fiscal year} \] [ccf/year]

\[ W(i,y) = \text{expected water use in the given combination of spatial area and the type of use } i \text{ in the fiscal year } y \text{ (Eq. 3.27)} \] [ccf/year]

\[ W_d(i,y,k) = \text{expected water consumption in the spatial area and the type of use category } i \text{ and in the fiscal year } y \text{ after } k \text{ months elapsed (Eq. 3.28)} \] [ccf/year]

\[ \beta(k) = \text{weights to be assigned the consumption forecast.} \]

After the fiscal year water consumption forecast is found by the weighted method, the monthly predictions must be modified for the remainder of the fiscal year to allow for the fact that part of the forecast consumption has already occurred. The expected water use in the remaining fraction of the fiscal year is determined from the following equation:

\[
W_R(i,y,k) = W_C(i,y,k) - \sum_{m=1}^{k} W(i,m,y) \quad (3.32)
\]

and then, water use can be subdivided into monthly values by taking the ratio of the demand fraction in each future month to the total fraction of demand which would normally occur during the remainder of the fiscal year:

\[
W_C(i,m,y,k) = W_R(i,y,k) \frac{d_w(m)}{\sum_{x=k+1}^{12} d_w(x)} \quad (3.33)
\]

where:

\[ W_R(i,y,k) = \text{expected water consumption for the spatial area /type of use category } i \text{ in the remaining months } m=k+1..12 \text{ of the fiscal year } y \]

\[ W_C(i,y,k) = \text{updated prediction of water use in the fiscal year } y \text{ (current fiscal year) (Eq. 3.31)} \] [ccf/year]
\( W(i,m,y) \) = observed water use in month \( m \) of the fiscal year \( y \) [ccf/month]

\( W_C(i,m,y,k) \) = expected water consumption in the spatial area/type of use category \( i \) in each remaining month \( m \) (\( m > k \)) of the fiscal year \( y \) [ccf/month]

\( k \) = number of months which elapsed.

### 3.6.2 Correction of monthly revenue from volume charges

The revenue generated by the volume charges can be calculated by the estimation of the amount of water use exceeding the Lifeline for each month remaining in the fiscal year. This amount of water multiplied by the appropriate volume charges will give the monthly revenues. This can be expressed by the following equation:

\[
R_{CV}(i,m,y,k) = W_C(i,m,y,k) \cdot h(m) \cdot N_v(m) \quad (3.34)
\]

where:

- \( R_{CV}(i,m,y,k) \) = updated revenue from the volume charge produced in the remaining month \( m \) of the fiscal year \( y \), for the given spatial area/user category \( i \) [$/month]
- \( W_C(i,m,y,k) \) = expected water consumption for the spatial area/user category \( i \) in each remaining month \( m \) of the fiscal year \( y \) (Eq. 3.33) [ccf/month]
- \( h(m) \) = projected proportion of water consumption above the Lifeline for the month \( m \) of the fiscal year (Eq. 3.19) [dimensionless]
- \( N_v(m) \) = volume charge [$/ccf].

The new estimate of the total–whole service area revenue produced by the volume charges corrected after \( k \) months of the fiscal year elapsed is determined from the relationship:

\[
R_{CV}(y,k) = \sum_{m=1}^{k} R_v(m,y) + \sum_{m=k+1}^{12} R_{CV}(m,y,k) \quad (3.35)
\]
where:

\[ R_{Cv}(y,k) = \text{corrected revenue from the volume charge produced in the fiscal year } y \text{ after } k \text{ months passed } [\$/year] \]

\[ R_v(m,y) = \text{recorded revenue generated by the volume charges in the month } m \text{ of the fiscal year } y \text{ [$/month]} \]

\[ R_{Cv}(m,y,k) = \sum_{i=1}^{6} R_{Cv}(i,m,y,k) \]

\[ R_{Cv}(i,m,y,k) = \text{corrected revenue from the volume charge produced in the remaining month } m \text{ of the fiscal year } y, \text{ for the given spatial area/type of use category } i \text{ [$/month]} \]

\[ k = \text{number of elapsed months}. \]

### 3.6.3 Update of revenue from base charges

The prediction of the revenue produced by the base charges is updated after \( k \) months of the current fiscal year elapsed by the summation of the recorded revenue in the months from 1 to \( k \) and the projected revenue for the months from \( k+1 \) to 12. Thus, the adjusted revenue from the base charges is calculated from formula:

\[ R_{Cb}(y,k) = \sum_{m=1}^{k} R_{b}(m,y) + \sum_{m=k+1}^{12} R_{Cb}(m,y) \quad (3.36) \]

where:

\[ R_{Cb}(y,k) = \text{corrected revenue from the base charge produced in the fiscal year } y \text{ after } k \text{ months passed } [\$/year] \]

\[ R_{b}(m,y) = \text{recorded revenue generated by the base charges in the month } m \text{ of the fiscal year } y \text{ [$/month]} \]

\[ R_{Cb}(m,y) = \text{predicted revenue from the base charge produced in the month } m \text{ of the fiscal year } y \text{ (Eq. 3.24) [$/month]} \]

\[ k = \text{number of elapsed months}. \]
3.7 PROJECTING WATER PRODUCTION

The prediction of the amount of water produced to meet the future demand is usually made by a simple multiplication of the projected water consumption by a coefficient, which represents the unaccounted for water. As an alternative, future water consumption and unaccounted for water are estimated by two separate models and then combined to give the production.

In this section a water production model is constructed by using the forecast consumption as described in the previous sections, while the unaccounted for water is modelled using historical data on the differences between the recorded water consumption and the water production. Water production and the water consumption are not measured at the same time because the production is measured as it occurs at the treatment plants while the consumption is accumulated over a monthly time period before it is read at the meter. Thus, a time shift takes place; the water consumption recorded for a given month is not contemporaneous with the production data for that month. Consequently the unaccounted for water cannot be determined by a simple subtraction of the water use from the production data as it is normally done; a shift has first to be corrected. The significance of this problem of the dislocated time intervals between production and consumption increases at times when significant seasonal variations in monthly water consumption are occurring, particularly in the Spring and the Fall.

In the following sections a method developed for the prediction of the unaccounted for water is introduced. The technique of the estimation of the time shift coefficient and the correction of water use to make the production and the consumption data compatible is discussed.
3.7.1 **Time-shift coefficient**

The time-shift coefficient is a value which represents the difference between the time when the consumption is recorded and the time when the production is gaged. An example of the time series when the water production readings are time-shifted in relation to the consumption measurements are presented in Figure 3.8.

If the production is measured at a time $T_p$ and the consumption is measured at a time $T_w$, the time-shift coefficient $\Delta T$ is determined by the following equation:

$$\Delta T = T_p - T_w \quad (3.37)$$

Whereas determining the point of time when the production is recorded is usually not difficult, the direct estimation of the point of time when the water consumption is recorded is in most of the cases impossible. Therefore, the time-shift coefficient must be estimated from the available production - consumption data.

![Graph showing water production and consumption time series before time-shifting.](image)

**FIGURE 3.8** Water production and consumption time series before time-shifting of water consumption (Phoenix, Arizona).
The basic idea of the time shift determination directly from the production and consumption data lies in the fact that the periodic changes in the monthly water consumption and production can be represented by the periodic functions. The time-shift coefficient can be approximated by the differences between the phases of these functions. Because of the existence of the trend and the different values of the average annual water consumption and the water production, it is useful to transform a set of raw data. For example, for each fiscal year the proportions of the monthly consumption and production to an average month of a given fiscal year can be calculated from the following relations:

\[ w(m,y) = \frac{12 \ W(m,y)}{W(Y)} \]  
\[ p(m,y) = \frac{12 \ P(m,y)}{P(Y)} \]

where:
- \( w(m,y) \) = proportion of the water consumption in the month \( m \) of the fiscal year \( y \) to the average monthly consumption
- \( W(m,y) \) = recorded water consumption in the month \( m \) of the fiscal year \( y \)
- \( W(y) \) = recorded water consumption in the fiscal year \( y \)
- \( p(m,y) \) = proportion of the water production in the month \( m \) of the fiscal year \( y \) to the average monthly production
- \( P(m,y) \) = recorded water production in the month \( m \) of the fiscal year \( y \)
- \( P(y) \) = recorded water production in the fiscal year \( y \).

The time series \( w(m,y) \) and \( p(m,y) \) have no annual trend and they have the same average which equals one. The series of the water use proportions and the water production proportions can be approximated with a sufficient degree of precision by the first harmonic of an annual Fourier series (the seasonal changes have frequency which is equal to once a year). The Fourier series representation of the time series of the production and consumption proportions are:
for the water consumption proportions:

\[ w_f(t) = w + A_w \left( \frac{2\pi}{12} t \right) \cos \left( \frac{2\pi}{12} t \right) + B_w \sin \left( \frac{2\pi}{12} t \right) \] (3.40)

for the water production proportions:

\[ p_f(t) = p + A_p \cos \left( \frac{2\pi}{12} t \right) + B_p \sin \left( \frac{2\pi}{12} t \right) \] (3.41)

where:
- \( w_f(t) \) = the Fourier approximation of the water use proportions \( w(m,y) \)
- \( w \) = average of the water consumption proportions (\( w = 1 \))
- \( p_f(t) \) = the Fourier estimate of the water production proportions \( p(m,y) \)
- \( p \) = average of the water production proportions (\( p = 1 \))
- \( t \) = time [month]
- \( \pi \) = 3.14159...
- \( A_w, B_w, A_p, B_p \) = coefficients of the Fourier series determined by analysis of the historical data.

The values of \( t = T_w \) and \( t = T_p \), can be determined by the equations (3.42) and (3.43) for the conditions that \( w_f(T_w) = w \) and \( p_f(T_p) = p \) respectively. These values of \( t \) correspond to times when the ratios are equal to one, their average value. \( T_w \) and \( T_p \), can be calculated from the formulas:

\[ T_w = \frac{12}{2\pi} \tan^{-1} \left( \frac{-A_w}{B_w} \right) \] (3.42)

and

\[ T_p = \frac{12}{2\pi} \tan^{-1} \left( \frac{-A_p}{B_p} \right) \] (3.43)

where:
\( T_w = \) time when the Fourier approximation of the water consumption proportions is equal to the average value [month]

\( T_p = \) time when the Fourier approximation of the water production proportions is equal to the average value [month].

The time shift coefficient, according to the formula (3.37), is then simply the difference between the \( T_w \) and \( T_p \) (\( \Delta T = T_w - T_p \)).

3.7.2 Time adjustment of water consumption record

As mentioned above the unaccounted for water can not be directly calculated from the raw data if the water production and the water consumption are measured at significantly different points of time. Consumption data ought to be corrected to account for the time displacement before calculating the unaccounted for water. The correction of the water consumption series can be performed by the application of the following formula:

\[
W'(m,y) = (1 - \Delta T) \ W(m,y) + \Delta T \ W(m+1,y) \quad (3.44)
\]

where:

- \( W'(m,y) \) = corrected water consumption in the month \( m \) of the fiscal year \( y \)
- \( \Delta T \) = time-shift coefficient
- \( W(m,y), W(m+1,y) \) = recorded (or predicted) water consumption in the month \( m \) and month \( m+1 \) of the fiscal year \( y \).

The unaccounted for water is calculated from the relationship:

\[
U_N(m,y) = P(m,y) - W'(m,y) \quad (3.45)
\]

where:

- \( U_N(m,y) \) = unaccounted for water in the month \( m \) of the fiscal year \( y \)
- \( P(m,y) \) = recorded water production in the month \( m \) of the fiscal year \( y \)
- \( W'(m,y) \) = corrected water consumption in the month \( m \) of the fiscal year \( y \).
3.7.3 Estimation of water production from water consumption

To estimate production from consumption, the relationship between the water consumption and the unaccounted for water must be determined. Different relationships $U_N(m,y) = f[W(m,y)]$ can be derived from the historical data. The simplest form of the expression of the relationship between the unaccounted for water and the water consumption is achieved by the usual percentage or proportion method. Here the proportions of the unaccounted for water are allowed to vary by months and they are calculated as a proportion of consumption (not production as is usually the case), since the consumption will be forecast. The proportions of unaccounted for water are:

$$u_N(m) = \frac{U_N(m,y)}{W'(m,y)}$$ \hspace{1cm} (3.46)

where:

- $u_N(m) =$ proportion of the unaccounted for water to the water consumption in the month $m$
- $U_N(m,y) =$ unaccounted for water in the month $m$ of the fiscal year $y$
- $W'(m,y) =$ corrected water use in the month $m$ of the fiscal year $y$.

Therefore having the value of the water consumption in the given month $m$, the production can be estimated from the formula:

$$P(m,y) = W'(m,y) \cdot [1 + u_N(m)]$$ \hspace{1cm} (3.47)

where:

- $P(m,y) =$ estimated water production in the month $m$ of the fiscal year $y$
- $W'(m,y) =$ corrected water use in the month $m$ of the fiscal year $y$
- $u_N(m) =$ proportion of the unaccounted for water to the water consumption in the month $m$. 

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4. STATISTICAL ESTIMATION OF MODEL PARAMETERS

This Section 3 presents the real estimation of the model parameters according to the methodology described mathematically in Section 3. Section 4.1 presents the data sets and the computer software applied in the process of the model parameters estimation. Section 4.2 presents the calculations of the water use per account, seasonal indices, and the demand adjustment factors. Section 4.3 discusses the distribution of the number of accounts and presents trends in meter size fractions. In Section 4.4 the estimation of the amount of water which falls under Lifeline limits is performed. The weighting coefficient, which is used to correct within-year predictions, is determined in Section 4.5. Section 4.6 shows the estimated value of the time-shift between the consumption time series and the production time series, and the values of the unaccounted for water.

4.1. DATA AND COMPUTER SOFTWARE DESCRIPTION

The model parameters have been determined from the data supplied by the Water Conservation and Resource Division, Water and Wastewater Department, Phoenix, Arizona. Reports: WCS 865.2 of 5/8/91 and WCS 825.1 constitute the main material used in this study. Forming the core data set, the Report WCS 865.2 covers the time period from January, 1986 to December, 1990. This report gives the monthly values of:

- total water consumption [ccf/month];
- water consumption in accounts with monthly use less than or equal to 6 ccf [ccf/month];
- water consumption in accounts with monthly use less than or equal to 10 ccf [ccf/month]; and
- total number of accounts [# acct].

The city is divided into three planning areas: on-project, off-project, and non-member. The data from three spatial categories are further subdivided into three user categories: single-family, multi-family, and non-residential. Each of the planning
area/type of user category is again subdivided into seven meter size classes: 5/8", 1", 1 1/2", 2", 3", 4", 6").

This report also gives monthly values of water consumption and the number of accounts for nine pressure zones--regions of the service area with similar pressure in the water distribution network. These zones are labelled from A-H, while one, designated as U, includes all accounts which do not fall into classifications.

The data for the off-project and non-member planning areas have been combined into one category which is referred to as off-project throughout this work. Consequently, only two spatial subdivisions of the City of Phoenix are considered in this analysis: on-project and off-project. Since the listing of this data set is very extensive (more than 70 pages), the data are not included in this thesis. Different subsets of the data given in the WCS 865.2 report have been used for the construction of individual parts of the models. Detailed description of these subsets is presented when particular model components are discussed.

The Report WCS 825.1 covers the period from July, 1981 to January, 1990. Monthly values of the total water consumption have been extracted and listed in Table A1.3.

The water production data and the water supply data have been taken from the report entitled The City of Phoenix Arizona, Water and Wastewater Department, Water Production and Consumption in Millions of Gallons. The monthly values of production from January, 1980 to June, 1989 are presented in Table A1.1 and the monthly values of water supply (water pumped into distribution network is called in the report "consumption") are listed in Table A1.2.

Weather observations from the Arizona State University Climatology Station have been used in this analysis:

a) for the time period from 1/1/53 to 12/31/1988, the daily values of
   - maximum temperature [°F]
   - minimum temperature [°F]
   - depth of the precipitation [inch/day]

b) for the time period from 7/1982 to 6/1990 the monthly values of evaporation [inch/month]

c) for the time period from 1/1989 to 12/1990 the monthly values of
   - mean maximum temperature [°F]
- mean minimum temperature [°F]
- number of rainy days (rainfall depth > 0.01”)
- total precipitation [inch/month].

Monthly values of the mean maximum temperature, number of rainy days (recorded precipitation depth is greater than 0.01”), and the total precipitation depth have been calculated from daily observations. They are given in Table A3.6, Table A3.7, and Table A3.8 respectively.

The data analysis and model parameter estimation were performed by the application of two computer programs: spreadsheet 1-2-3 (1-2-3 is a registered trademark of Lotus Development Corporation) and statistical software—Statgraphics version 4.0 (Statgraphics is a registered trademark of Statistical Graphics Corporation). Lotus 1-2-3 was used to do simple calculations and data manipulations--sorting and selection--it was used mainly as a preprocessor. All the statistical analysis which is presented in this thesis, was performed by Statgraphics. Following Statgraphics procedures were utilized:

**Data Management**

- **Import Data Files** - was used to import Lotus 1-2-3 files. Statgraphics version 4.0 can import only files created by Lotus 1-2-3 version 2.01 or earlier.
- **Export Data Files** - was applied to create Lotus 1-2-3 format files which contained the results of the Statgraphics analysis, e.g. coefficients of regression lines or seasonal indices.

**Regression analysis**

- **Simple Regression** - was used to test different forms of models (linear, multiplicative, exponential, and reciprocal ) which contain only one independent variable, e.g. trend models of meter size fractions or number of accounts.
- **Multiple Regression** - was used in the preliminary analysis of the applicability of dummy variables in the model.
- **Stepwise Variable Selection** - was applied to select statistically significant variables (t-statistic greater than 2.0) in the core models of water consumption per account. The forward selection was used. The F-statistics
was selected as a value above which the variable was included into the model.

**Forecasting**

Seasonal Decomposition of time series was used to estimate seasonal indices. The classic *multiplicative* ratio-to-moving-average method was selected.

The whole analysis was performed on the IBM-compatible 386 computer with 8 Mb of RAM memory. An IBM-AT 286 machine with 2 Mb RAM memory appeared to be too small for the analysis of the relatively large data set by the Statgraphics.

### 4.2 MODELLING WATER CONSUMPTION PER ACCOUNT

The water consumption per account $V(i,m,y)$ has been calculated by a simple division of the sum of water use for all meter size categories $W(i,m,y)$ by the corresponding sum of the number of accounts $A(i,m,y)$:

$$V(i,m,y) = \frac{W(i,m,y)}{A(i,m,y)}.$$  

The calculated values are listed in Table A2.1.

Four fiscal years (1986, 1987, 1988 and 1989) have been used for the estimation of the seasonal indices. The full set of data (five calendar years) could not be used because the data for the first few months of the 1986 are questionable: an extremely high water consumption per account for the off-project/multi-family category occurred in January and March, and for off-project/non-residential category in January. In addition, in every month from February to June of 1986 the sum of water use for all the pressure zones was about 0.01% to 48% higher (and in two cases lower) than the sum of water consumption for all the planning areas and all the users. The respective sum of the number of accounts in all pressure zone categories was identical to the sum of the total number of accounts in all planning area/user categories. All detected differences are recorded in Table A1.4.
4.2.1 Seasonal indices

The sets of 12 indices have been calculated separately for each planning area and user category by the application of the Statgraphics' seasonal decomposition procedure. The values are listed in Table A2.2 and graphically presented in Figure 4.1. The seasonal indices demonstrate that on average the water use per account varies seasonally: from about 64% (January) to the 153% (July) of the normal weather trend consumption for the single-family category, and 79% (February) to 129% (July) for the multi-family category. These variations are similar for both planning areas. Similar seasonal variations of non-residential water use occur, but show distinct patterns for different regions of the city. The seasonal indices for on-project area are as low as 74% (February) and as high as 130% (July) while for off-project area they assume values between 57% (February) and 145% (July).

Since there are no significant differences in the indices between the planning areas for single-family and multi-family user categories, the indices for these categories have been calculated for the whole planning area and the same results used for on-project and off-project areas. The seasonal indices of the non-residential/off-project category are, however, significantly different from those of the non-residential/on-project. Therefore for the non-residential category separate sets of seasonal indices are applied for each planning area. Table A2.2 presents the percentage differences between seasonal indices estimated for different spatial/user categories and the results of t-tests of the differences between the indices for each month of the fiscal year. The t-tests indicate that for the single-family category, the differences between on-project and off-project seasonal indices are significant on the five percent level only for July. For the multi-family category, significantly different seasonal fractions among planning areas occur only for January. However, for the non-residential category the differences between on-project and off-project seasonal indices are statistically significant for eight months of the year. This statistical testing confirms the visual comparison of Figure 4.1.
FIGURE 4.1 Seasonal indices of the water consumption per account for different user categories and planning areas: (a) single-family, (b) multi-family, and (c) non-residential.
Table 4.1 illustrates a full set of seasonal indices applied in the model of water consumption per account.

**TABLE 4.1** Seasonal indices applied in modelling seasonal variations of water consumption per account.

<table>
<thead>
<tr>
<th>Month</th>
<th>Single-family</th>
<th>Multi-family</th>
<th>Non-residential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Area</td>
<td>Total Area</td>
<td>On-project</td>
</tr>
<tr>
<td>Jul</td>
<td>1.53182</td>
<td>1.28568</td>
<td>1.30053</td>
</tr>
<tr>
<td>Aug</td>
<td>1.30918</td>
<td>1.19947</td>
<td>1.26752</td>
</tr>
<tr>
<td>Sep</td>
<td>1.19857</td>
<td>1.17667</td>
<td>1.21610</td>
</tr>
<tr>
<td>Oct</td>
<td>1.02297</td>
<td>1.04368</td>
<td>1.09765</td>
</tr>
<tr>
<td>Nov</td>
<td>0.81050</td>
<td>0.96086</td>
<td>0.95124</td>
</tr>
<tr>
<td>Dec</td>
<td>0.73518</td>
<td>0.90158</td>
<td>0.83800</td>
</tr>
<tr>
<td>Jan</td>
<td>0.63973</td>
<td>0.81967</td>
<td>0.74330</td>
</tr>
<tr>
<td>Feb</td>
<td>0.64379</td>
<td>0.79466</td>
<td>0.73678</td>
</tr>
<tr>
<td>Mar</td>
<td>0.72368</td>
<td>0.79696</td>
<td>0.76935</td>
</tr>
<tr>
<td>Apr</td>
<td>0.88396</td>
<td>0.85716</td>
<td>0.88918</td>
</tr>
<tr>
<td>May</td>
<td>1.09517</td>
<td>0.98298</td>
<td>0.99758</td>
</tr>
<tr>
<td>Jun</td>
<td>1.40545</td>
<td>1.18066</td>
<td>1.19278</td>
</tr>
</tbody>
</table>

4.2.2 **Weather Patterns**

The weather data have been studied for the existence of a long term trend. The results of this analysis are presented in Appendix 3. Two seasons have been distinguished: a winter season (from November to April), and a summer season (July to October from the beginning of a fiscal year and May and June from the end of that fiscal year). The annual and seasonal values of weather variables:

- average daily maximum temperature [°F]
- average daily minimum temperature [°F]
- average daily mean temperature [°F]
- total precipitation depth [inch/month]
- depth of pan evaporation (1982-1989 only) [inch/month]
were assembled from the data described in Section 4.1 and they are presented in Tables A3.1 to A3.5 respectively. The annual data here refer to the fiscal year, July 1 of the current year to the June 30 of the following calendar year. The maximum temperature and the precipitation do not show significant trends through time (See: Figures A3.1.1-A3.1.3 and Figures A3.4.1-A3.4.3, and the results of the regression analysis in Tables A3.1.1-A3.1.3 and Tables A3.4.1-A3.4.3). The minimum temperature and the mean temperature, for both the annual data and the seasonal data, have a jump occurring at about 1980-1982 of about 8 [°F] in the minimum temperature and 4 [°F] in the mean temperature (See: Figures A3.2.1-A3.2.3 and Figures A3.3.1-A3.3.3). The relocation of the Arizona State University weather station was the cause of this jump. Prior to May, 1981 this station was located in a citrus grove. In January, 1982 the station was moved to its current Tempe location, where it has a parking lot and buildings nearby. In the new location, the surroundings of the weather station are periodically flood irrigated. These factors affect the daily minimum and mean temperature observations but apparently they do not affect the maximum temperature. The results of the regression analysis, presented in Tables A3.5.1-A3.5.3, indicate that there is an increasing trend in the pan evaporation data, but the period of the record for the eight years is not long enough to conclude whether this trend is permanent or temporary.

Daily maximum air temperature was selected as the variable indexing heat effect on water demand in preference to daily mean or minimum air temperature (because they have a jump in the record) and pan evaporation (because it has a trend in the record). Since the study of the fiscal year and seasonal data on rain and maximum temperature has not shown a trend, it has been assumed that such a trend does not exist for any given month of the year. For instance, no trend has been assumed in the individual weather variables for January, February, March, e.t.c., between 1953 and 1990. The averages (over the study period from June 1986 to December 1990) of the monthly mean maximum temperature, the monthly precipitation depth, and the monthly number of days with the rainfall greater than 0.01 inch for each month of the year have been calculated. These values are represented in Table A3.6, Table A3.7 and Table A3.8 respectively. For the period of July, 1986 to December, 1990 the differences between the observed monthly weather
parameters and the respective averages have been determined. These values are listed in Table A3.9.

4.2.3 Normalization of Core Models

The normalization of the water consumption per account has been performed according to the procedure described in Section 3.1.3. The seasonal component has been removed from the water use per account series by the application of the previously calculated seasonal indices (Eq. 3.5). Then the deseasonalized consumption for the period from July, 1986 to December, 1990 has been regressed against four independent variables (Eq. 3.6 with time variable included):

- \( m_y \), a dummy variable indicating monthly time step (\( m_y = 7 \) for July, 1986 and progressively up to \( m_y = 60 \) for December, 1990)
- \( \Delta T_{\text{max}} \), deviations of the observed monthly mean maximum temperature from the short term average for July, 1986 to December, 1990 [°F]
- \( \Delta P_d \), deviations of the observed monthly precipitation depth from the short term average [inch/month]
- \( \Delta P_{0.01} \), deviations of the observed monthly number of days with the rainfall greater than 0.01 inch from the short term average.

The "short term" refers here to the period from July, 1986 to December 1990, which was used because this is the period of the water use data set. The departures of all climatic variables were calculated with respect to this mean value also because this makes the regression unbiased with respect to the climate over the data period.

The data entered into the Statgraphics stepwise regression analysis are listed in Table A2.3 (the deseasonalized water consumption per account), and Table A3.9 (weather related data). The computer printouts are presented in Table A2.4. Some results of the regression analysis are introduced in Table 4.2.

The monthly values of the deviations from the average of the total precipitation depth and the deviations of the monthly number of days with rainfall
greater than 0.01 inch have little influence on the consumption per account. The positive sign of the regression coefficients suggest that the off-project/non-residential and on-project/multi-family water consumers use more water during months when the number of rainy days is greater. The consumption by the off-project/non-residential users increases during the months of higher monthly precipitation depth. Although these results are statistically significant, they do not make physical sense: rainfall effects are also imbeded in the temperature data, so temperature is the main "weather modifier" of water use and the rainfall further adjusts the water demand--the effect of highly correlated variables.

The coefficients of the deviations of the mean maximum temperature have the expected sign, and they are statistically significant (for 95% confidence limits) embracing all the categories, except on-project/non-residential and off-project/multi-family categories, which do not have a significant relationship with any weather related variable.
TABLE 4.2  Parameters of the normalized deseasonalized water consumption per account (Eq. 3.6). All significant coefficients are listed (t-statistics greater than two). For on-project/single-family category and off-project/multi-family category the significant trend has been detected. T-statistics for variables are shown in parentheses.

<table>
<thead>
<tr>
<th>User category</th>
<th>Coefficients</th>
<th>( m_y )</th>
<th>( \Delta T_{\text{max}} )</th>
<th>( \Delta P_d )</th>
<th>( \Delta P_{0.01} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_1 )</td>
<td>( a_2 )</td>
<td>( a_3 )</td>
<td>( a_4 )</td>
<td>( a_5 )</td>
</tr>
<tr>
<td>On-project</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-family</td>
<td>18.996548</td>
<td>-0.032126</td>
<td>0.3742044</td>
<td>0.2834253</td>
<td>0.2034769</td>
</tr>
<tr>
<td></td>
<td>(57.57)</td>
<td>(-3.58)</td>
<td>(4.87)</td>
<td>(3.94)</td>
<td>(2.006)</td>
</tr>
<tr>
<td>Multi-family</td>
<td>86.200456</td>
<td></td>
<td>0.9022198</td>
<td></td>
<td>0.7493875</td>
</tr>
<tr>
<td></td>
<td>(220.34)</td>
<td></td>
<td>(4.16)</td>
<td></td>
<td>(2.63)</td>
</tr>
<tr>
<td>Non-resident.</td>
<td>108.39956</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(164.36)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Off-project</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-family</td>
<td>19.672349</td>
<td>0.1628777</td>
<td>0.278686</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(114.33)</td>
<td>(2.7871)</td>
<td>(3.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multi-family</td>
<td>122.83397</td>
<td></td>
<td></td>
<td>2.2337401</td>
<td>5.9527204</td>
</tr>
<tr>
<td></td>
<td>(56.88)</td>
<td></td>
<td></td>
<td>(3.78)</td>
<td>(2.561)</td>
</tr>
<tr>
<td>Non-resident.</td>
<td>128.29037</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(132.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The declining \( a_2 = -0.032 \) trend found in the on-project/single-family water use per account data is probably caused by a relatively small data set and extremely low values observed in 1990. A regression analysis of the data from which the year 1990 has been excluded does not indicate any time related tendencies. Therefore, the stepwise regression has been repeated for the full data set, but with the time step variable \( m_y \) eliminated from the analysis. During the second regression analysis the coefficient describing the relationship of water use per account with the deviations of number of rainy days has been found insignificant. Figure 4.2 shows the
deseasonalized water consumption for single-family users from the on-project area and the two regression lines which have been determined respectively with and without the time related independent variable ($m_y$). The estimated coefficients of both lines are given in Table 4.2. These are equivalent to the mean water use per account in each of the six core models.

The second category for which the significant trend, this time increasing, has been detected by stepwise regression is off-project/multi-family ($a_2 = 0.163$). The existence of this increasing trend in average water use per account (average for all meter size classes) does not mean that such a trend exists for individual meter size classes. Most likely the accounts with the same meter diameter have similar water consumption through the time and the growth in the average consumption per account is produced by a decreasing number of accounts with a small water use (small meter sizes) and an increasing number of accounts with higher water consumption (larger meter sizes). The analysis of the number of accounts presented in the Section 4.3 shows that for this user category the number of accounts with 1" meters and larger is increasing, whereas the number of accounts with 5/8" meters is declining. As has been done in case of the on-project/single family category, no trend is assumed in the off-project/multi-family water consumption per account. This trend is left in the unexplained part of water consumption per account.

Since the trend may also results from the changes in water price or conservation programs, further studies are required to find out the correct reason for detected trends in on-project/single-family and off-project/multi-family water consumption per account. The University of Southern Illinois has performed an additional analysis of the influence of price and conservation programs on the water use per account.
FIGURE 4.2 Deseasonalized water consumption per account and normal weather average [ccf/month/acct] for on-project planning area and users: (a) single-family, (b) multi-family, and (c) non-residential.
FIGURE 4.3 Deseasonalized water consumption per account and normal weather average [ccf/month/acct] for off-project planning area and users: (a) single-family, (b) multi-family, and (c) non-residential.
4.2.4 Demand Adjustment Factors

The demand adjustment factors have been calculated by the application of Eq. (3.9) for the period from July 1986 to December 1990. The values are listed in Table A2.5 and plotted in Figure 4.4 (on-project category) and Figure 4.5 (off-project category). These adjustment factors can be directly applied into the process of water use modelling to simulate the conditions of the selected fiscal years from 1986 to 1989, or the calendar years 1987 to 1990.

By the visual analysis of Figures 4.4 and 4.5 some consistent water use patterns can be detected, which indicate that water use is influenced by some factors common to all consumer groups. For example, all users increased water consumption in March, 1989 and then they almost constantly decreased usage until August (single-family), September (multi-family), and October (non-residential) of 1990. At the end of 1990 another jump in water use may be observed. Note that "decrease" here is a demand adjustment factor so what is occurring is that higher than normal usage in March but this departure from normality decreased later in the year to a value of about 1.0 showing that demand was at normal levels.

Although the water use patterns for a given consumer class are very similar for both the planning areas, differ among user categories, although they show some similarities. These similarities can be found among all three user categories, indicating that some variations in water use are related to conditions undetermined at this time, which affected the water consumption. It would be useful the isolate this part of water fluctuations from the random ones.

As mentioned in Section 3.1.4, a further study has been performed at Southern Illinois University to explain the variability of the adjustment factors and therefore to explain the deviations of the water consumption from the normal values by such variables as weather, water price, and conservation programs.
FIGURE 4.4 Demand adjustment factors for on-project planning area and users: (a) single-family, (b) multi-family, and (c) non-residential.
FIGURE 4.5 Demand adjustment factors for off-project planning area and users: (a) single-family, (b) multi-family, and (c) non-residential.
4.3 FORECASTING NUMBER OF ACCOUNTS

The precision of the prediction of water consumption, water production, and revenues depend on the accuracy of the number of accounts projections. Therefore a detailed study of the number of accounts has been performed for all categories: spatial area, type of user, and meter size.

4.3.1 Distribution of number of accounts

Figure 4.6 presents the distribution of the average number of accounts in each planning area/type of water use category recorded in 1990. More than 86% of the total number of accounts belongs to the single-family users. The residential consumers (single-family plus multi-family) have 92% while the non-residential users only 8% of all accounts. The number of accounts in the on-project planning area is slightly higher than the number of accounts in the off-project area--55.3% versus 44.7%.

FIGURE 4.6 Distribution of average number of accounts in each planning area/type of water use category recorded in 1990.
Figure 4.7 and Table 4.3 show the distribution of the average number of accounts, observed in each meter size category in calendar year 1990 for given planning area/user category.

**On-project**

- **Single-family**
  - 1" (1.49%)
  - 1 1/2" and 2" (0.21%)
  - 1/2" (98.30%)

- **Off-project**
  - 1" (3.35%)
  - 1 1/2" and 2" (0.22%)
  - 5/8" (96.42%)

**Multi-family**

- **On-project**
  - 2" (15.35%)
  - 3", 4", and 6" (0.26%)
  - 1 1/2" (9.74%)
  - 1" (14.11%)
  - 5/8" (60.53%)

- **Off-project**
  - 2" (37.87%)
  - 3", 4", and 6" (0.73%)
  - 5/8" (33.02%)
  - 1 1/2" (10.43%)
  - 1" (17.95%)

**Non-residential**

- **On-project**
  - 3" (1.07%)
  - 4" (1.01%)
  - 2" (19.20%)
  - 1 1/2" (15.65%)
  - 1" (21.02%)
  - 5/8" (41.40%)

- **Off-project**
  - 4" (1.03%)
  - 6" (0.64%)
  - 3" (5.28%)
  - 2" (30.54%)
  - 5/8" (24.26%)
  - 1 1/2" (17.62%)
  - 1" (20.47%)

**FIGURE 4.7** Distribution of average number of accounts in the calendar year 1990.
TABLE 4.3  Distribution of the number of accounts falling into particular meter size classes within spatial area/type of use categories--average values for the calendar year 1990.

<table>
<thead>
<tr>
<th>Meter Size</th>
<th>Single-family</th>
<th>Multi-family</th>
<th>Non-residential</th>
<th>Single-family</th>
<th>Multi-family</th>
<th>Non-residential</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/8&quot;</td>
<td>124969</td>
<td>7434</td>
<td>6815</td>
<td>111601</td>
<td>1183</td>
<td>1537</td>
</tr>
<tr>
<td></td>
<td>98.30%</td>
<td>60.53%</td>
<td>41.40%</td>
<td>96.42%</td>
<td>33.02%</td>
<td>24.26%</td>
</tr>
<tr>
<td>1&quot;</td>
<td>1888</td>
<td>1732</td>
<td>3459</td>
<td>3878</td>
<td>643</td>
<td>1297</td>
</tr>
<tr>
<td></td>
<td>1.49%</td>
<td>14.11%</td>
<td>21.02%</td>
<td>3.35%</td>
<td>17.95%</td>
<td>20.47%</td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>201</td>
<td>1197</td>
<td>2576</td>
<td>205</td>
<td>374</td>
<td>1116</td>
</tr>
<tr>
<td></td>
<td>0.16%</td>
<td>9.74%</td>
<td>15.65%</td>
<td>0.18%</td>
<td>10.43%</td>
<td>17.62%</td>
</tr>
<tr>
<td>2&quot;</td>
<td>62</td>
<td>1885</td>
<td>3161</td>
<td>55</td>
<td>1357</td>
<td>1935</td>
</tr>
<tr>
<td></td>
<td>0.05%</td>
<td>15.35%</td>
<td>19.20%</td>
<td>0.05%</td>
<td>37.87%</td>
<td>30.54%</td>
</tr>
<tr>
<td>3&quot;</td>
<td>0</td>
<td>9</td>
<td>176</td>
<td>0</td>
<td>11</td>
<td>334</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td>0.07%</td>
<td>1.07%</td>
<td>0%</td>
<td>0.31%</td>
<td>5.28%</td>
</tr>
<tr>
<td>4&quot;</td>
<td>0</td>
<td>12</td>
<td>167</td>
<td>0</td>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td>0.10%</td>
<td>1.01%</td>
<td>0%</td>
<td>0.11%</td>
<td>1.03%</td>
</tr>
<tr>
<td>6&quot;</td>
<td>0</td>
<td>11</td>
<td>105</td>
<td>0</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td>0.09%</td>
<td>0.64%</td>
<td>0%</td>
<td>0.31%</td>
<td>0.78%</td>
</tr>
<tr>
<td>No meter</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0.005%</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.003%</td>
<td>0%</td>
<td>0.02%</td>
</tr>
<tr>
<td>Total</td>
<td>127126</td>
<td>12282</td>
<td>16460</td>
<td>115742</td>
<td>3582</td>
<td>6334</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The single-family residential consumers nearly all use 5/8" meters, which constitute more than 98% (on-project area) and 96% (off-project area) of all accounts recorded for these categories, and more than 93% of total number of accounts equipped with 5/8" meters (Table A2.6, Appendix 2, presents the percentage distribution of accounts in each meter size category and for total service area number of accounts). Although the meters of 1" make a very small percentage of all the accounts in single-family category (1.5% on-project and 3% off-project planning area), they represent about 45% of the total number of 1" meters. The remaining accounts of the single-family category have 1 1/2" and 2" meters, and they
constitute a negligible percentage of accounts in both single-family category and a
given meter size category.

The 5/8” meter accounts form a significant portion of multi-family residential
accounts (60% on-project and 33% off-project area), but they make up less than 3%
of total number of 5/8" meter accounts. The percentage distribution of multi-family
accounts with 1", 1 1/2", and 2" meters varies between 10% and 38%. The remaining
accounts of this user category (3", 4", and 6" meters) are negligible. For the on-
project planning area: the 3" meter size category has had a constant number of 9
accounts since January 1988, the 4" category has had a constant number of 12
accounts recorded throughout the study period (86-90) with an occasional drop
down to 10 or 11 accounts, and the 6" category has had 11 accounts in almost all
months of 1990. For the off-project area: eleven 3" meter accounts have been
recorded in almost every month since summer 1989, four 4" meter accounts have
been observed since the beginning of 1988, and eleven 6" meter accounts have been
recorded since the summer of 1988.

The distribution of accounts for the non-residential users has a meaning
similar to the one for multi-family consumers. The accounts with meter size 5/8" to
2" constitute the majority of accounts in this planning area/type of use categories.
Though the accounts with large meters (3"- 6") do not constitute a large portion of
the non-residential accounts, they do include almost all the accounts in some meter
size categories. Ninety six percent of accounts with 3" meters, the same percentage of
accounts with 4" meters, and 88 percent of accounts with 6" meters belong to the
non-residential consumers.

4.3.2 Analysis of meter size fractions

The meter size fractions have been calculated by dividing the number of
accounts falling into a given meter size category by the total number of accounts for
each planning area/type of use category (Eq. 3.13). Then these fractions have been
averaged in the 6 month periods--January to June and July to December--over the
historical data horizon. The results are plotted in Figure 4.8 to Figure 4.11 and listed
in Table A2.7. All fractions show an increasing or decreasing trend. The meter size fraction estimator calculated from the most recent data set (the last six months of the 1990) is the most reliable indicator of meter size fractions in the near future.

The trend in the total number of meters in each of the six categories is explicitly modelled by the trend equations in the core models, whereas the different trends of proportions in the various meter size categories are not modelled. (Their modelling would require a shift-share type of analysis similar to that used in studies of trends in the population distribution of a given region.) The forecasting model has options so that different distributions of the proportions of meter sizes within each of the six core account models and the data in Table A2.7 can be used as a guide to trends in these proportions if different types of distributions in the future need to be analyzed in the future.
FIGURE 4.8 Proportions of accounts given meter size to total number of accounts (meter size fractions) for single-family category averaged in six month intervals.
FIGURE 4.9 Proportions of accounts given meter size to total number of accounts (meter size fractions) for multi-family category averaged in six month intervals (proportions of meter size categories: 5", 6", and 7" are not shown).
FIGURE 4.10 Proportions of accounts given meter size to total number of accounts (meter size fractions) for on-project/non-residential category averaged in six month intervals.
FIGURE 4.11 Proportions of accounts given meter size to total number of accounts (meter size fractions) for off-project/non-residential category averaged in six month intervals.
4.3.3 **Trends in number of accounts by meter size category**

The existence of the trend in meter size fractions results from the different growth rate observed in a number of accounts falling into individual meter size categories. In this section some results of trend analysis in number of accounts are presented.

Two periods of time have been assumed for the analysis. One of them covers the two most recent calendar years 1989 and 1990; the other contains July, 1987 to December, 1990. These two overlapping periods were chosen to determine the difference between the stable growth rate of the number of accounts which was observed during 1989 and 1990 and the average growth rate for the longest time interval in which the number of accounts follows consistent trend pattern. The data up to 1987 have not been applied into a regression analysis because of the high accounts growth rate, drastically different from the one observed in the most recent years. This pivotal point of growth rate change took place around the middle of 1987.

Each planning area/type of use category has been analyzed separately. The number of accounts falling into a particular meter size category and the total number of accounts (for planning area/user category being analyzed) have been used in the regression analysis. The dummy variables have been created to subdivide data into: total, 5/8", 1", 1 1/2", 2", 3", 4", and 6" classes. The regression analysis has been repeated for all the planning area/type of use categories, and for the three time periods--1/88 to 12/90, 7/87 to 12/90, and 7/86 to 6/87. The calculated coefficients of the regression lines are listed in Table A2.8. These lines are shown in Figures A2.1 to A2.12. The interpretation of the t-statistics of dummy variables is not applicable in this case, and therefore, the t-statistics (and standard errors of coefficients) are not listed in Table A2.8.

During the analysis of the data from the last two calendar years, an exception has been made: the number of accounts from the off-project/multi-family/1.5" meter category declined by more than 8 percent between April and May, 1989, thus, for this category the regression line has been determined on the data which covers 5/89 to 12/90.
The regression analysis of the fiscal year 1986 observations has been performed to estimate of the growth rate which occurred few years ago and which may occur in future.

The quite different development of the on-project and off-project areas is observed: for the on-project service region each month on average about 26 new accounts are established for single-family users, 12 accounts are cancelled for multi-family consumers, and 7 new accounts are created for non-residential consumers, whereas for the off-project region each month about 240 new accounts are installed for single-family users, 3 new accounts for multi-family consumers, and 19 new accounts for non-residential consumers.

The numbers presented above have been compared to the ones estimated for the fiscal year 1986: on average in each month of that fiscal year, 120 new accounts were added in the on-project/single-family category, seven accounts were added in the on-project/multi-family category, and 18 new accounts were added in the on-project/non-residential category. The numbers of new accounts per month for the off-project area were 502, 19, and 35 respectively.

The average growth rate in number of accounts observed during the last two calendar years (89 and 90) is consistent with the average growth rate observed during the last 3.5 years (July, 87 to December, 90). Only the off-project/single-family category exhibits decrease in the average growth rate of the number of accounts when comparing the last two-year period and the last 3.5-year period.

### TABLE 4.4  Growth rates of number of accounts by planning area/user category [#acct/month].

<table>
<thead>
<tr>
<th>Planning area</th>
<th>User category</th>
<th>Single-family</th>
<th>Multi-family</th>
<th>Non-residential</th>
</tr>
</thead>
<tbody>
<tr>
<td>average for period from July 86 to June 87</td>
<td>On-project</td>
<td>120</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Off-project</td>
<td>503</td>
<td>19</td>
<td>35</td>
</tr>
<tr>
<td>average for period from July 87 to December 90</td>
<td>On-project</td>
<td>27</td>
<td>-12</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Off-project</td>
<td>255</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>average for period from January 89 to December 90</td>
<td>On-project</td>
<td>27</td>
<td>-12</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Off-project</td>
<td>240</td>
<td>3</td>
<td>19</td>
</tr>
</tbody>
</table>
4.3.4 Comparison of selected models

To estimate the range of a possible error produced by the application of constant meter size fractions, two sets of predictions of the number of accounts in each planning area/type of use/meter size category have been made for December 1995. One of them has been performed by the application of the six core account models estimated from the 89-90 data set as well as by the application of average meter size fractions calculated from the 7/90-12/90 data. (The core models have been used to predict number of accounts falling into a given planning area/type of user category. These predictions have been multiplied by the meter size fractions to calculate the number of accounts for each individual meter size category). The second set of predictions have been produced by the application of individual planning area/user/meter size category regression models determined from the 89-90 observations (for each individual meter size category a regression line has been estimated). The coefficients of the individual models and core account models are listed in Table A2.8. The meter size fractions (extracted from Table A2.7) and the predicted values are listed in Table 4.5. The differences in predictions as a percentage of the average projected number of accounts, and the difference in predictions as a percentage of the average of projected all accounts falling into given meter size category (the sum for all the planning area/user categories) are also listed in Table 4.5.

The results of comparison show that for the residential user categories the trend in meter size fractions can be neglected. It has no practical influence on the precision of the five year predictions of the number of accounts falling into a given meter size category: the error which may be introduced by the assumption of the constant meter size fractions is less than two percent (of all accounts belonging to the given meter size category).

Relatively high differences (up to 30%) are indicated for the predictions of the accounts from the non-residential/meter size 3", 4" and 6" categories. The high number of accounts with small meters (1" to 2") forms the pattern of the growth rate of the core account models. The small number of accounts with large meter diameters (3", 4" and 6"), does not show the same growth pattern as the 1" to 2" meters.
Therefore, trends of the accounts with 3", 4", and 6" meters are not compatible to the trend in the total number of accounts. These errors are not critical because of the high variability in large meters data. For example, the number of accounts for the off-project/non-residential/meter size 3" category decreased in the 89-90 period by about 60 accounts, whereas during eight months of the 1986 the number of accounts increased by about 280 (Figure A2.11).

The 5-year forecast of the number of accounts by two methods discussed above, does not bring strong evidence against the application of the constant meter size fractions. The results obtained by the two distinct methods are not significantly different. It must be noticed that the comparison presented above assumes that the trends in 89-90 will continue for five years. Historical data shows that this assumption is highly questionable. The premise is unprovable that the regression of individual meter size categories is "better" than just a trend applied to the total number of meters in each category.
TABLE 4.5  Comparison of predicted number of accounts for December 1995 by application both: individual meter size regression models and core account models with meter size fractions.

<table>
<thead>
<tr>
<th>Meter size</th>
<th>Meter Size Fraction (avg. for Jul-Dec, 1990)</th>
<th>Prediction by:</th>
<th>Difference as percent. of:</th>
<th># acct</th>
<th>%</th>
<th># acct</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>individual regression lines</td>
<td>core model and meter size fractions</td>
<td>sum of avg. predictions in given meter size category</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-family</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>0.9830731</td>
<td>126627</td>
<td>126646</td>
<td>-0.02</td>
<td>-0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.0148620</td>
<td>1931</td>
<td>1915</td>
<td>0.83</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>0.0015798</td>
<td>200</td>
<td>204</td>
<td>-1.57</td>
<td>-0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&quot;</td>
<td>0.0004851</td>
<td>69</td>
<td>62</td>
<td>9.79</td>
<td>0.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>128827</td>
<td>128827</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Multi-family</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>0.6047662</td>
<td>6819</td>
<td>6950</td>
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<td>-0.05</td>
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<td></td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.1411280</td>
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<td>1622</td>
<td>0.68</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>0.0974029</td>
<td>1107</td>
<td>1119</td>
<td>-1.12</td>
<td>-0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&quot;</td>
<td>0.1541163</td>
<td>1886</td>
<td>1771</td>
<td>6.26</td>
<td>1.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3&quot;</td>
<td>0.0007351</td>
<td>10</td>
<td>8</td>
<td>15.32</td>
<td>0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4&quot;</td>
<td>0.0009666</td>
<td>12</td>
<td>11</td>
<td>4.38</td>
<td>0.16</td>
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</tr>
<tr>
<td>6&quot;</td>
<td>0.0008849</td>
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<td>10</td>
<td>88.62</td>
<td>6.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>11492</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Non-resid.</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>0.4125648</td>
<td>6195</td>
<td>6977</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.2101670</td>
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<td>3554</td>
<td>1.11</td>
<td>0.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>0.1571446</td>
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<td>2657</td>
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<tr>
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<td>0.1927968</td>
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<td>3260</td>
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<td>181</td>
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<td></td>
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<td>110</td>
<td>34.72</td>
<td>19.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>16911</td>
<td>16911</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single-family</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>0.9641779</td>
<td>126535</td>
<td>126694</td>
<td>-0.13</td>
<td>-0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.0335784</td>
<td>4543</td>
<td>4412</td>
<td>2.92</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>0.0017712</td>
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<td>233</td>
<td>9.18</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&quot;</td>
<td>0.0004725</td>
<td>67</td>
<td>62</td>
<td>8.06</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
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<td>131401</td>
<td>131401</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multi-family</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>0.3284285</td>
<td>1095</td>
<td>1251</td>
<td>-13.30</td>
<td>-0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1&quot;</td>
<td>0.1792056</td>
<td>669</td>
<td>683</td>
<td>-1.95</td>
<td>-0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2&quot;</td>
<td>0.1043392</td>
<td>401</td>
<td>397</td>
<td>0.99</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2&quot;</td>
<td>0.3807634</td>
<td>1615</td>
<td>1450</td>
<td>10.75</td>
<td>1.77</td>
<td></td>
<td></td>
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<tr>
<td>3&quot;</td>
<td>0.0030729</td>
<td>13</td>
<td>12</td>
<td>10.08</td>
<td>0.23</td>
<td></td>
<td></td>
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<tr>
<td>4&quot;</td>
<td>0.0011174</td>
<td>4</td>
<td>4</td>
<td>-6.21</td>
<td>-0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6&quot;</td>
<td>0.0030729</td>
<td>11</td>
<td>12</td>
<td>-5.18</td>
<td>-0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>3809</td>
<td>3809</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Meter size</td>
<td>Meter Size Fraction (avg. for Jul-Dec, 1990)</td>
<td>Prediction by: individual regression lines</td>
<td>Prediction by: core model and meter size fractions</td>
<td>Difference as percent of: average prediction</td>
<td>Difference as percent of: sum of avg. predictions in given meter size category</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>---------------------------------------------------</td>
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<td># acct</td>
<td>%</td>
<td>%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-resid.</td>
<td>5/8&quot;</td>
<td>0.2410075</td>
<td>1620</td>
<td>1829</td>
<td>-12.10</td>
<td>-0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1&quot;</td>
<td>0.2050776</td>
<td>1640</td>
<td>1556</td>
<td>5.26</td>
<td>0.61</td>
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<td></td>
<td>1 1/2&quot;</td>
<td>0.1770222</td>
<td>1488</td>
<td>1343</td>
<td>10.25</td>
<td>2.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2&quot;</td>
<td>0.3061519</td>
<td>2404</td>
<td>2323</td>
<td>3.41</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3&quot;</td>
<td>0.0525174</td>
<td>250</td>
<td>398</td>
<td>-45.75</td>
<td>-27.60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4&quot;</td>
<td>0.0102979</td>
<td>108</td>
<td>78</td>
<td>31.65</td>
<td>9.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6&quot;</td>
<td>0.0079254</td>
<td>78</td>
<td>60</td>
<td>25.74</td>
<td>7.68</td>
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</tr>
<tr>
<td>Total</td>
<td>5/8&quot;</td>
<td>268891</td>
<td>270346</td>
<td>-0.54</td>
<td></td>
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<td></td>
</tr>
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<td></td>
<td>1&quot;</td>
<td>14010</td>
<td>13741</td>
<td>1.94</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1/2&quot;</td>
<td>6339</td>
<td>5954</td>
<td>6.27</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2&quot;</td>
<td>9694</td>
<td>8929</td>
<td>8.21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3&quot;</td>
<td>476</td>
<td>600</td>
<td>-23.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4&quot;</td>
<td>346</td>
<td>265</td>
<td>26.44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6&quot;</td>
<td>271</td>
<td>192</td>
<td>34.32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An additional comparison is presented in Table A2.9. This table contains the change in the number of accounts during a 12-month period (a year) for each planning area/type of use/meter size category. Those values have been calculated by the application of the individual regression lines and by the application of the core regression lines and meter size fractions. Three time periods have been selected for this comparison: fiscal year 1986, 7/87-12/90, and 1/89-12/90. For each period, a separate set of regression models has been estimated. The set of the meter size fractions which have been applied, was estimated from the 7/90-12/90 observations. Only the non-residential categories show large discrepancies between the annual change in the number of accounts modelled by the individual regression lines and modelled by meter size fractions.
4.4 LIFELINE PROPORTIONS OF WATER CONSUMPTION

The data from fiscal years: 1986-1989 have been applied to estimate the set of the monthly average proportions of the water consumed below or equal to the Lifeline 6 ccf for each planning area/type of use category. A similar set of proportions has been calculated for the Lifeline 10 ccf. This proportions are listed in Table A2.10 (Appendix 2) and presented in Figure 4.12 and Figure 4.13.

It is possible to determine the relationships between the proportions of the water consumption for different Lifelines and to describe these relationships by appropriate functions. Once the basic set of proportions is defined, other set of proportions can be calculated from these functions. This calculation allows for a quick and precise estimation of both the water consumed above the Lifeline and the revenue collected from the volume charges for various combinations of the Lifeline levels and the months of the year. The results of these estimations are helpful in determining the optimal selection of the Lifeline values throughout the year. Table A2.10 contains an additional column of values of the proportions for the Lifeline 10 ccf which have been directly estimated, by the application of linear function from the proportions of water consumed below the Lifeline 6 ccf.
FIGURE 4.12 Proportions of water consumed below the Lifeline 6 [ccf] to total water consumption for users: (a) single-family, (b) multi-family, and (c) non-residential.
FIGURE 4.13 Proportions of water consumed below the Lifeline 10 [ccf] to total water consumption for users: (a) single-family, (b) multi-family, and (c) non-residential.
Table 4.6 shows the coefficients determined by the regression analysis of the linear relationship between the proportions specific for the Lifeline 6 [ccf] and the proportions specific for the Lifeline 10 [ccf]. This relationship is described by the formula:

\[ h'_{10}(i,m) = a + b h'_6(i,m) \quad (4.1) \]

where \( h'(i,m) \) is the proportion of water consumed below the specified Lifeline and \( h'(i,m) = 1 - h(i,m) = \) one minus the proportion of water consumed above the Lifeline, and it has been used to calculate the values given in Table A2.10, column "10 ccf-estimated".

\[ TABLE \ 4.6 \quad \text{Results of the regression analysis of the linear relationship between the proportions specific for the Lifeline 6 [ccf] and the proportions specific for the Lifeline 10 [ccf] (Eq. 4.1)}. \]

<table>
<thead>
<tr>
<th>Planning Area</th>
<th>User Category</th>
<th>Constant (a) (Std Err of Y Est)</th>
<th>Coefficient (b) (Std Err of Coef.)</th>
<th>R Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-project</td>
<td>Single-family</td>
<td>0.066800 (0.006581)</td>
<td>1.326694 (0.020979)</td>
<td>0.9975</td>
</tr>
<tr>
<td></td>
<td>Multi-family</td>
<td>0.008573 (0.001323)</td>
<td>1.513771 (0.035328)</td>
<td>0.9946</td>
</tr>
<tr>
<td></td>
<td>Non-residential</td>
<td>0.003054 (0.000647)</td>
<td>1.515276 (0.026414)</td>
<td>0.9970</td>
</tr>
<tr>
<td>Off-project</td>
<td>Single-family</td>
<td>0.046479 (0.004588)</td>
<td>1.397244 (0.016947)</td>
<td>0.9985</td>
</tr>
<tr>
<td></td>
<td>Multi-family</td>
<td>0.001212 (0.000400)</td>
<td>1.633404 (0.017837)</td>
<td>0.9988</td>
</tr>
<tr>
<td></td>
<td>Non-resid. residential</td>
<td>0.001004 (0.000925)</td>
<td>1.591289 (0.028232)</td>
<td>0.9969</td>
</tr>
</tbody>
</table>

No. of Observations 12
For example, based on the results of the regression analysis presented in Table 4.6 the relation between the proportions $h'_6(m,i)$ and the proportions $h'_10(m,i)$ for off-project/single-family category is: $h'_10 = 0.04648 + 1.397244 h'_6$. These observed and modelled relationships are shown in Figure 4.14:

![Graph showing the relationship between $h'_6$ and $h'_10$.](image)

**FIGURE 4.14** Example of the relationship between the proportions for the Lifelines 6 [ccf] and 10 [ccf] (off-project/single-family category).

### 4.5 REVENUE UPDATE WITHIN CURRENT FISCAL YEAR

The method of the update of current fiscal year water consumption and revenue which is described in Section 3.6, utilizes the fact that the new value can be calculated as a weighted average of predictions made from two different procedures. One of the predictions can be made by using a core water consumption models, and the second one may be made by applying the measured water use and monthly water use fractions. This can be expressed by the equation (after Eq. 31):
\[ W_{C}(i,y,k) = [1 - \beta(k)] \ W(i,y) + \beta(k) \ W_d(i,y,k) \]  
(4.2)

where:

- \( W_{C}(i,y,k) \) = updated prediction of water use in the current fiscal year [ccf/year]
- \( W(i,y) \) = expected water use in the given combination of spatial area and the type of use \( i \) in the fiscal year \( y \) (Eq. 3.27) [ccf/year]
- \( W_d(i,y,k) \) = expected water consumption in the spatial area and the type of use category \( i \) and in the fiscal year \( y \) after \( k \) months elapsed (Eq. 3.28) [ccf/year]
- \( \beta(k) \) = weights to be assigned to the consumption forecasts.

The determination of weight coefficients \( \beta(k) \) has been performed for the longest available data-set--total monthly water supplied into the distribution network in Phoenix, Arizona in fiscal years from 1980/1981 to 1988/1989. These data has been extracted from the report entitled *City of Phoenix Arizona, Water and Wastewater Department, Water Production and Consumption in Millions of Gallons* (for various fiscal years) and they are presented in Table A1.3.

For each fiscal year the total water supplied to the distribution system has been calculated by the summation of the monthly values. Then, the regression lines have been determined for the six periods: 1980/81 - 1983/84, 1980/81 - 1984/85, 1980/81 - 1985/86, 1980/81 - 1986/87, 1980/81 - 1987/88, and 1980/81 - 1988/89. The parameters of fitted lines:

\[ W_a(y) = a + b \cdot y \]  
(4.3)

where:

- \( W_a(y) \) = predicted water use in year \( y \) [million gallons]
- \( y \) = fiscal year [for 1980/81 \( y = 1980 \)]
- \( a, b \) = coefficients

are listed in Table A2.11. Based on these regression lines the prediction of water use has been made for the year following the time period used in the regression analysis. For example, the prediction of water use in fiscal year 1986/87 has been made from the regression line which has been estimated from data from fiscal years 1980/81 to 1985/86. The predictions of water supply \( W_a(y) \) in years 1984/85 to 1988/89 and
observed values $W(y)$ are presented in Table A2.12. Differences between observed and predicted values are also shown. The variance of the fiscal year water supply predictions $C_w^2$ has been estimated as 3831548.48 [million gallons]$^2$.

For the same time periods used to determine the regression lines, the average monthly fractions of water consumption have been calculated. For each month of the fiscal year, the monthly fractions of water consumption have been determined by dividing the recorded monthly water use by the total consumption in the given fiscal year $\frac{W(m,y)}{W(y)}$. Then, the average monthly fractions $d_w(m,Y)$ have been calculated for the same time periods (fiscal years $Y$) as used for the estimation of regression lines. The distribution of these average fractions is presented in Table A2.13. This process can be expressed by the equation:

$$d_w(m,Y) = \frac{1}{Y} \sum_{y=1}^{Y} \frac{W(m,y)}{W(y)} \tag{4.4}$$

where:

- $d_w(m,Y)$ = average monthly water consumption fraction for the month $m$
- $W(m,y)$ = observed water use in month $m$ of the fiscal year $y$ [ccf/month]
- $W(y)$ = observed water consumption in the fiscal year $y$ [ccf/year]
- $Y$ = historical years of record used for the estimation of average monthly fraction.

For each month of the fiscal years 1984/85 to 1988/89 the cumulative average monthly fractions and the cumulative water supply have been calculated (Table A2.14 and Table A2.15 respectively). Then, the respective fiscal year predictions have been made from division of cumulative water supply by the cumulative monthly fractions:

$$W_d(y,k) = \frac{\sum_{m=1}^{k} W(m,y)}{\sum_{m=1}^{k} d_w(m,Y)} \tag{4.5}$$
where:

\[ W_d(y,k) = \text{expected water supply in the fiscal year } y \text{ after } k\text{-month observations} \]

\[ W(m,y) = \text{observed water supply in month } m \text{ of the fiscal year } y \text{ [ccf/month]} \]

\[ d_w(m,y) = \text{average monthly fraction for month } m \text{ (determined for preceding year } y \text{ period) (Eq. 4.4)} \]

\[ k = \text{number of cumulated values}. \]

Calculated predictions are shown in Table A2.16. The differences between the predicted and recorded water supply, and the variance of these differences for each month are presented in Table A2.17.

Finally, the weights \( b(k) \), which are assigned to the consumption forecasts, have been calculated by applying formula:

\[
b(k) = \frac{C_w^2}{C_d(k)^2 + C_w^2}
\]  

(4.6)

where:

\[ C_w^2 = \text{sample variance of differences between the observed water supply and the predicted one from the regression lines} \]

\[ = \text{var} \{ W(y) - W_a(y) \} \text{ for } y = 1984/85 \text{ to } 1988/89 \]

\[ = 3831548.48 \text{ [million gallons]}^2 \]

\[ C_d(k)^2 = \text{sample variance of differences between the recorded water supply and the estimated one from } k\text{-month observations} \]

\[ = \text{var} \{ W(y) - W_d(y,k) \} \text{ for } y = 1984/85 \text{ to } 1988/89. \]

The values of the coefficient \( b(k) \) are listed in Table 4.7 and plotted in Figure 4.15.
TABLE 4.7  Estimated values of the coefficient $\beta(k)$.

<table>
<thead>
<tr>
<th>Month</th>
<th>Coefficient $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul</td>
<td>0.08</td>
</tr>
<tr>
<td>Aug</td>
<td>0.14</td>
</tr>
<tr>
<td>Sep</td>
<td>0.16</td>
</tr>
<tr>
<td>Oct</td>
<td>0.18</td>
</tr>
<tr>
<td>Nov</td>
<td>0.25</td>
</tr>
<tr>
<td>Dec</td>
<td>0.28</td>
</tr>
<tr>
<td>Jan</td>
<td>0.37</td>
</tr>
<tr>
<td>Feb</td>
<td>0.46</td>
</tr>
<tr>
<td>Mar</td>
<td>0.53</td>
</tr>
<tr>
<td>Apr</td>
<td>0.77</td>
</tr>
<tr>
<td>May</td>
<td>0.95</td>
</tr>
<tr>
<td>Jun</td>
<td>1.00</td>
</tr>
</tbody>
</table>

FIGURE 4.15 Weight coefficient $\beta(k)$.

The analysis of the behavior of the coefficient $\beta(k)$ in time for total consumption in Phoenix showed that it can be approximated by the relationship:
where \( k \) is the number of elapsed months. This means that an estimate of annual consumption produced using 4 months of measured consumption should be weighted by 4/12 and the estimate obtained by projecting trends from the previous year by 8/12. Likewise, if the consumption forecast has 7 months of recorded values available for analysis, the weight assigned to the annual forecast computed from these values is 7/12 and weight 5/12 is attached to the trend forecast values. After all the consumption in a year has been recorded, \( \beta(k) = 1 \) as it should do, since then no forecast is needed.

4.6 UNACCOUNTED FOR WATER AND WATER PRODUCTION

4.6.1 Data availability

The following analysis of the unaccounted for water has been based on water production data taken from the report entitled *City of Phoenix Arizona, Water and Wastewater Department, Water Production and Consumption in Millions of Gallons* (for various fiscal years). These data are listed in Table A1.1. The consumption data applied in the analysis have been taken from the WCIS 825.1 Report. The set of the available water use data is presented in Table A1.3.

The fiscal years 1981 (1981/82), 1983 (1983/84), and 1984 (1984/85) have been excluded from the analysis. The consumption data from the calendar year were questionable. For example the recorded water consumption in the June and in the July of 1984 were \( 22 \times 10^9 \text{ gal} \) and \( 16.5 \times 10^9 \text{ gal} \), respectively, whereas the magnitude of water use for the both months should be about \( 10 \times 10^9 \text{ gal} \). The fiscal year 1981 has been excluded from the data set because the preliminary analysis indicated four months for which the consumption has been higher than the production (months: September, October, November, and February). The data applied in the estimation of the time shift coefficient and the unaccounted for water are listed in Table A2.18, Appendix 2.
4.6.2 Determination of time-shift coefficient

In Phoenix, Arizona, the water consumption is not measured at the same time when the water production is recorded. The plot of the recorded water consumption and the water production presented in Figure 4.16 clearly indicates the time shift between these two series. The line representing the consumption is shifted to the right of the line representing the production. For the clarity of the plots only the data from three fiscal years are shown.

For each fiscal year the proportions of the monthly consumption and production to the average month of the given fiscal year have been calculated from the relations (3.38) and (3.39). The first harmonics of the Fourier series (Eq. 3.40) and (Eq. 3.41) have been determined and the time $T_w$ when the Fourier approximation of the water consumption proportions is equal to the average value and the time $T_p$ when the Fourier approximation of the water production proportions is equal to the average value have been calculated.

![Chart showing monthly water production and consumption from 1986 to 1988.](chart.png)

The values of the Fourier series first harmonic coefficients and the time when this harmonic equals to one for the water production proportions are as follows:

- $A_p = 0.37024501$
- $B_p = 0.18233074$
- $T_p = 3.87394625$

while for the water consumption data these values are:

- $A_w = 0.2687274$
- $B_w = 0.24668353$
- $T_w = 4.41836601$.

The estimated time-shift coefficient $\Delta T$ is equal to 0.544 [month], e.g. approximately 16 days. This implies that on average consumption lags behind production by approximately 16 days on average. For example, if the January production is measured on the first day of February (period covered: 1/1-2/1), the consumption which corresponds to this production is that which is recorded between January 16 and February 15, on average.

The next step is the correction of the recorded water consumption to make them compatible with the production data. The new corrected consumption series has been estimated by applying the estimated time shift coefficient into the weighted averaging formula (3.44). Therefore the corrected water consumption can be computed from the equation:

$$W'(m,y) = (1 - 0.544) W(m,y) + 0.544 W(m+1,y) \quad (4.8)$$

where:
- $W'(m,y) =$ corrected water use in the month $m$ of the fiscal year $y$
- $W(m,y), W(m+1,y) =$ recorded water consumption in the month $m$ and the month $m+1$ of the fiscal year $y$. 
To find out if the corrected water use series $W'(m,y)$ are compatible with the production record, the proportions $w'(m,y)$ of the corrected monthly consumption to the average monthly consumption in the fiscal year have been calculated:

$$w'(m,y) = \frac{12 W'(m,y)}{W'(Y)}.$$ (4.9)

Then, the coefficients of the first harmonic of the Fourier function:

$$w'_f(t) = w' + A_w' \cos\left(\frac{2\pi}{12} t\right) + B_w' \sin\left(\frac{2\pi}{12} t\right)$$ (4.10)

have been determined to be $A_w' = 0.31712233$ and $B_w' = 0.15535865$.

According to the relationship:

$$T_w' = \frac{12}{2\pi} \tan^{-1}\left(\frac{-A_w'}{B_w'}\right)$$ (4.11)

the time $T_w'$ when the Fourier approximation of the corrected water use proportions is equal to the average value ($w'_f(t) = w' = 1$) has been estimated ($T_w' = 3.87000982$).

The differences between the $T_p$ (production series) and the $T_w'$ (corrected consumption series) is about 0.0039 of a month (about 3 hours), therefore there is no shift in time between the production and the corrected consumption series.

The first harmonics of the Fourier series fitted to the water production proportions, water consumption proportions, and the corrected water consumption proportions are presented in Figure 4.17.
FIGURE 4.17 Comparison of the Fourier series approximation of production proportions, water consumption proportions, and corrected water consumption proportions. The corrected consumption has been brought back in time by 16 days to make it contemporaneous with the production data.

There is a visible shift between the production and the consumption curve, whereas there is no difference between the phases of the production plot and the corrected consumption line.

4.6.3 **Unaccounted for water as function of month of the year**

The annual values of the unaccounted for water have been determined by the subtraction of the corrected consumption values from the production data for each fiscal year, and then the proportions of the determined unaccounted for water to the
annual corrected water consumption have been calculated. These proportions are listed in Table A2.19, Appendix 2 and drawn in Figure 4.18.

![Graph showing unaccounted consumption over fiscal years 1982 to 1988]

**FIGURE 4.18** Proportions of annual unaccounted for water to annual water use.

Although the plot presented in Figure 4.18 shows some evidence of trend the regression analysis does not support the hypothesis of trend existence. The slope of the regression line is insignificant. The fitted line is as follows:

\[
\begin{align*}
\text{u}_N(y) &= -0.4095 + 0.006104 y \\
& (-1.322) \quad (1.6868)
\end{align*}
\]

where values in brackets are the t-statistics. Since the trend in the proportions of the unaccounted for water is not significant, the average monthly proportions of the unaccounted to the corrected consumption have been calculated from the relationship:
\[ u_N(m,y) = \frac{U_N(m,y)}{W'(m,y)} \]  \hspace{1cm} (4.13)

where:

- \( u_N(m,y) \) = average proportions of the unaccounted for water for the month \( m \) of the fiscal year \( y \)
- \( U_N(m,y) \) = unaccounted for water in the month \( m \) of the fiscal year \( y \)
  \[ = P(m,y) - W'(m,y) \]
- \( W'(m,y) \) = corrected water consumption in the month \( m \) of the fiscal year \( y \).

The observed monthly amounts of the unaccounted for water for Phoenix, Arizona are listed in Table A2.18. It can be noticed that a few months had a negative value of the unaccounted for water. February is a special month during which the water consumption is usually greater than the water production or during which these two amounts are very close. These impossible observations are probably caused by the errors introduced by the measurement devices. Generally the proportions of the unaccounted for water are higher for summer months and lower for winter months.

It is convenient to express monthly proportions of the unaccounted for water as a function of the month. Therefore the coefficients of all the six harmonics of Fourier series have been determined from the calculated proportions of the unaccounted for water. These coefficients are listed in Table 4.8 and the smoothed value of unaccounted for water by month are shown in Table A2.20. They have a mean value of 10.2% (expressed as a percent of consumption given by WCS 825.1 Report) and vary from a low of 3% in the winter months to a high of 17% in the summer months. When expressed as percent of production, unaccounted for water represents 9.2% of production on average for these years, rising to a maximum of 15% during the summer and falling to a minimum of about 3% during the winter. These data represent the average of computations for data from 1982 and 1985-88.
TABLE 4.8 The Fourier series coefficients of the monthly proportions of the unaccounted for water. (The asterisk indicates the harmonic used in construction of the unaccounted for water model.

<table>
<thead>
<tr>
<th>i</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06237</td>
<td>0.033062 *</td>
</tr>
<tr>
<td>2</td>
<td>-0.00199</td>
<td>-0.00467</td>
</tr>
<tr>
<td>3</td>
<td>-0.01055</td>
<td>-0.0265</td>
</tr>
<tr>
<td>4</td>
<td>-0.01065</td>
<td>0.020181</td>
</tr>
<tr>
<td>5</td>
<td>0.001711</td>
<td>-0.0211</td>
</tr>
<tr>
<td>6</td>
<td>-0.01504</td>
<td>0</td>
</tr>
</tbody>
</table>

\( u_N = 0.101818 \)

The analysis indicates that from 1982 to 1989, the unaccounted for water averaged 10.1818% of consumption. This means that 1/1.101818, or 90.8% of water produced, was delivered and metered in Phoenix over that period, so unaccounted for water averaged 9.2% of production.

Only the first harmonic have been chosen to model the monthly proportions of the unaccounted. Therefore, the Fourier approximation function can be written as follows:

\[
 u_{Nf}(m) = u_N + A_1 \cos\left(\frac{2\pi}{12} m\right) + B_1 \sin\left(\frac{2\pi}{12} m\right) \tag{4.14}
\]

where:

- \( u_{Nf}(m) \) = the Fourier approximation of the unaccounted for water proportions
- \( u_N \) = average of the unaccounted for water proportions (Table 4.8)
- \( m \) = month of the fiscal year \([m=1 \text{ for July}]\)
- \( \pi \) = 3.1415...
- \( A_1, B_1 \) = coefficients (Table 4.8).

The plot of the presented above Fourier series approximation of the monthly proportions of the unaccounted for water with the calculated proportions of unaccounted for water to the corrected consumption are drawn in Figure 4.19.
4.6.4 Forecasting water production

The recorded water consumption has been corrected by applying the equation (4.8), and then the production has been determined from the following relationship:

\[
P'(m,y) = W'(m,y) \ [1 + u_{NF}(m)]
\]  \hspace{1cm} (4.15)

where:

- \(P'(m,y)\) = water production estimated from the corrected water consumption for the month \(m\) of the fiscal year \(y\)

- \(W'(m,y)\) = corrected water consumption in the month \(m\) of the fiscal year \(y\)

- \(u_{NF}(m)\) = the Fourier approximation of the unaccounted for water proportions \(u_N(m,y)\).
The plots of the recorded water production and the water production estimated from the historical record of the water consumption are presented in Figure 4.20.

![Figure 4.20 Plot of the recorded water production and the water production estimated from the water consumption record.](image)

The standard error of monthly water production estimation is $372.9545 \times 10^6$ gal, and the relative error is 4.9%. 

\[ \text{Estimated from the water consumption record.} \]
5 SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER STUDY.

In this thesis a model of water consumption, water production and the revenue generated from water delivery for the City of Phoenix, Arizona is developed. The traditional modelling of water use called "cascade modelling" has been applied. Two new procedures have been developed in this research: one of them, based on Bayesian theory, has been used in the process of the updating predictions after new data are available. The second procedure, based on Fourier series, enables estimation of the unaccounted for water when the production observations are not compatible. The production readings are time shifted in relation to the consumption measurements.

The model considers the water use in two planning areas: on-project and off-project. Three types of users are distinguished for each planning area: single-family, multi-family, and nonresidential. Each planning area/type of user category is further subdivided into seven meter size classes: 5/8", 1", 1&1/2", 2", 3", 4", and 6".

Six values represent the average-month water consumption per account. The water use per account for two categories: on-project/single-family category and off-project/multi-family category showed small but significant trend. Since the factors causing these trends are not clear at this stage of analysis, a constant values of monthly water use per account [in ccf/acct/month] have been assumed as follows: for on-project planning area and for single-family consumers 17.92, for multi-family consumers 86.20, and for non-residential consumers 108.40. For off-project service area these values are slightly higher and they are respectively: 19.67, 128.29, and 152.41 [ccf/acct/month].

The seasonal pattern of the water consumption for the residential consumers is similar for the two planning areas. The single-family monthly water use varies from 153% of average month in July (maximum) to 64% of average month in December (minimum). The highest multi-family consumption is in July--128% while the lowest multi-family consumption is in February--79% of an average month. For the non-residential category the seasonal variation of water use in off-project area is significantly higher than the one for the on-project area: For this category the water
use in on-project area varies from 74% (February) to 130% (July) whereas in off-project area it varies from 57% (February) to 140% (July).

The growth of the city is represented by the six regression lines in numbers of water accounts, each for a given type of user/planning area category (a month is assumed as a time unit). The number of accounts in Phoenix, Arizona was increasing but at a declining rate during the study period (July, 1986 to December, 1990). For the on-project/single family category the growth rate estimated in the fiscal year 1986/87 was 120 accounts per month, whereas for the same category the growth rate in 1989 and 1990 years was 27 accounts per month. The respective growth rates for the multi-family category were 8 and -12 accounts per month (where a negative number indicates decrease in number of accounts). For the non-residential category, the respective growth rates were 18 and 7 accounts per month. The off-project service area showed even higher decreases in growth rates (For a single-family user category, the growth rate changed from 503 accounts per month in fiscal year 1986/87 to average 240 accounts per month in calendar years 1989-1990. For the multi-family user category, the growth rate changed from 19 account per account in fiscal year 1986/87 to about 3 accounts per month in 1989-90. For the non-residential category, the growth rate decreased from 35 accounts per month in fiscal year 1986/87 to 19 accounts per month in years 1989-1990). The regression lines estimated from the 1989-1990 data are assumed as most reliable for predicting water use in the immediate future. Yet the analysis of the growth rates in different periods showed considerable uncertainty about predictions of number of accounts, especially those which are made for 5-10 year horizon.

For each planning area/type of user category the proportions of monthly water consumption below the two lifelines--6 ccf and 10 ccf--were determined in order to estimate the revenue generated by the volume charges. The analysis showed that there is a linear relationship between the proportions of water used below the lifeline 10 ccf and the proportions of water used below the lifeline 6 ccf. This feature can be used to derive additional sets of proportions from the base one for different lifelines.

To estimate the revenue generated from the base charges, the number of accounts in each meter size category must be modelled. This is achieved by the multiplication of estimated number of accounts by meter size fractions, where the
meter size fractions are equal to the number of accounts falling into a given meter size category divided by the total number of accounts. For all categories (planning-area/user/meter size), the meter size fractions have shown trends. Therefore the monthly fractions used in estimating revenue have been calculated from most recent observations available, from July, 1990 to December,1990. This approach precisely estimates the number of accounts falling into categories of small meter size: 5/8", 1", 1&1/2", and 2", whereas the estimates of the number of accounts falling into categories of large meters: 3", 4", and 6" are questionable by this approach. Since the accounts with 5/8" - 2" meters constitute more than 99.5% of the total number of accounts (5/8"-90%, 1"-4.6%, 1&1/2"-2%, and 2"-3%), it has been concluded that the errors in modeling trends in accounts with 3", 4", and 6" meters have no significant influence on the projected revenue produced from base charges.

To predict water production, the relationship between total consumption and production for the city as a whole for data from 1982 and 1985-88 has been analyzed. It has been demonstrated that on average in these years, recorded consumption lags recorded production by about 16 days and that unaccounted for water averages 9.2% of production annually, varying in a smooth fashion from a high of 15% in the summer months to a low of 3% in the winter months. Using these figures, projections of monthly total production can be computed from projections of monthly total consumption.

Since the parameters of the water forecasting model have been determined from the five year data set, the maximum reasonable time horizon for the projections of the water use, water production, and the revenues generated by the volume and base charges is about five years.

The conclusions reached as a result of the presented in this thesis research are presented below.

The research shows that the disaggregated cascade model introduced here is applicable for forecasting water consumption, water production, and revenue associated with water delivery system.

Subdivision of the total service area into different subareas and into separate water consumer groups allows detailed modeling of the dynamics and patterns of city development. Traditional methods of demand forecasting are only able to model the growth of city as a unit. The analysis of the Phoenix water delivery system shows that
although the subdivision of water consumers into three user categories (single-family residential, multi-family residential and nonresidential) is sufficient for water forecasting purposes, it might be useful to recognize more than two service subareas. Detailed modelling of water consumption in specified city regions is important for the water distribution network and water production facilities planning and maintenance. Further study of the proper degree of city disaggregation needs to be performed.

In the case of Phoenix, the growth in the total water consumption is related exclusively to the growth of the number of accounts. The consumption per account does not show significant trends in time over the years 1986 to 1990. Seasonal patterns in water consumption are practically independent of the number of accounts.

Whereas revenue generated by volume charges depends on the precision of water use estimation, the revenue generated by base charges depends on the precise modelling of the number of accounts with meters of a given size. Since the model is more precise at this level of detail, disaggregation of the water system into meter size categories allows better estimation of the number of accounts and therefore a better forecast of revenue from base charges.

Although the subdivision of the water delivery system into spatial and consumer type categories advances the process of modelling water consumption, water production, and related revenues, further improvements may be introduced by the application of models in which the correlation between water use in adjacent regions and different user types is modelled explicitly instead of each series being treated separately. This requires a further study of correlation between the water use in different city regions and different water consumers.