

Cooperative Game Theory for Transboundary River Basins: The Syr Darya Basin

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Abstract

Cooperative game theory can be used for management of water resources in transboundary river basins. Applications of game theory which have been used for conflict analysis in water resources management include metagame analysis and graph models. These games are not considered classical game theory. These games allow strategies to evolve over time through repeated play and the players typically do not have communication prior to play. In cooperative game theory the players, or decision makers, have communication prior to the game and are allowed to make joint agreements. Cooperative games are considered non-zero-sum games where the costs and benefits of decisions are allocated to all players. Unlike metagames or graph models, the user must specify all possible strategy sequences that could occur. In a transboundary river basin setting, to generate the possible strategies, a water resources model is used to generate the payoffs from the sequence of strategies. These payoffs are then entered into a payoff matrix for calculations. In this paper, an application of cooperative game theory for the Syr Darya basin is presented.

Introduction

In this paper we consider the cooperative Game Theory analysis of a transboundary river basin and explore the extent to which cooperation may exist between riparian countries under various circumstances and what the value of that cooperation may be. An idealized schematic of the Syr Darya basin in Central Asia is shown in Fig. 1, with the river flowing through three riparian countries (Kyrgyzstan-Kg, Uzbekistan-Uz, and Kazakhstan-Kz) along the river. This is a simplified view as it does not include Tajikistan, which is also a riparian country, but it is a relatively small water user. In addition, several tributaries and annual reregulation reservoirs are also neglected.

Kyrgyzstan is a mountainous, upstream country where the majority of flow in the basin is generated from snow and glacier melt runoff. This country has a small, relative to the demands of the other two countries, demand for municipal and agricultural water and uses the river mainly to generate energy from a large hydroelectric facility associated with Toktogul reservoir. Kyrgyzstan is experiencing increasing population and energy demand and is concerned about meeting winter energy needs once the demand exceeds the hydroelectric capacity of Toktogul reservoir (if it hasn't already). Thus, this country is interested in negotiating with

Uzbekistan and Kazakhstan to receive compensation (in the form of cash payment or equivalent energy sources – electricity or fossil fuels) for providing irrigation water in the irrigation period, instead of releasing water for electricity production in the winter.

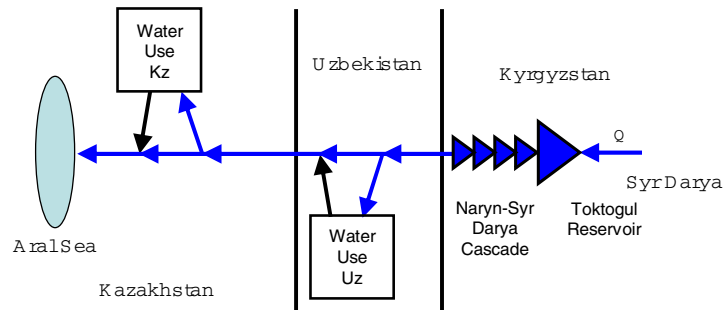


Figure 1. Syr Darya basin shared between three countries Kyrgyzstan, Uzbekistan and Kazakhstan.

Uzbekistan is a middle basin country with an agricultural economy that has a large irrigation water demand. This country is dependent on the multi-year storage capacity of Toktogul reservoir to supply its irrigation water demand during drier than average years. Thus, Uzbekistan is interested in negotiating with Kyrgyzstan for an appropriate storage and release regime for the reservoir to meet its irrigation water needs and it has adequate energy resources to make payments of fuel to Kyrgyzstan in the winter in compensation for the irrigation season releases. Under low to normal flow conditions in the river, Uzbekistan has the ability to divert all of the water out of the river and use it for irrigation, thus leaving no water in the river for Kazakhstan. However, this diversion ability is limited to about 50% by longstanding Soviet rules and recent international agreements.

Kazakhstan is the downstream riparian of the basin. It also has an agricultural economy with an associated irrigation water demand. Kazakhstan receives water from releases out of Toktogul reservoir storage as well as agricultural return flows from Uzbekistan. Kazakhstan would like to negotiate with Kyrgyzstan and Uzbekistan for adequate flows to supply its irrigation needs and it has adequate energy resources to make payments of fuel to Kyrgyzstan in the winter in compensation for the irrigation season releases.

Downstream of Kazakhstan, the river terminates in the Northern Aral Sea which has a delta region that is sensitive to the freshwater inflows from the Syr Darya and needs an adequate hydrograph of environmental flows to ensure its ecosystem health. The international community has shown a great desire to see environmental flows reach this delta and lake.

The purpose of the work outlined here is to determine the value of cooperation between the countries, if that cooperation exists. Each country has some options that it can exercise depending on the degree of trust and cooperation with the neighboring countries. For instance, given reasonable assurance of an adequate water supply, Uzbekistan could decide to expand its irrigated area. On the other hand, if there is poor cooperation, Kyrgyzstan could decide to release increased wintertime flows to generate electricity. As a result, Uzbekistan could consider building additional storage in the middle section of the basin to capture these flows for use in

the growing period. Kazakhstan is likewise dependent on the strategy of upstream Kyrgyzstan as well as the plans of Uzbekistan. It can also consider building additional storage to capture releases that occur in times and volumes that exceed irrigation water demands.

In the remainder of this paper, we describe the basic equations and data of a simple river basin model to calculate the value of various strategies of the countries in their negotiations. Then the various coalitions of a cooperative game theory application in this basin are presented with the characteristic values of the countries based on the results of the model.

The River Basin Model

The model considers the allocation of water to energy and agricultural production in the basin for one year with monthly time steps beginning on January 1 and ending on December 31 of an average flow year. The river network illustrated in Fig. 1 contains a storage reservoir located in Kyrgyzstan with capacity for multi-year regulation of the river. The capacity of Toktogul reservoir is 19.5 billion m³; however, the dead capacity is 5.5 billion m³, so the effective (or “active”) storage volume of the reservoir is 14.5 billion m³. Toktogul reservoir is accompanied by a cascade of reservoirs (the Naryn-Syr Darya Cascade also owned by Kyrgyzstan) whose volume and water elevation do not change over time (these are “pass-through” or “run-of-the-river” reservoirs).

The model contains mass balance constraints on the volume of water in storage in the reservoir in any month. At each junction point in the river basin where water is diverted from, or returned to, the river, a flow balance constraint exists for each month. All of the flows in the model are expressed in units of million m³ per month. The return flow coefficients are used to represent the fraction of the water diverted to the water use areas in each country that is returned to the river. Aggregate crop water requirements for each country are input data to the model and cropped areas are decision variables in the model. Toktogul reservoir is the only reservoir in the cascade for which the elevation of the water surface varies over time, depending on the volume of water in storage. For Toktogul reservoir, the energy generated is a nonlinear function of the water in storage, but a linear relationship exists for the run of the river reservoirs in the cascade below Toktogul reservoir.

The energy demand for Kyrgyzstan is satisfied by hydropower from Toktogul reservoir with energy being purchased for any deficit of hydroelectric energy (sometimes fuels can be obtained through energy – water exchanges with Uzbekistan and Kazakhstan). The total annual energy demand is 11,220 million kWh with peak demand in the winter months.

Irrigation is the primary consumptive water use in the basin and there are two large irrigation areas; one area in Uzbekistan (about 1.6 million ha) and another area in Kazakhstan (about 1.0 million ha). A number of different crops can be grown in each irrigation zone, but here we only consider an aggregate of the crops. The annual irrigation water requirements are 11,900 and 11,100 m³/ha for Uzbekistan and Kazakhstan, respectively. The profit (\$/ha) for growing crops in each country varies, with Uzbekistan receiving \$449 per ha and Kazakhstan receiving \$208 per ha. The

return flow coefficient (the fraction of applied irrigation water returning to the river) for each country is 0.40. The annual inflow to Toktogul reservoir, Q in Fig. 1, is 11.9 billion m^3 in an average year.

A multi-objective, weighting method (Loucks *et al.*, 1981) is used to formulate the objective function in the model representing tradeoffs between supplying power to Kyrgyzstan (minimizing squared deficits from energy demand) and irrigation water to Uzbekistan and Kazakhstan (maximizing agricultural profit). The model is programmed in the General Algebraic Modeling System (GAMS) language (Brooke *et al.*, 2006). A brief introduction to this system and the model are available with the model (Dinar *et al.*, 2007).

The Game

Consider the game with 3 players: Kyrgyzstan, Uzbekistan and Kazakhstan, and the following 7 possible coalitions (ranging from non-cooperation to full cooperation of the 3 countries): {Kg}; {Uz}; {Kz}; {Kg, Uz}; {Kg, Kz}; {Uz, Kz}, and {Kg, Uz, Kz}. The river basin model is used to calculate, for each of the possible coalitions, a value that is the net payoff from any of these possible arrangements. We will consider each coalition in turn.

Coalition {Kg}: Coalition {Kg} represents the situation in which Kyrgyzstan acts alone, without any cooperation with Uzbekistan and Kazakhstan. This coalition emphasizes the top priority energy production interests of Kyrgyzstan, but it also includes the profit maximizing behavior of Uzbekistan and Kazakhstan utilizing the flows that they receive in the middle and downstream regions of the basin. Under Coalition {Kg}, Kyrgyzstan releases sufficient water to cover only its internal power demands and it receives no compensating payment if it generates surplus energy, so there is no incentive to release excess water over what is needed to meet its national energy demand. In addition, Kyrgyzstan must purchase any energy deficit on the open market at a cost of \$0.08 per kWh, and they must pay for the operation, maintenance and replacement costs on the hydropower generation equipment at Toktogul reservoir - estimated at \$0.01 per kWh. If Kyrgyzstan did not own the hydroelectric facility at Toktogul reservoir, then they would have to satisfy all of their energy needs through purchases on the energy market, so this foregone cost is their gross benefit. The benefit that Kyrgyzstan receives is the market value of the energy demand minus (1) the cost of purchasing energy to make up any deficit and (2) the cost of hydro-energy generation O&M (see Table 1). The downstream countries receive only the water that is released to meet the energy demand of Kyrgyzstan—named “residual” water (Fig.2). Uzbekistan is the first country to receive the water and by existing treaty can use no more than 58% of it to produce as much profit as possible from irrigated agriculture. Kazakhstan receives the remainder of the flow plus the return flow from Uzbekistan’s agriculture. The agricultural profits to Uzbekistan and Kazakhstan are shown in Table 1. The value of Coalition {A} is \$785 million. Table 1 shows the costs and benefits to the countries.

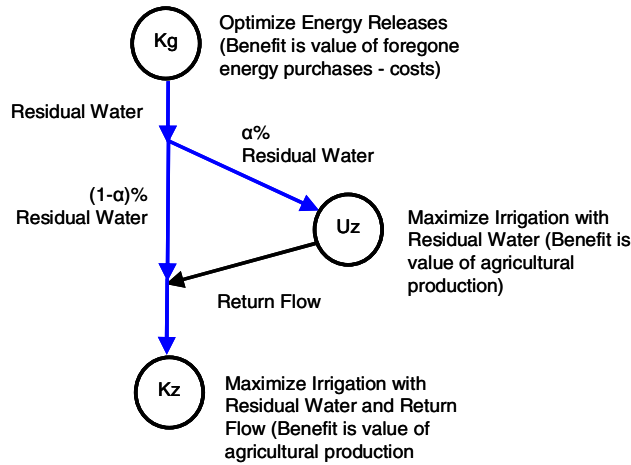


Figure 2. Logic of Coalitions {Kg}, {Uz}, and {Kz}.

Table 1. Results for Coalitions {Kg}, {Uz} and {Kz}.

Entity	Category	Unit	Amount
Kyrgyzstan	Foregone energy cost	million \$	898
	Hydro-energy cost	million \$	112
	Deficit energy cost	million \$	0
	Total energy cost	million \$	112
	Total benefit	million \$	785
Uzbekistan	Agricultural Profit	million \$	83
	Total Benefit	million \$	83
Kazakhstan	Agricultural Profit	million \$	35
	Total Benefit	million \$	35

Coalition {Uz}: Coalition {Uz} represents the situation in which Uzbekistan acts alone, without any cooperation with Kyrgyzstan or Kazakhstan, who each act to serve their own needs with the water available to them. Kyrgyzstan releases sufficient water to cover its power demand (see Fig. 2). Uzbekistan receives $\alpha\%$ (say, 58%) of the residual water and uses this water in irrigation for agricultural production. Kazakhstan receives the other $(1-\alpha)\%$ of the residual water. Since the optimizing behavior of the individual countries is the same, the same results as Coalition {Kg} are obtained (see Table 1). The value of Coalition {Uz} is \$83 million.

Coalition {Kz}: Coalition {Kz} represents the situation in which Kazakhstan acts alone, without any cooperation with Kyrgyzstan or Uzbekistan, who each act to serve their own needs with the water available to them. Since the behavior of the countries is the same, the same results as Coalition {Kg} are obtained (see Table 1). The value of Coalition {Kz} is \$35 million.

Coalition {Kg, Uz}: Coalition {Kg, Uz} represents the situation in which Kyrgyzstan and Uzbekistan cooperate in the form of Uzbekistan compensating Kyrgyzstan for its energy deficit in cash. In return for this compensation, Uzbekistan receives irrigation water according to its needs plus any surplus energy generated by Kyrgyzstan during the release of the irrigation flows. Uzbekistan may sell this

surplus energy at a price of \$0.08 per kWh (see Fig. 3). Under this coalition, Kazakhstan still receives return flows from Uzbekistan, and it acts to maximize its agricultural production with these flows. From the model, the results shown in Table 2 are obtained. The surplus energy transferred from Kyrgyzstan to Uzbekistan is sold by Uzbekistan for a value of \$406 million. The compensating payment from Uzbekistan to Kyrgyzstan to cover the energy deficit is \$400 million. The value of Coalition {Kg, Uz} is \$1,130 million.

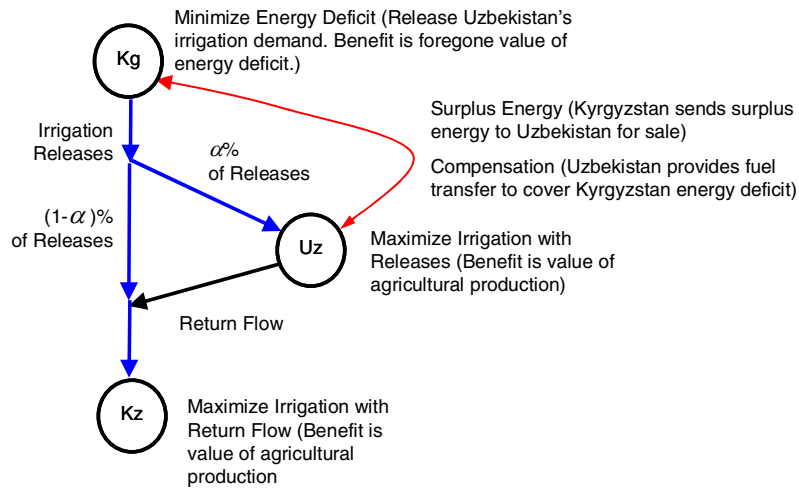


Figure 3. Logic of Coalition {Kg, Uz}

Table 2. Results for Coalition {Kg, Uz}.

Entity	Category	Unit	Amount
Kyrgyzstan	Foregone energy cost	million \$	898
	Hydro-energy cost	million \$	113
	Deficit energy cost	million \$	400
	Compensation from Uzbekistan	million \$	400
	Total energy cost	million \$	113
	Total benefit	million \$	785
Uzbekistan	Agricultural Profit	million \$	340
	Surplus Energy	million \$	406
	Compensation to Kyrgyzstan	million \$	400
	Total Benefit	million \$	345
Kazakhstan	Agricultural Profit	million \$	142
	Total Benefit	million \$	142

Coalition {Kg, Kz}: Coalition {Kg, Kz} represents the situation in which Kyrgyzstan and Kazakhstan cooperate through Kazakhstan compensating Kyrgyzstan for its energy deficit and Kyrgyzstan delivering irrigation water according to Kazakhstan's needs plus any surplus energy generated during the release of these flows. The results shown in Table 3 are obtained. The surplus energy transferred from Kyrgyzstan to Kazakhstan is sold by Kazakhstan for a value of \$328 million. The compensating payment from Kazakhstan to Kyrgyzstan to cover the energy deficit is \$435 million. The value of Coalition {AC} is \$932 million.

Table 3. Results for Coalition {Kg, Kz}.

Entity	Category	Unit	Amount
Kyrgyzstan	Total benefit	million \$	785
Uzbekistan	Agricultural Profit	million \$	339
	Total Benefit	million \$	339
Kazakhstan	Agricultural Profit	million \$	142
	Surplus Energy	million \$	406
	Compensation to Kyrgyzstan	million \$	400
	Total Benefit	million \$	147

Coalition {Uz, Kz}: This coalition represents the situation in which Uzbekistan and Kazakhstan cooperate, but Kyrgyzstan acts alone without any cooperation with the other countries. Similar to Coalition {Kg}, Kyrgyzstan releases sufficient water to cover only its internal power demands. Uzbekistan and Kazakhstan share the residual water for agricultural production. Table 1 shows the results. The value of the Coalition {Uz, Kz} is \$118 million.

Coalition {Kg, Uz, Kz}: This coalition represents the situation in which Kyrgyzstan, Uzbekistan and Kazakhstan all cooperate (the so-called *Grand Coalition*). This cooperation takes the form of Uzbekistan and Kazakhstan sharing the compensation payment to Kyrgyzstan for its energy deficit (see Fig. 4). In return for these compensating payments, the downstream countries receive irrigation water according to their needs plus any surplus energy. Uzbekistan receives $a\%$ (say, 58%) of the surplus energy and shares the other $(1 - a)\%$ with Kazakhstan. Uzbekistan receives $\beta\%$ (say, 50%) of the surplus energy and shares the other $(1 - \beta)\%$ with Kazakhstan. The results are shown in Table 4. The surplus energy transferred from Kyrgyzstan has a value of \$203 million for each country and the compensating payment to cover the energy deficit is \$200 million each. The value of Coalition {Kg, Uz, Kz} is \$1,272 million.

Characteristic Function: The Characteristic function of the game, v , assigns to each coalition the maximum value of the game between the coalition under consideration and the other countries that are not in that coalition. For the non-cooperative coalitions, $v(Kg)$, $v(Uz)$, and $v(Kz)$ are what each country may obtain acting on its own; $v(Kg, Uz)$, $v(Kg, Kz)$ and $v(Uz, Kz)$ are the values that the partial coalitions may obtain if they form these subgroups; and $v(Kg, Uz, Kz)$ is the value of the grand coalition of all countries in the negotiation. The characteristic function values for all possible coalitions are presented in Table 5, where we see that Kyrgyzstan and Uzbekistan can gain an additional \$262 million per year if they cooperate. Similarly, Kyrgyzstan and Kazakhstan can gain an additional \$165 million and all three countries can gain \$369 million if they form the Grand Coalition. The main question remaining for the countries is how to divide up these additional gains, should they decide to cooperate and join one of the coalitions.

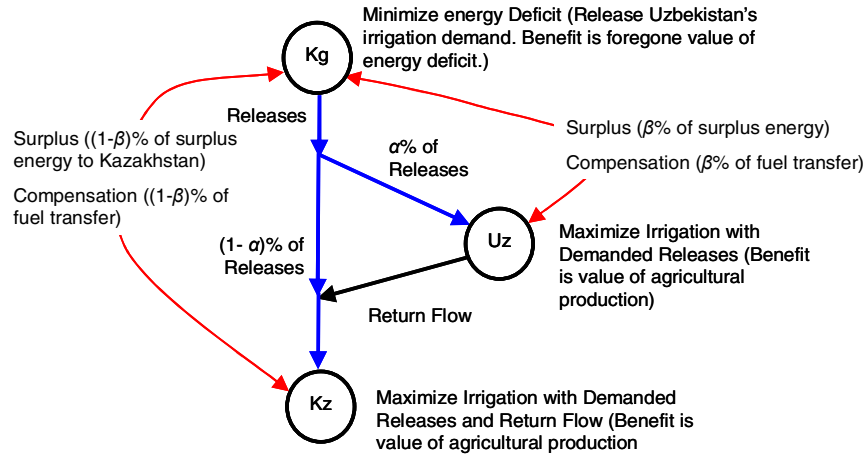


Figure 4. Logic of Coalition {Kg, Uz, Kz}.

Table 4. Results for Coalition {Kg, Uz, Kz}.

Entity	Category	Unit	Amount
Kyrgyzstan	Total benefit	million \$	785
Uzbekistan	Agricultural Profit	million \$	339
	Surplus Energy	million \$	203
	Compensation to Kyrgyzstan	million \$	200
	Total Benefit	million \$	342
Kazakhstan	Agricultural Profit	million \$	142
	Surplus Energy	million \$	203
	Compensation to Kyrgyzstan	million \$	200
	Total Benefit	million \$	145

Table 5. Characteristic function values for coalitions.

Coalition	Marginal Contribution of Country to Coalition (million \$)			Value of Coalition	$\sum v\{j\}$	Incremental gains for coalition
	Kg	Uz	Kz			
{Kg}	785			785	746	0
{Uz}		83		83	83	0
{Kz}			35	35	35	0
{Kg, Uz}	785	83		1130	868	262
{Kg, Kz}	785		35	932	767	165
{Uz, Kz}		83	35	118	118	0
{Kg, Uz, Kz}	785	83	35	1272	903	369

Core: The Core is a set of gains from allocating the resource among the players that is not dominated by any other allocation set; that is, it provides a range for the possible allocation solutions. The core provides a bound for the maximum allocation each player may request in the negotiation. Let $\Omega_j; j = Kg, Uz, Kz$ be each country's allocation of the gains from a coalition. We can plot this information on a simplex

where the distance from any point to the side opposite a vertex is that (vertex) country's allocation of the gains. The distance from any vertex to the opposite side is 1272 in Fig. 5.

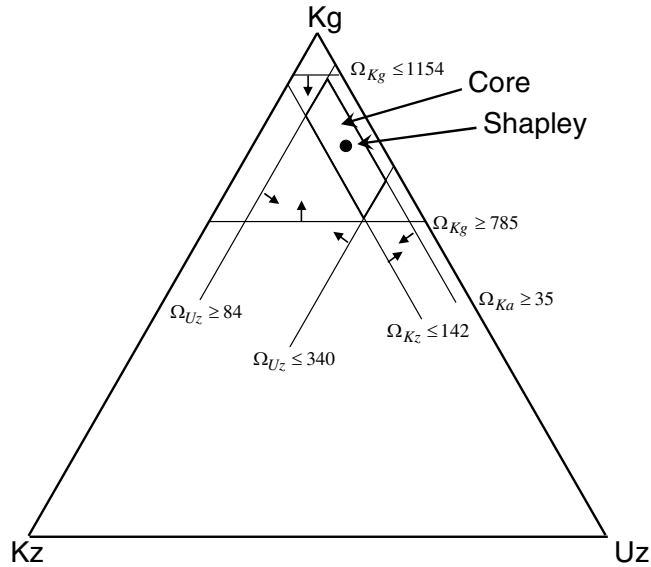


Figure 5. The Core of the Game.

The Core represents the set of non-dominated allocations in the game. The question remains, which point inside this set represents the most reasonable allocation of the gains from the game? Several methods have been developed to identify point allocations which are within the core of the game. The *Shapley value* provides one attractive alternative. Considering the sequence of establishment of each coalition and the allocation of the incremental gain generated by each country's entry into an existing coalition we can calculate the Shapley value. Consider Table 6 where the countries are assumed to form the grand coalitions {Kg, Uz, Kz} step by step, starting with country and then adding the second and finally the third (the permutations in Table 6). As each country joins, we award that country the value that they add to the growing coalition (Straffin, 2004).

We know that without cooperation, Kyrgyzstan, Uzbekistan and Kazakhstan can expect to gain \$785, \$83 and \$35 million per year, respectively. Can they do better than this through cooperation? From Table 6 we see that the gain for Kyrgyzstan through cooperation could be in the range (\$785, \$1154) million per year with an average value of \$970 million per year. Similarly, the gain for Uzbekistan is in the range (\$83, \$345) million per year with an average value of \$212 million per year, and the gain for Kazakhstan is in the range (\$35, \$142) million per year with an average value of \$89 million per year. The average value is the Shapley allocation: $\Omega_{Kg} = \$970$, $\Omega_{Uz} = \$212$, $\Omega_{Kz} = \$89$ million per year (see Fig. 6).

Table 6. Value of Various Permutations of Coalition Formation.

Permutation	Unit	Marginal contribution of Country to the coalition		
		Kg	Uz	Kz
Kg Uz Kz	Million \$	785	345	142
Kg Kz Uz	Million \$	785	340	147
Uz Kg Kz	Million \$	1047	83	142
Uz Kz Kg	Million \$	1154	83	35
Kz Kg Uz	Million \$	897	340	35
Kz Uz Kg	Million \$	1154	83	35
Total	Million \$	5822	1274	536
Shapley value	Million \$	970	212	89
Percent of profit	%	0.76	0.17	0.07

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