

Effect of Reinforcement Spacing on the Behavior of Geosynthetic-Reinforced Soil

Amr M. Morsy, M.Sc., S.M.ASCE¹; Dov Leshchinsky, Ph.D., M.ASCE², and Jorge G. Zornberg, Ph.D., P.E., M.ASCE³

¹Dept. of Civil, Architectural, and Environmental Engineering, Univ. of Texas at Austin, 301 E. Dean Keeton St. Stop C1792, Austin, TX 78712. E-mail: amr.morsy@utexas.edu

²ADAMA Engineering, Inc., 12042 SE Sunnyside Rd., Suite 711, Clackamas, OR 97015. E-mail: adama@geoprograms.com

³Dept. of Civil, Architectural, and Environmental Engineering, Univ. of Texas at Austin, 301 E. Dean Keeton St. Stop C1792, Austin, TX 78712. E-mail: zornberg@mail.utexas.edu

Abstract

While significant emphasis has been placed in the technical literature on the interaction between soil backfill and geosynthetic reinforcement, companion phenomena that may develop in a reinforced soil mass due to reinforcement vertical spacing may have been overlooked. This paper integrates the results of experimental and field evaluations aimed at identifying such phenomena. Both evaluations were in turn complemented with numerical simulations. The experimental program, conducted on geosynthetic-reinforced soil (GRS) cells, indicated that the soil confined between subsequent reinforcement layers acts as a monolithic block. The field evaluation, which included assessment of the behavior of two GRS walls, showed responses consistent with those in the experimental component. Numerical simulation of these walls indicated that the effect of closely-spaced reinforcement increases with increasing backfill shear strength. Overall, the effect of reinforcement vertical spacing may have a relevant impact on the behavior of GRS that is often not accounted for in design.

INTRODUCTION

The interaction between soil backfill and geosynthetic reinforcement may be affected by phenomena that are related to the reinforcement vertical spacing. Such phenomena developing in a reinforced soil mass may be related to soil arching, as described by Terzaghi's classic trap-door theory (Terzaghi 1936). Soil arching develops during soil deformation and can take different arching shapes (e.g. Chen et al. 2008, Costa et al. 2009, Iglesias et al. 2013, Rui et al. 2016). This phenomenon may also take place in reinforced soil, especially in cases involving closely-spaced reinforcement. Such phenomenon is expected to depend on the soil density, grain size distribution, overburden pressure, and interface characteristics. Previous studies have been conducted on GRS to study the impact of closely-spaced reinforcement. Specifically, an experimental testing program was conducted by Leshchinsky et al. (1994) on GRS unit cells to study the impact of reinforcement vertical spacing with focus on the soil arching phenomenon. Specifically, a pullout testing device was developed to evaluate the displacement and strain fields within a reinforced soil unit cell. The testing program included pullout of single reinforcement layers and of two reinforcement layers connected to a rigid facing panel. This paper presents a reevaluation of the

experimental results obtained by Leshchinsky et al. (1994) and their integration to assess the performance of field monitoring and numerical results, which were also conducted to evaluate the effect of geosynthetic reinforcement vertical spacing. The field research component involves the evaluation of two GRS walls, and was complemented with numerical simulations conducted to extrapolate the findings of the field study with focus on the effect of reinforcement spacing. The integrated experimental, field, and numerical results aim at assessing the interaction of the various wall components that may affect wall performance with varying reinforcement vertical spacing.

EXPERIMENTAL AND ASSOCIATED NUMERICAL COMPONENTS: REEVALUATION OF RESULTS

Leshchinsky et al. (1994) conducted an experimental study to evaluate the effect of vertical reinforcement spacing on the failure mechanism in geosynthetic-reinforced structures. The motivation of their study was to assess failure mechanisms based on limit state analysis, which involve development of a failure slip surface extending from the toe to the crest of the structure. The reinforcement must extend beyond the slip surface to tie back the unstable zone to the stable zone. Limit equilibrium analysis does not account for the interaction occurring in soil and reinforcement layers considering spacing. For instance, the interaction among reinforcement layers may increase with decreasing vertical reinforcement spacing. In this case, the interaction between largely-spaced reinforcement layers would be comparatively minor, making the limit state a practical design approach. However, for closely-spaced reinforcement, the assumption may no longer be valid as the interaction (or load shedding) would increase with decreasing reinforcement spacing.

Two testing series were performed: (1) pullout of single reinforcement layer embedded in a confined soil mass, which assessed the performance of a reinforcement layer in a soil mass in conventional testing conditions; and (2) pullout of two reinforcement layers embedded in a confined soil mass, which assessed the effect of interaction between reinforcement layers. Two devices were used to evaluate the behavior of single and double reinforcement layers embedded in soil mass, respectively. Figure 1a shows a schematic view of the device where a single reinforcement layer was employed. The device involved a steel frame that accommodates samples that were 60 cm long, 19 cm wide, and 30 cm high. The reinforcement layers were of the same width as the box. A normal confining pressure was applied to the top surface of the reinforced soil mass using a pressurized air bag. The second device was similar to the first one except that it was twice as high (i.e., 60 cm high), as shown in Figure 1b. The side walls of both devices were made of transparent Plexiglas to enable photogrammetric measurement of soil movements as the pullout load increases. This allowed evaluation of the interaction between the reinforcements and the soil mass. The transparent walls also allowed evaluation of the kinematics of the shear band that developed upon generation of shear stresses at the soil-reinforcement interface. The second device allowed placement of two reinforcement layers, enabling assessment of the interaction between two contiguous reinforcement layers. The vertical spacing of the reinforcement layers was 20 cm. A horizontal force was applied to a panel connected to the reinforcement layers. Accordingly, the test was conducted by imposing lateral displacements to a facing unit located between two reinforcement layers (rather than by increasing the overburden pressure on the reinforced soil mass). The test results suggest that the

vertical reinforcement spacing influences the stiffness of the reinforced soil mass composite. For closely-spaced reinforcement of typical stiffness and strength, the failure surface was not likely to develop within the reinforced soil mass. Instead, the failure surface developed behind the reinforced soil zone. Closely-spaced reinforcement allowed formation of composite material that behaved as monolithic mass.

The backfill material used in the testing program was Ottawa sand, which classifies as poorly graded sand (SP according to the Unified Soil Classification System). The average and maximum particle sizes were 0.26 and 0.90 mm, respectively. The backfill was compacted dry to a relative density of 70%, which corresponds to an average unit weight of 16.8 kN/m^3 . The backfill was placed in six lifts by pluviation and was densified by slight tapping on the walls of the box. Triaxial tests conducted on specimens prepared at a 70% target relative density resulted in peak and residual friction angles of 38 and 34 degrees, respectively. The reinforcement used in this study was polypropylene biaxial geogrid with a tensile strength of 45.2 kN/m in the testing direction.

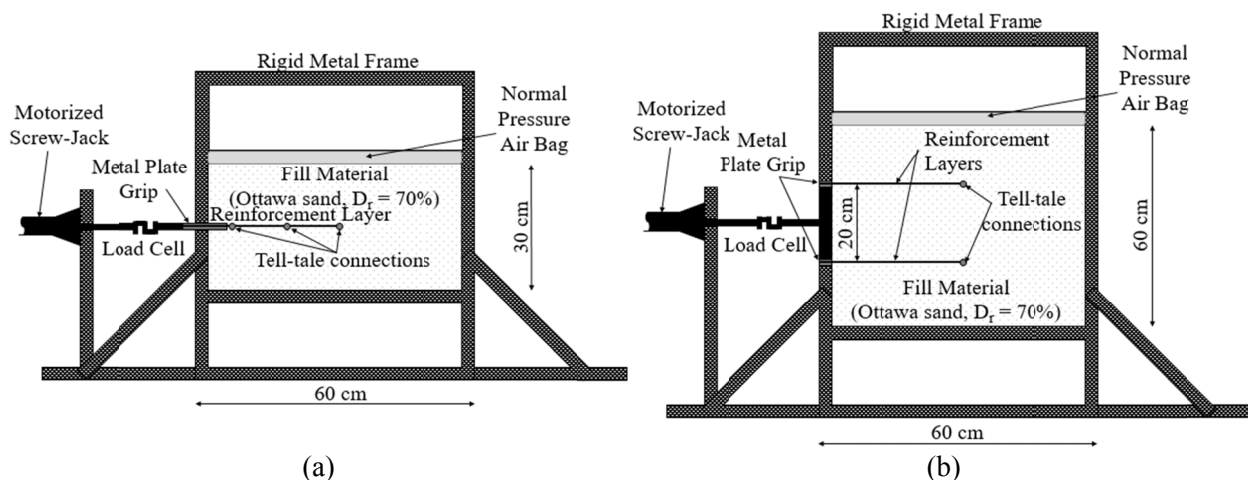


Figure 1. Pullout equipment: (a) single-reinforcement test; and (b) double-reinforcement test (redrawn after Leshchinsky et al. 1994).

Overall, the pullout process was observed to progressively propagate from the front end of the reinforcement to its rear end. Accordingly, the portions of the reinforcement closer to the line of load application reached pullout resistance capacity prior to the further portions. Accordingly, the soil-reinforcement interface strength in the front zones reached residual condition before the rear zones, which may still mobilize strength pre-peak, reach peak, and ultimately post-peak shear strength.

The results of single-reinforcement tests showed that the shear stresses generated at soil-reinforcement interface influenced a soil region ranging in thickness from 2.5 to 5 cm on each side of the reinforcement. This zone can be referred to as shear band and is schematically shown in Figure 2a. This pattern was found to be independent of the confinement. Note that since measurements are those observed on the latex membrane assumed to deform in unison with the adjacent soil. It was concluded that, for the geogrid and sand used in the study, the zone of influence of a single deforming geogrid is about 3 cm on each side. This implies that two deforming geogrids (i.e., two geogrid subjected to tension load) will interact with each other (i.e., behave as a composite soil-geogrid material) if the vertical spacing is at most 6 cm. Note that this is

valid for the type of backfill employed in the study, for which $D_{50} = 0.26$ mm, $\phi_{\text{peak}} = 38^\circ$ and $\phi_{\text{residual}} = 34^\circ$. Backfill with particles larger than the sand used in these tests are expected to have larger effects. The tests conducted in this study were not intended to simulate pullout performance but rather to identify mechanisms and a response that could be deemed as composite material behavior. The results of the double-reinforcement tests showed that the pullout resistance was essentially the same as that obtained using a single-reinforcement of the same length and confinement configuration. That is, the soil between the reinforcement layers was found to stiffen, resulting in the soil/reinforcement unit to behave as a monolithic block. This block involves two outer interfaces on which shear stresses develop against the adjacent soil, while no shear displacements (and associated shear stresses) could be identified on the two inner interfaces adjacent to the stiffened soil block. This resulted in a pullout resistance in the double-reinforcement tests equivalent to that in the single-reinforcement tests. However, this response was found to apply only at comparatively high confining pressures, which is when the soil between the reinforcements is stiff enough to behave as a monolithic block. On the other hand, at low confining pressure the pullout resistance in the double-reinforcement tests was higher than that in the single-reinforcement layer tests. This is probably due to the generation of shear stresses at the inner interface between the reinforcement layers and the soil between the reinforcement layers. In addition, the tensile stiffness in the double-reinforcement test was higher than that in the single-reinforcement test. The observed deformation field is schematically represented in Figure 2b.

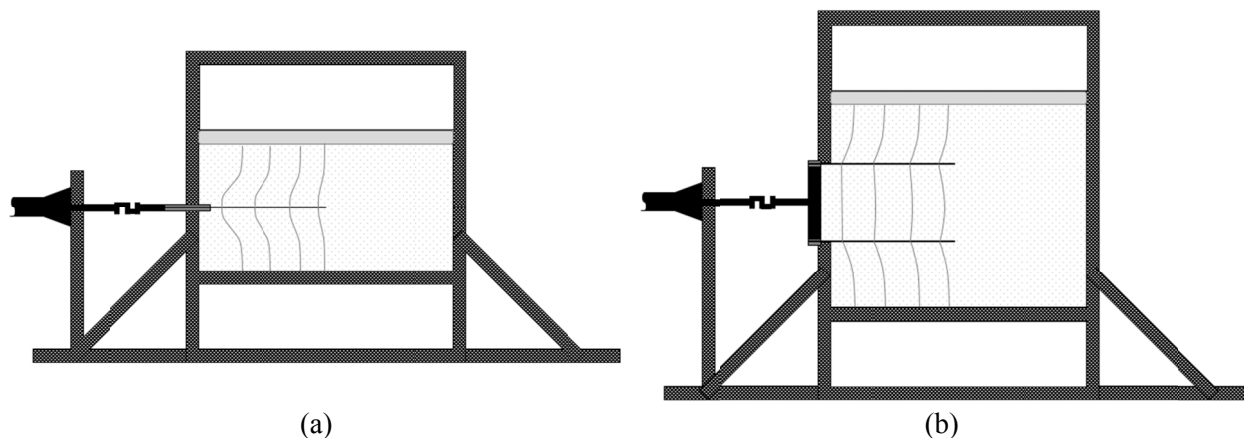


Figure 2. Schematic representation of shear band: (a) single-reinforcement test; and (b) double-reinforcement test (redrawn after Leshchinsky et al. 1994).

Figure 3a shows the expected failure mechanism, consistent with current design methodologies, when the active state is reached. However, the active state mechanism depicted in Figure 3a was not observed in the double-reinforcement tests. In addition, the load measured in the load cell after completion of the expected pullout test is compressive, consistent with those predicted by active earth pressure theory. Instead, they were zero. Accordingly, it appears that some ‘silo’ or arching effects developed, which resisted the lateral pressures that were expected to act in the block between the two geogrids. Figure 3b shows the actual failure mechanism observed in the double-reinforcement tests. As shown in Fig 3b, deformation in the soil mass followed the facing movement. For the spacing and geogrid stiffness used in this experimental program, only an external failure occurred (i.e., in a soil mass outside that bounded by the two layers). The soil

between the two geogrids moved ‘rigidly’ with the geogrids, without developing an active slip surface.

A numerical evaluation was conducted to study the effects of vertical reinforcement spacing (Leshchinsky and Vulova 2001). Extensive parametric studies were conducted using a finite difference software, which employs a finite difference approach. The numerical model adopted “moving reference” algorithm where every new soil layer and block row are placed on top of a preceding layer that is allowed to deform during construction. This allowed the wall facing to undergo lateral outward deformation cumulatively as construction progresses. Simulated construction continued until a prevailing mode of failure occurred.

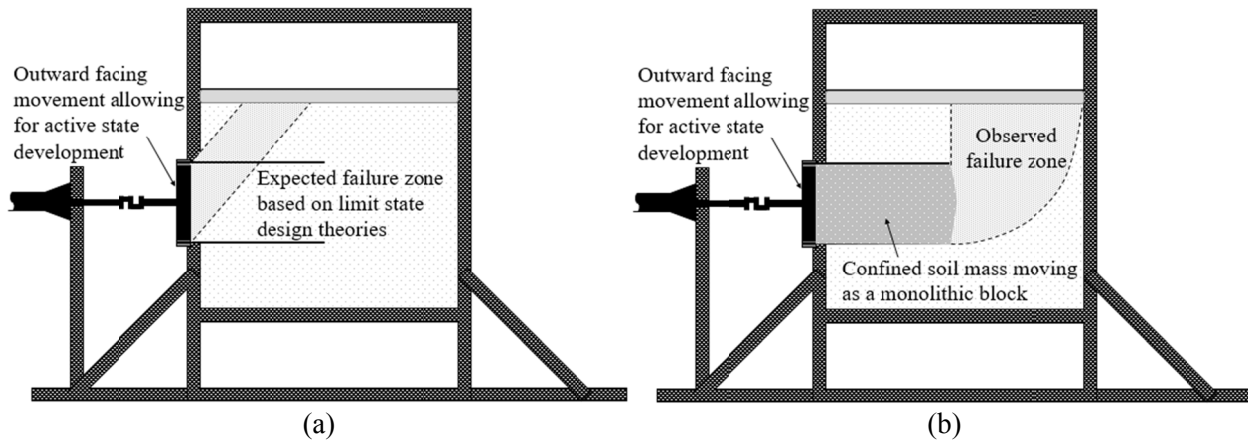


Figure 3. Failure mechanism: (a) postulated failure mechanism in active-state design; and (b) observed failure (redrawn after Leshchinsky et al. 1994).

The results indicated that the effect of closely-spaced reinforcement increases with increasing backfill shear strength. This trend was found to be more pronounced if the foundation soil is stiff (i.e., competent foundation). For reinforcement spacing values below 200 mm, the reinforced soil mass was found to behave as a coherent mass and did not develop internal plastic zones. On the other hand, comparatively large spacing (beyond 600 mm) were found to lead to connection failure. Reinforcement spacing was found to play a major role in wall behavior and, inconsistent with current design approaches, it significantly affected the prevailing mode of failure. Overall, the numerical results indicated that interaction of all wall components (i.e., facing, foundation, retained soil, reinforced soil, and reinforcement properties) may affect wall performance. Also, the numerical results implied that, for high quality backfill, “close spacing” corresponds to values below 400 mm; although this value was found to be highly dependent on multiple factors. Also, the parametric studies indicated that, for closely spaced reinforcement, commonly used methods for external stability analysis (e.g., direct sliding, toppling, deep-seated failure, and compound) are adequate for design. However, current design guidelines may not be accurate for the case of predicting reinforcement strength requirements.

FIELD AND ASSOCIATED NUMERICAL COMPONENTS

Based on the findings of the experimental component of this research, a field evaluation was conducted, which involved two GRS retaining walls constructed in Stockbridge, Georgia. Construction started in November 1994 and was completed in August 1995. The walls utilized

segmental concrete blocks; the walls are referred to herein as WALL 1 and WALL 2. The walls were 6.84 m-high (36 block rows) and were reinforced at vertical spacing values of 0.4 and 0.8 m (i.e., every two and four block courses), respectively. The walls were subjected to a surcharge corresponding to a 0.76-m thick soil layer. The geosynthetic reinforcement used in the walls involved uniaxial geogrids with an ultimate tensile strength of 70 and 114 kN/m for WALL 1 and WALL 2, respectively. The reinforced backfill material, which was the same as the retained soil, was a concrete sand characterized by an average grain size, D_{50} , of 0.79 mm. The reinforcement length to wall height ratio, L/H , was approximately 0.3, which is significantly lower than the minimum ratio of 0.7 established by the American Association of State Highway and Transportation Officials (AASHTO) requirements and of 0.6 established by the National Concrete Masonry Association (NCMA) requirements. However, an L/H ratio of 0.3 had already been adopted by Tatsuoka (1994) while using rigid facing. Short reinforcement was deemed acceptable, particularly considering that planar reinforcements (i.e. geosynthetic sheets) are used. This reinforcement enhances the stability of the structures due to its large contact area with backfill, unlike strip reinforcements that should be longer in order to transfer similar loads in a smaller contact area (Tatsuoka 1994). The comparatively large contact area results in a comparatively large pullout resistance as long as the tensile capacity is comparatively high. The short reinforcement length adopted in these walls was defined based on external stability calculations assuming factors of safety of 1.5 for sliding and overturning. It should be noted that AASHTO requires a factor of safety of 2.0 for overturning. The foundation soil was competent, so bearing capacity was not a governing design issue. The premise was that the proximity of layers in the walls under investigation was deemed close for the particle size and the friction angle of the well-graded, angular sand in the walls. Consistent with the results of the previous experimental component of this study, a consistent performance of the full-scale walls would be expected to show no development of internal failure surfaces.

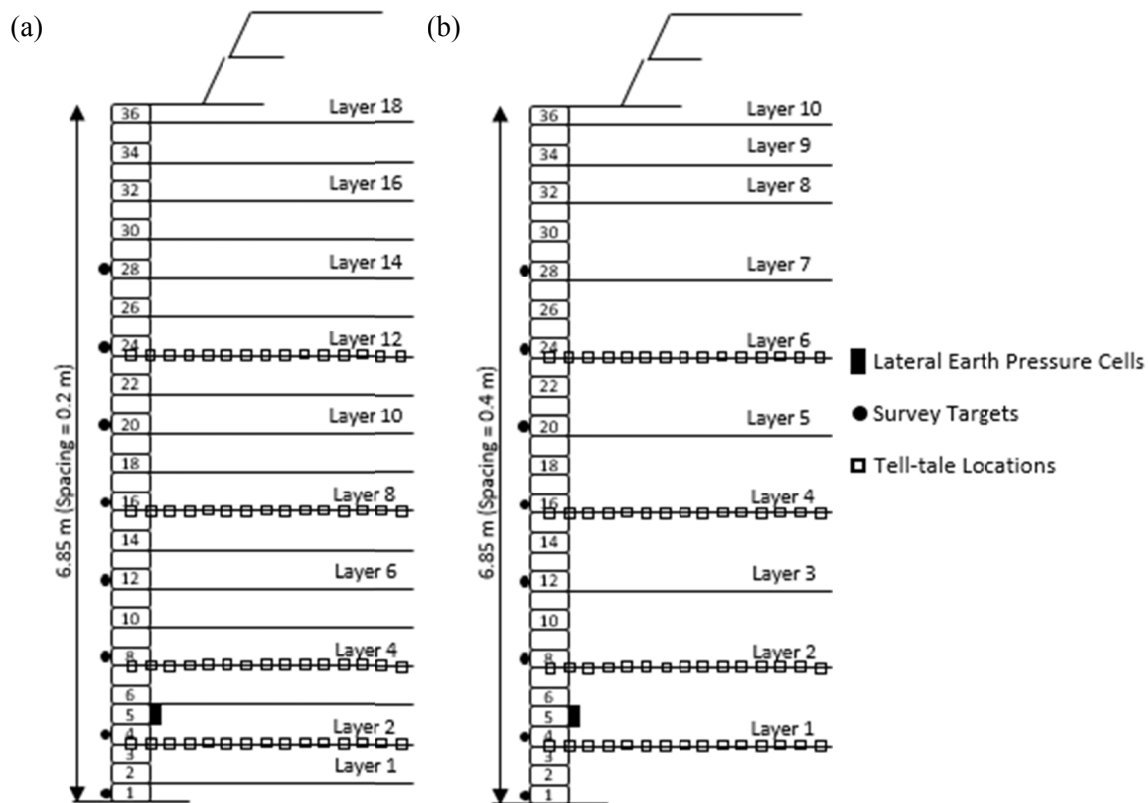


Figure 4: Instrumented cross-sections of the GRS walls: (a) WALL 1; and (b) WALL 2

Figure 4 shows the instrumented cross-sections of the constructed walls. Both walls were boosted with eight survey targets on the facing units (rows 1, 4, 8, 12, 16, 20, 24, and 28). Four reinforcement layers were instrumented by 15 displacement sensors attached to the layers along their length using tell-tales. Layers instrumented in WALL 1 were layers 2, 4, 8, and 12; whereas, those instrumented in WALL 2 were layers 1, 2, 4, and 6. Note that the instrumented layers in both walls are placed at the same elevation. In addition, two lateral earth pressure cells were installed at the back of facing block 5 in both walls.

The two walls were modelled using a two-dimensional soil-structure interaction code originally developed by Seed and Duncan (1984), and subsequently modified by Boulanger et al. (1991) and Bray (1995). Backfill and facing blocks were represented by 4-node quadrilateral elements and the geogrid reinforcement layers were represented by 2-node bar elements. Interface elements were assigned at possible slippage surfaces between the various structure components. The model simulated construction to the walls full height using 18 increments for WALL 1 wall and 10 increments for WALL 2 wall. The foundation soils were represented by a 3-m thick soil layer of the same material as the backfill material. A description of the properties adopted in the numerical simulations is described next.

Backfill material and facing blocks properties: A set of drained triaxial compression tests were conducted on the backfill material under confinement levels ranging from 34 to 103 kPa, which were deemed representative to those in the field. The obtained failure envelope was nonlinear, stress-level dependent. The behavior of the backfill material was represented by a hyperbolic model (Duncan and Chang 1970). Since the hyperbolic model does not consider dilation, the bulk modulus (B) was used by specifying Young's modulus (E) and Poisson's ratio

(v). Soil compaction was not considered in the simulations. The behavior of the facing blocks was also simulated using a linear elastic hyperbolic model. Table 1 summarizes the hyperbolic model parameters assigned to the backfill and facing block materials.

Table 1: Hyperbolic model parameters for backfill material and facing blocks.

Parameter	Backfill Material	Facing Blocks
Unit weight, γ (kN/m ³)	18	20
Young's modulus number, K	542	2×10^7
Young's modulus exponent, n	0.18	0
Failure ratio, R_f	0.78	0
Bulk modulus number, K_B	4517	952380
Bulk modulus exponent, m	0	0
Cohesion, c (kN/m ²)	0	0
Friction angle at 1 atm, Φ (deg)	46.7	50
Friction angle reduction, $\Delta\Phi$ (deg)	11	0
At-rest lateral earth pressure coefficient, K_o	0.5	0.1
Unload-reload modulus number, K_{ur}	542	2×10^7

Reinforcement properties: While a nonlinear model would be appropriate to simulate the behavior of the reinforcement layers, a linear model was adopted, as the finite element code used in this study could only simulate linear elastic bar elements. The stiffness at 2% axial strain was adopted in analysis to represent the linear stiffness of the reinforcement layers. The various geogrid reinforcement properties adopted in the simulations are summarized in Table 2.

Table 2: Geogrid reinforcement properties.

Parameter	WALL 1	WALL 2
Young's modulus, E (kN/m ²)	11526	11205
Cross-section area, A (m ² /m)	0.0018	0.0028
Axial stiffness, EA (kN/m)	20.7	31.4

Interface and linkage elements properties: Generally, interface elements simulate potential slippage between two different materials. Interface elements were assigned at the possible slippage surfaces: (1) geogrid-backfill interfaces; (2) geogrid-facing block interfaces; and (3) facing block-block interfaces. Standard values for the normal, shear spring, and unloading shear spring coefficients were employed (Boulanger et al. 1991). Interface friction angles adopted considered full-scale block-block and geogrid-facing block shear tests. The interface stress-displacement behaviors were simulated by nonlinear hyperbolic models. The interface element properties adopted in the simulations are summarized in Table 3. Linkage elements were assigned to reinforcement layers. Linkage elements are springs that allow pullout while enforcing compatible displacements of the bar elements nodes linked. The linkage elements are described by two parameters: (1) normal stiffness coefficient (K_n), which was assumed as 1×10^8 ; and (2) shear stiffness coefficient (K_s), which has a standard value of 1×10^5 .

Table 3: Interface element properties.

Parameter	Geogrid-Backfill	Geogrid-Facing Block	Facing Block-Block
Adhesion (kN/m ²)	0	0	0
Friction angle at 1 atm, Φ (deg)	35	45	45
Friction angle reduction, $\Delta\Phi$ (deg)	0	0	0
Normal spring coefficient, K_n	1×10^8	1×10^8	1×10^8
Shear spring coefficient, K_s	5×10^5	5×10^5	5×10^5
Unloading shear spring coefficient, K_{su}	5×10^3	5×10^3	5×10^3
Modulus exponent, n	0.2	0.2	0.2
Failure ratio, R_f	0.7	0.7	0.7

WALL 1 was represented by 988 nodes, 580 quadrilateral elements, 167 bar elements, 18 link elements, and 324 interface elements. On the other hand, WALL 2 was represented by 868 nodes, 612 quadrilateral elements, 95 bar elements, 10 link elements, and 180 interface elements. It should be noted that the difference between the two walls is the reinforcement vertical spacing, which is 0.4 and 0.8 m for WALL 1 and WALL 2, respectively. The outward facing displacement profiles, as measured in the field for WALL 1 and WALL 2, are presented in Figs. 5a and 5b, respectively. The maximum displacement for both walls was observed at one third of the wall height. The outward displacements for WALL 1 were found to be slightly smaller than those for WALL 2. Numerical predictions of the outward displacement profiles are also presented in Figs. 5a and 5b for WALL 1 and WALL 2, respectively. While the predicted displacement values are lower than those measured in the field, the profile shapes are fairly similar. Accordingly, the measured and predicted displacement profiles are deemed consistent. Overall, the measured and predicted displacement in both walls were comparatively small (less than 1.5 cm at the facing's mid-height).

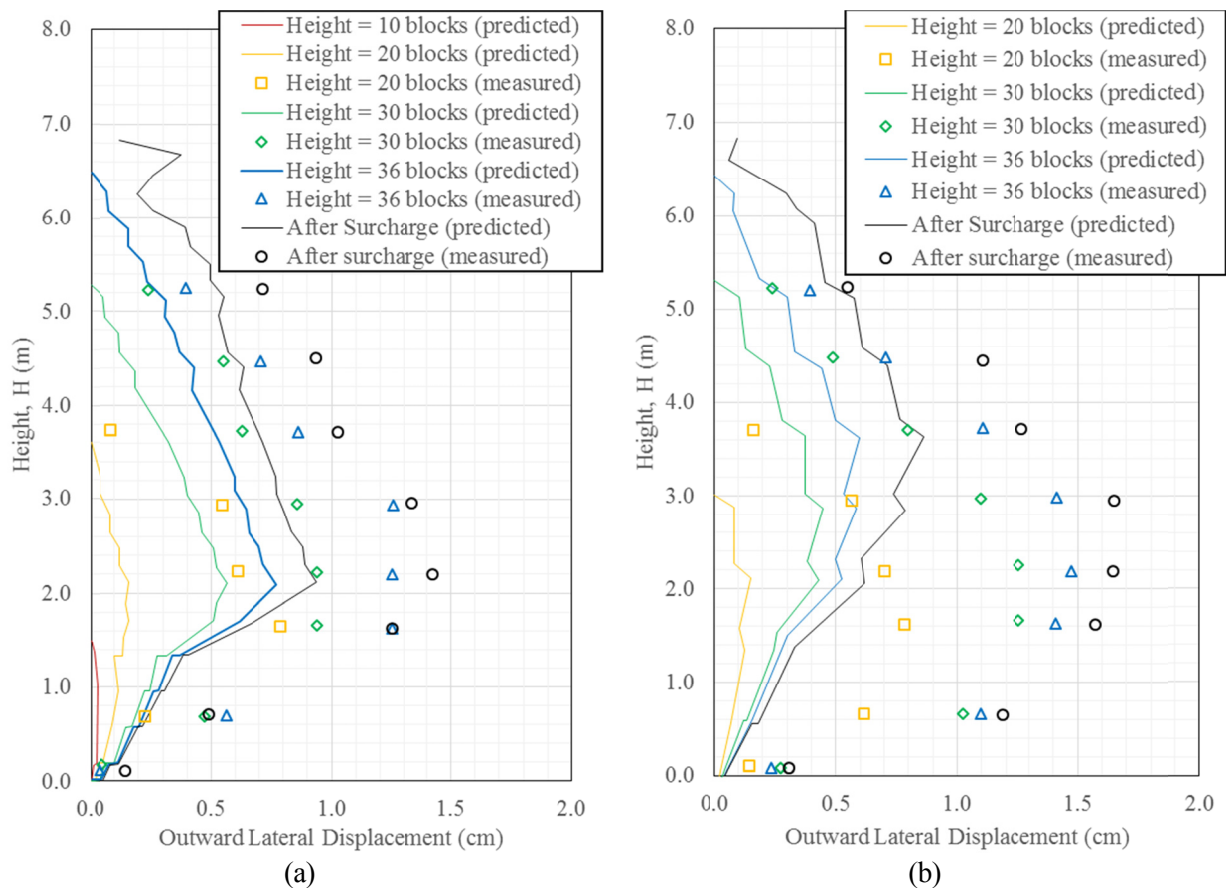


Figure 5. Outward lateral displacement profile at various construction stages: (a) WALL 1; and (b) WALL 2.

Figs. 6 and 7 show the reinforcement straining at various levels for WALL 1 and WALL 2, respectively. Figs. 6a through 6d show the reinforcement tensile strains at various construction stages for layers 2, 4, 8, and 12, respectively. On the other hand, Figs. 7a through 7d show the reinforcement tensile strains at various construction stages for layers 1, 2, 4, and 6, respectively. That is, the elevations where strains were measured in WALL 1 correspond to the same elevations where some of the reinforcements were also measured in WALL 2. The measured strain values fluctuate somewhat between tension and compression. However, as shown in Figs. 6 and 7, overall reinforcement strains were below 0.4%. It should be noted that reinforcement strains increased from 0.1% to less than 0.4% after adding the surcharge. The largest value of tensile strain was observed at reinforcement layers close to one third of the wall height. The strains predicted using numerical simulations were found to be smaller than those measured strains.

Lateral pressure transducers showed comparatively small stresses acting against the block facing. Similarly, results from the numerical simulations indicated significantly low pressures against the block facing elements. When the height of the wall was reduced after completion of the field tests, no collapse occurred as the facing units were removed. It was concluded that the soil confined between the geogrids (in both walls) acted as a monolithic block, which is consistent with the experimental findings of the experimental component of the study. This is in spite of differences in backfill materials and reinforcement vertical spacing. In fact, the D_{50} in the experimental component

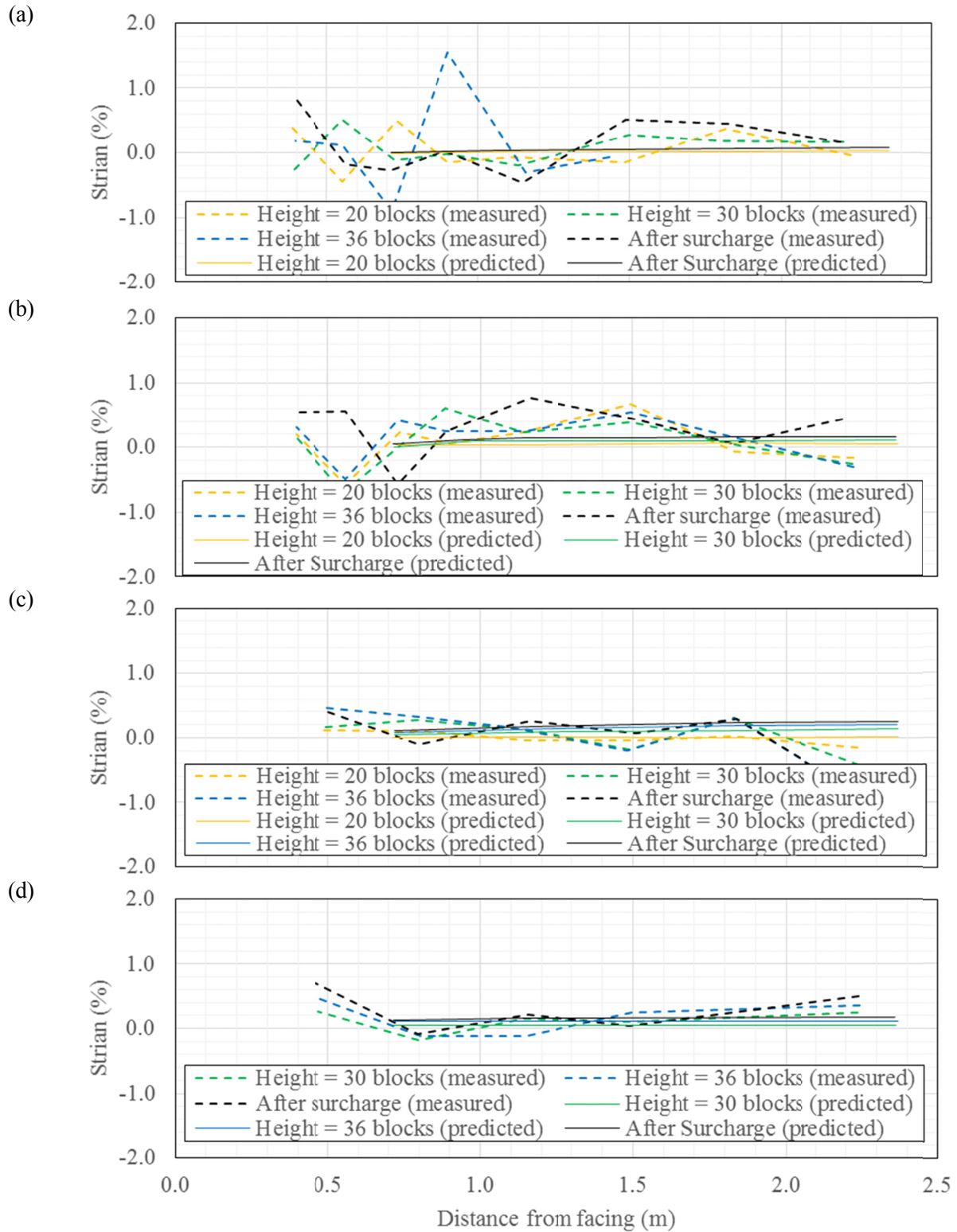


Figure 6. Measured and predicted reinforcement straining: (a) Layer 2; (b) Layer 4; (c) Layer 8; and (d) Layer 12.

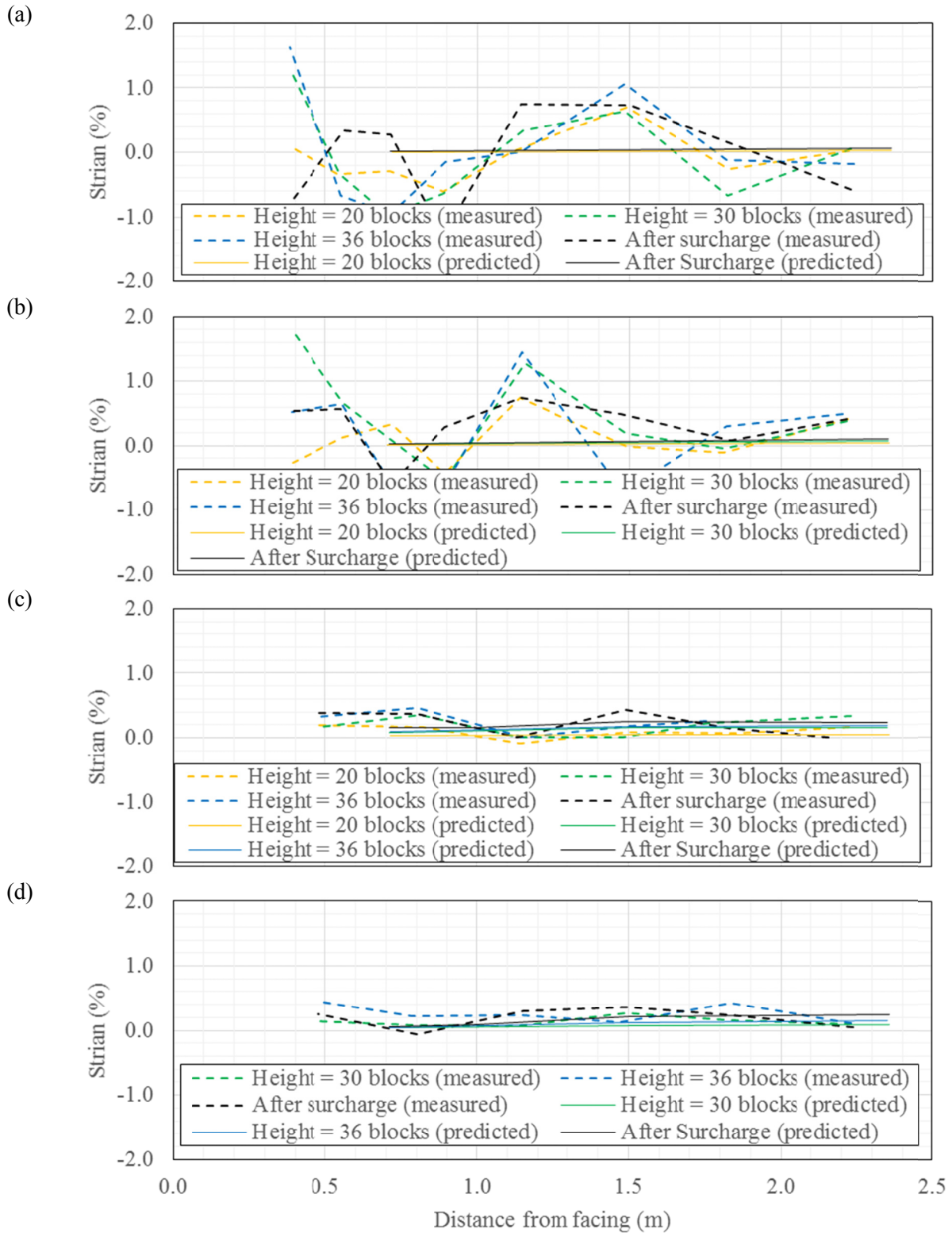


Figure 7. Measured and predicted reinforcement straining: (a) Layer 1; (b) Layer 2; (c) Layer 4; and (d) Layer 6.

was 0.26 mm, or approximately 3 times finer than in the field test. Also, the reinforcement vertical spacing in the experimental component was 20 cm, or approximately half the spacing in WALL 2 and a fourth of the spacing in WALL 1. The effect of closely-spaced reinforcement is expected to be proportional to the D_{50} of the backfill material as shear band is also a function of the median grain size. In this case, it may be concluded that field and experimental observations related to composite behavior of GRS structures are in reasonable agreement. The relationship between the reinforcement vertical spacing and particle size is supported by the work initially conducted in Cambridge in the 60's showing that, generally, the thickness of a shear band is about 15 to 20 times D_{50} . This response may be construed as the 'arching' influence zone being directly proportional to D_{50} . It was noted that increased reinforcement vertical spacing led to larger lateral displacements as well as to larger loads being carried by the reinforcement. Slight decrease in reinforcement stiffness may result in rapid increase of internal movements thus potentially invalidating the composite wall approach.

CONCLUSIONS

An experimental evaluation was conducted, which indicated that the interaction of reinforcement layers in a geosynthetic-reinforced structure may be significant and could render a composite material behavior. For the conditions evaluated in this experimental component, which used a sand backfill, a mobilization of a single geosynthetic reinforcement indicated that a spacing of 6 cm would render such behavior, although mobilization of a double geosynthetic reinforcement system indicated that 20 cm may also be adequate to render composite behavior. Results of the double geosynthetic reinforcement system indicated that the soil mass between reinforcements was mobilized as a monolithic system.

A field evaluation, involving monitoring of two geosynthetic-reinforced walls with different vertical reinforcement spacing, was also conducted. The results showed that wall displacements, reinforcement strains, and lateral pressure on facing were comparatively small. This observation implied that the soil confined between reinforcement acted as a monolithic block, which is consistent with the observations gathered in the experimental program. Field results indicated that the composite behavior occurred but was limited to reinforcement spacings below 0.6 m for the geogrids used in this research component.

Overall, results of the experimental and field components of this investigation, jointly point towards the beneficial impact of closely-spaced reinforcement on the performance of reinforced soil structures and, particularly, on the impact of closely-spaced reinforcement on the stresses acting against the wall facing components. While a value was not established for the reinforcement vertical spacing below which a composite behavior should be expected, the following practical recommendations can be drawn: (1) composite behavior is not expected for reinforcement vertical spacing values beyond 0.6 m, although this value is expected to correspond to a minimum value of geosynthetic reinforcement stiffness; (2) the length of geosynthetic reinforcement is expected to be governed by external stability considerations (e.g. direct sliding, overturning/eccentricity); and (3) the impact of closely-spaced reinforcement on decreasing the stresses acting against the wall facing components is significant.

ACKNOWLEDGEMENTS

Support received for this study from Tensar (experimental and field components) and Federal Highway Administration (numerical component) is greatly appreciated.

REFERENCES

- Boulanger, R.W., Bray, J.D., Chew, S.H., Seed, R.B., Mitchell, J.K., & Duncan, J.M. (1991). *SSCOMPPC: Finite element analysis program for evaluation of soil-structure interaction and compaction efforts*. Report No. UCB/GT/91-02, Univ of California, Berkeley.
- Bray, J.D. (1995). Personal Communications, Univ of California, Berkeley.
- Broms, B.B., (1978). "Design of fabric reinforced retaining structures." *Proc. of the Symposium on Earth Reinforcement*, ASCE, Pittsburgh, PA, April 27, 1978, 282-304.
- Chen, Y.M., Cao, W.P., & Chen, R.P. (2008). "An experimental investigation of soil arching within basal reinforced and unreinforced piled embankments." *J. G&G*, 26(2):164-74.
- Costa, Y.D., Zornberg, J.G., Bueno, B.S., & Costa, C.L. (2009). "Failure Mechanisms in Sand over a Deep Active Trapdoor." *JGGE*, ASCE, 135(11):1741-1753.
- Duncan, J.M. & Chang, C-Y. (1970). "Nonlinear analysis of stress and strain in soils." *J. of the Soil Mechanics & Foundations Division*, ASCE, 96(SM5):1629-1653.
- Iglesias G.R., Einstein H.H., & Whitman R.V. (2013). "Investigation of soil arching with centrifuge tests." *JGGE*, 140(2):248-56.
- Leshchinsky, D., (1997). *Design procedure for geosynthetic reinforced steep slopes*. Technical Report REMR-GT-23, Jan. 1997, WES, Vicksburg, MS.
- Leshchinsky, D., Kaliakin, V., Bose, P., & Collin, J. (1994). "Failure Mechanism in Geogrid-Reinforced Segmental walls: Experimental Implications." *Soils & Foundations*, Journal of the Japanese Society of Soil Mechanics & Foundation Engineering, 34(4):33-41.
- Leshchinsky, D. & Vulova, C. (2001). "Numerical investigation of the effects of geosynthetic spacing on failure mechanisms in MSE block walls." *Geosynthetics Int.*, 8(4):343-365
- Ling, H.I., Cardany, C.P., Sun, L-X., & Hashimoto, H. (2000). "Finite Element Study of a Geosynthetic-Reinforced Soil Retaining Wall with Concrete-Block Facing." *Geosynthetics Int.*, 7(2):137-162.
- Rui, R., van Tol, F., Xia, X. L., van Eekelen, S., Hu, G., & Xia, Y. Y. (2016). "Evolution of soil arching; 2D DEM simulations." *Journal of Computers & Geotechnics*, 73, 199-209.
- Seed, R.B. & Duncan, J.M. (1984). *A finite element analysis program for evaluating soil-structure interaction & compaction efforts*. Report No. UCB/GT/84-02, Univ of California, Berkeley.
- Tatsuoka, F., Tateyama, M., Murata, O. & Tamura, Y. (1994). "Closure on 'Geosynthetic-Reinforced Soil Retaining Walls with Short Reinforcement and a Rigid Facing.'" Recent Case Histories of Permanent GRS Retaining Walls, Balkema, Proc. of Seiken Sym. 11, Tokyo, Japan, 323-344.
- Terzaghi K. (1936). "Stress distribution in dry and in saturated sand above a yielding trap-door." *Proc. of 1st int. conference on soil mechanics & foundation engineering*. Cambridge, MA.
- Terzaghi, K. & Peck, R.B. (1967). *Soil mechanics in engineering practice*. 2nd edition, John Wiley & Sons, Inc.