Chapter 2. Economic Analysis of Water Resources

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2.1 Cost – Benefit Analysis

2.1.1 Choosing Among Feasible Alternatives

Economic analysis, or the understanding and prediction of decision making under conditions of resource scarcity, plays a major role in the planning, design and management of sustainable water resource systems. Allocation of water among competing uses to obtain an optimum value in terms of market or welfare measures is one of the main problems of water resources planners. Price theory is very relevant where markets are operating efficiently, whereas welfare economics seeks to maximize human welfare in situations where desirable social gains and undesirable social costs are not fully accounted for in a profit maximizing, market economy (North, 1985). Price theory and welfare economics tend to focus on static analyses of projects, whereas, financial analysis considers the time value of investments and decisions. In this section, we will consider some aspects of financial analysis.

Choice is governed by economic and financial feasibility and acceptability with respect to social and environmental impacts. Here we want to consider investment analysis which serves as a guide for allocating resources between present and future consumption. The process consists of:

Identifying alternatives to be considered;
Predicting the consequences resulting from these alternatives;
Converting the consequences into some commensurable units (e.g., $'s); and
Choosing among the alternatives

One project may produce one type of output, while another project produces another kind of output. In order to compare the projects and make investment decisions, common units must be used to express the outputs of each alternative before any comparison can be made. Monetary units are the most commonly used units.

Some projects will provide outputs in the near future and other projects may delay outputs for an appreciable time or distribute them uniformly over the project lifetime. Outputs today do not have the same value as outputs tomorrow and the following observations are appropriate:

- Investors often prefer early return on investments since it provides them with more flexibility in making future investment decisions;
- Benefits and costs at different times should not be directly compared or combined, since they are not in common units;
- Future benefits and costs must be multiplied by a factor that becomes progressively smaller for times further into the future. This multiplicative factor is called the <u>discount rate</u> and it has a great impact on the alternative selected;
- Future benefits and costs are given more weight with lower discount rates and less weight with higher discount rates; and

- Committing resources to one project may deny the possibility of investing in some other project. This brings up the question of opportunity costs, or what must be foregone in order to undertake some alternative.

One should always keep in mind that different points of view may be adopted in analyzing alternatives, e.g., project sponsors; people in a specific area or region; and an entire nation. Each point of view may value benefits and costs differently and even define items differently (i.e., one person's cost may be another person's benefit.).

2.1.2 Cost-Effectiveness Analysis

A program is cost-effective if, on the basis of *life cycle cost* analysis of competing alternatives, it is determined to have the lowest costs expressed in present value terms for a given amount of benefits. Cost-effectiveness analysis is appropriate whenever it is unnecessary or impractical to consider the dollar value of the benefits provided by the alternatives under consideration. This is the case whenever (i) each alternative has the same annual benefits expressed in monetary terms; or (ii) each alternative has the same annual affects, but dollar values cannot be assigned to their benefits. Analysis of alternative defense systems often falls in this category (OMB, 1992).

2.1.3 Benefit-Cost Analysis

Financial benefit-cost analysis evaluates the effect of a project on the water sector or utility by providing projected balance, income, and sources and applications of fund statements (ADB, 2005). This can be distinguished from economic benefit-cost analysis which evaluates the project from the viewpoint of the entire economy. In financial benefit-cost analysis, which we will consider here, the unit of analysis is the project and not the entire economy nor the entire water sector or utility. Therefore, it focuses on the additional financial benefits and costs to the water sector, attributable to the project.

Benefit – Cost Analysis is a systematic quantitative method of assessing the desirability of government projects or policies when it is important to take a long view of future effects and a broad view of possible side-effects (OMB, 1992).

- Both costs and benefits of a project must be measured and expressed in commensurable units;
- It is the main analytical tool used to evaluate water resource and environmental decisions;
- Benefits of an alternative are estimated and compared with the total costs that society would bear if that action were undertaken; and
- Viewpoint is important - some groups are only concerned with benefits, others are concerned only with costs

In benefit – cost analyses, any costs and benefits that are unaffected by which alternative is selected should be neglected. That is, the differences between alternatives need only be considered.

Estimates of benefits and costs are typically uncertain because of imprecision in both underlying data and modeling assumptions. Because such uncertainty is basic to many analyses, its effects should be analyzed and reported (OMB, 1992). Uncertainty may exist in: objectives, constraints, public response, technological change, or extreme events and recurrence.

2.1.3.1 Interest Rate Calculations

Consider investing \$100 at a rate of 5%. At the end of one year the value of the investment would be:

$$F_1 = \$100(1+0.05) = \$105 \tag{2.1.1}$$

Similarly, at the end of 2 years, the value would be

$$F_2 = \$105(1+0.05) = \$100(1+0.05)^2 = \$110.25 \tag{2.1.2}$$

or, generalizing this, we have at the end of t years that an initial investment of \$$P$ would be worth

$$F_t = P(1+i)^t \tag{2.1.3}$$

Put another way, a single payment of F_t available t years in the future is worth

$$P = \frac{F_t}{(1+i)^t} \tag{2.1.4}$$

and a series of (not necessarily equal) payments F_t available t years in the future is worth

$$P = \sum_{t=1}^{T} \frac{F_t}{(1+i)^t} \tag{2.1.5}$$

Example 1. Assume that an initial investment of \$50 disbursement at t=0 is required and that this will result in \$200 receipt at t= 1 year and \$150 receipt at t = 2 years with an interest rate of 7% annually (see figure).

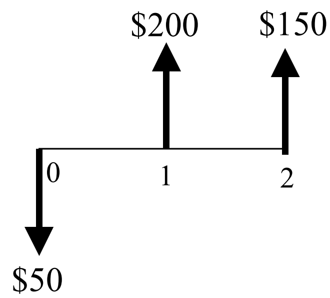

Figure 2.1. Cash flow diagram

What is the Present Value, *P*, of the given cash flow?

$$P = C + \sum_t F_t (1+i)^{-t} = C + F_1(1+i)^{-1} + F_2(1+i)^{-2} \tag{2.1.6}$$

Fill in the blanks:

P = ( ) + ( ) + ( )

P = ( ) $

What is the Future Value of the given cash flow at the end of year 2?

$$F = C(1+i)^2 + F_1(1+i)^{+1} + F_2 \tag{2.1.7}$$

Fill in the blanks:

F = ( ) + ( ) + ( )

F = ( ) $

2.1.4 Financial Analysis

Financial analyses use cash flow analysis and discounting techniques in benefit-cost analyses to maximize the rate of return to capital. Capital is the limiting factor here and maximizing profit is not necessarily achieved. The basic principle is that an investment will be made if revenues in the future will repay the cost at a positive rate of interest (North, 1985).

The standard criterion for deciding whether a government program can be justified on economic principles is *net present value* -- the discounted monetized value of expected net benefits (i.e., benefits minus costs). Net present value is computed by assigning monetary values to benefits and costs, discounting future benefits and costs using an appropriate discount rate, and subtracting the sum total of discounted costs from the sum total of discounted benefits. Discounting benefits and costs transforms gains and losses occurring in different time periods to a common unit of measurement. Programs with positive net present value increase social resources and are generally preferred. Programs with negative net present value should generally be avoided (OMB, 1992).

The process proceeds as:

- Define each alternative and predict their consequences
- Place monetary value on consequences
- Select a discount rate
- Convert time streams of benefits and costs
    - Construct cash flow diagrams
    - Convert values of costs and benefits at one date to equivalent values at the present (or another convenient) time.

$$NB_t = B_t - C_t \tag{2.1.8}$$

where NB_t = Net benefits at time *t*, B_t = benefits at that time, and C_t = costs at that time. The Present Value of net benefits is

$$P = \sum_{t=1}^{T} \frac{NB_t}{(1+i)^t} \tag{2.1.9}$$

Often it is convenient to convert a present value to an equivalent annual value. For this we can use the Capital Recovery Factor (CRF)

$$CRF_T = \frac{i(1+i)^T}{(i+1)^T - 1} \tag{2.1.10}$$

Then the equivalent annual value is

$$A = P \cdot CRF_T \tag{2.1.11}$$

2.1.5 Discount Rate

In order to compute net present value, it is necessary to discount future benefits and costs. This discounting reflects the time value of money. Benefits and costs are worth more if they are experienced sooner. All future benefits and costs, including nonmonetized benefits and costs, should be discounted. The higher the discount rate, the lower is the present value of future cash flows. For typical investments, with costs concentrated in early periods and benefits following in later periods, raising the discount rate tends to reduce the net present value (OMB, 1992).

The discount rate measures the rate at which current consumption will be sacrificed to ensure consumption (production) later. Greater sacrifices mean more resources can be devoted to future production. Alternatives discount rates include:

- Zero;
- Interest paid to borrow funds for project financing;
- Internal rate of return;
- Market interest rate for risk free investments -- Interest on recently issued government bonds having a maturity date approximately equal to the project life
- Most productive investments (opportunity cost of capital) -- If funds were committed to a project yielding the highest rate of return first, then to subsequent projects in order of rate of return, the IRR of the last project selected before funds run out is the MIRR
- Interest paid on borrowed funds for governments using bond financing
- US federal practice (Senate Doc. 97, 1962) -- "Average rate of interest payable by the US Treasury on interest-bearing marketable securities outstanding at the end of the fiscal year preceding computation which had terms to maturity of 15 years or more". See http://www.treasury-investing-101.com/Treasury-Index.html for current and historic rates.

2.1.6 Examples

Example 2. (after Linsley, et. al., 1979) Two alternative plans are considered for a section of an aqueduct. Plan A uses a tunnel, and Plan B uses a lined canal and steel flume. Both plans yield the same revenues over the life of the project. The interest rate is 6 % per year and the study period is 100 years.

Table 2.1. Information for Example 1.

	Plan A		**Plan B**	
	Tunnel	Canal	Canal lining	Flume
Life	100 yr	100 yr	20 yr	50 yr
First cost	\$450,000	\$120,000	\$50,000	\$90,000
Annual O&M cost	\$4000		\$10,500	

Compare the equivalent annual costs of the two plans

$$CRF = \frac{i(1+i)^N}{(i+1)^N - 1} \tag{2.1.12}$$

where i = 0.06 and N = 20, 50, and 100 years

Table 2.2. Calculations for Example 1.

Plan A		
Capital recovery cost for the tunnel	\$450,000 x 0.060177	\$27,080
Annual maintenance cost		\$4,000
Total annual cost		\$31,080
Plan B		
Capital recovery cost for canal	\$120,000 x 0.060177	\$7,221
Capital recovery cost for canal lining	\$50,000 x 0.087184	\$4,359
Capital recovery cost for flume	\$90,000 x 0.063444	\$5,710
Annual maintenance cost		\$10,500
Total annual cost		\$27,790

Total investment is \$450,000 and \$260,000, respectively, for the two projects. Even though the annual O&M costs are lower for Plan A, the annual cost comparison tells us that the extra investment is not justified. Thus Plan B should be selected.

Where the capacity of the project is to be determined, we simply determine the project with the maximum net benefits (difference of benefits over costs). That is:

$$Max\, NB(x) = B(x) - C(x) \tag{2.1.13}$$

where x is the capacity of the project. Thus

$$\frac{dB(x)}{dx} = \frac{dC(x)}{dx} \tag{2.1.14}$$

or

$$\frac{dB(x)}{dC(x)} = 1 \tag{2.1.15}$$

That is, we increase the capacity up to the point where the marginal (incremental) benefits just exceed the marginal (incremental) costs and then stop.

Example 3. (after North, 1985, Ex. 5-1) A flood control district can construct several alternative control works to alleviate flooding. These alternatives include the construction of dam A, dam B, and a system of levees C. Each of these works can be built to function alone or together with any other or all other projects. Thus we have a possibility of the following combinations: ABC, A, B, C, AB, AC, and BC. The life of each dam is 80 years and the life of the levee system is 60 years. The cost of capital is 4% . Information on total investment, operation and maintenance costs, and average annual flood damages is given in the table. Which flood control undertaking is the most economical?

Table 2.3. Flood Control Project Data

Project	Total Investment (million $)	Annual Operation and Maintainence (thous. $)	Average Annual Flood Damages (million $)
A	6	90	1.10
B	5	80	1.30
C	6	100	0.70
AB	11	170	0.90
AC	10	190	0.40
BC	9	180	0.50
ABC	15	270	0.25
Do nothing	0	0	2.00

The annual investment costs can be computed for each alternative by multiplying the investiment cost by the appropriate capital recovery factor:

$$CRF_T = \frac{i(1+i)^T}{(i+1)^T - 1} \tag{2.1.16}$$

where T = 80 years for dams and 60 for levees, respectively.

Table 2.4. Flood Control Project Calculations

Project	Total Investment ($ mln)	CRF	Annual Investment Costs ($ mln)	Annual Operation and Maintainence ($ mln)	Total Annual Cost ($ mln)
A	6	0.04181	0.251	0.090	0.341
B	5	0.04181	0.209	0.080	0.289
C	6	0.04420	0.265	0.100	0.365
AB	11		0.460	0.170	0.630
AC	10		0.516	0.190	0.706
BC	9		0.474	0.180	0.654
ABC	15		0.725	0.270	0.995

The **Incremental Benefit – Cost Ratio Method** compares the additional benefit to the cost of any alternative compared to other alternatives to find the solution. The procedure is:

1. Discard any alternative with B/C < 1
2. Rank order the alternatives from lowest to highest cost
3. Compute the incremental benefit - cost ratio for the contender versus current best alternative. If that ratio is greater than 1, contender becomes current best.
4. Repeat until all alternatives have been tested. Final current best is preferred alternative.

Table 2.5. Further Flood Control Project Calculations

Comparison	Project	Benefits ($ mln)	Cost ($ mln)	B/C Ratio	ΔB ($ mln)	ΔC ($ mln)	$\Delta B/\Delta C$	Conclusion
$\varnothing \to$ B	B	0.7	0.289	2.42	0.7	0.289	2.4	$\varnothing$ < B
B $\to$ A	A	0.9	0.341	2.64	0.2	0.052	3.8	A > B
C $\to$ A	**C**	**1.3**	**0.365**	**3.56**	**0.4**	**0.024**	**17**	**C > A**
AB $\to$ C	AB	1.1	0.63	1.75	-0.2	0.265	-0.75	C > AB
BC $\to$ C	BC	1.5	0.654	2.29	0.2	0.289	0.69	C > BC
AC $\to$ C	AC	1.6	0.706	2.27	0.3	0.341	0.88	C > AC
ABC $\to$ C	ABC	1.75	0.995	1.76	0.45	0.63	0.71	C > ABC

Example 4. (after Mays and Tung, Example 2.2.1) Determine the optimal scale of development of a hydroelectric project using benefit – cost analysis. Various alternative size projects and corresponding benefits are shown in the table below.

Table 2.6. Hydropower Project Data

Scale (MW)	Benefits B ($ mln)	Costs C ($ mln)	Net Benefits $B - C$ ($ mln)
50	18.0	15.0	3.0
60	21.0	17.4	3.6
75	26.7	21.0	5.7
90	29.8	23.4	6.4
100	32.7	26.0	6.7
125	38.5	32.5	6.0
150	42.5	37.5	5.0
200	50.0	50.0	0.0

The following figures show plots of the (1) project benefits and costs versus capacity, and (2) project benefits versus costs. Using a marginal analysis we find that the optimal capacity is 100 MW. The following table shows the incremental benefit – cost ratio method to find the same solution as before.

Table 2.6. Incremental Cost-Benefit Analysis for Hydropower Project

	Comparison	Scale (MW)	B ($ mln)	C ($ mln)	Incremental ΔB ($ mln)	ΔC ($ mln)	$\Delta B/\Delta C$	Conclusion
I	$\varnothing \to$ I	50	18.0	15.0	18	15	1.20	$\varnothing$ < I
II	I $\to$ II	60	21.0	17.4	3.0	2.4	1.25	II > I
III	II $\to$ III	75	26.7	21.0	5.7	3.6	1.58	III > II
IV	II $\to$ IV	90	29.8	23.4	3.1	2.4	1.29	IV > III
V	IV $\to$ V	100	32.7	26.0	2.9	2.6	1.11	V > IV
VI	V $\to$ VI	125	38.5	32.5	5.8	6.5	0.89	VI <V
VII	V $\to$ VII	150	42.5	37.5	9.8	11.5	0.85	VII < V
VII	V $\to$ VIII	200	50.0	50.0	17.3	24.0	0.72	VIII < V

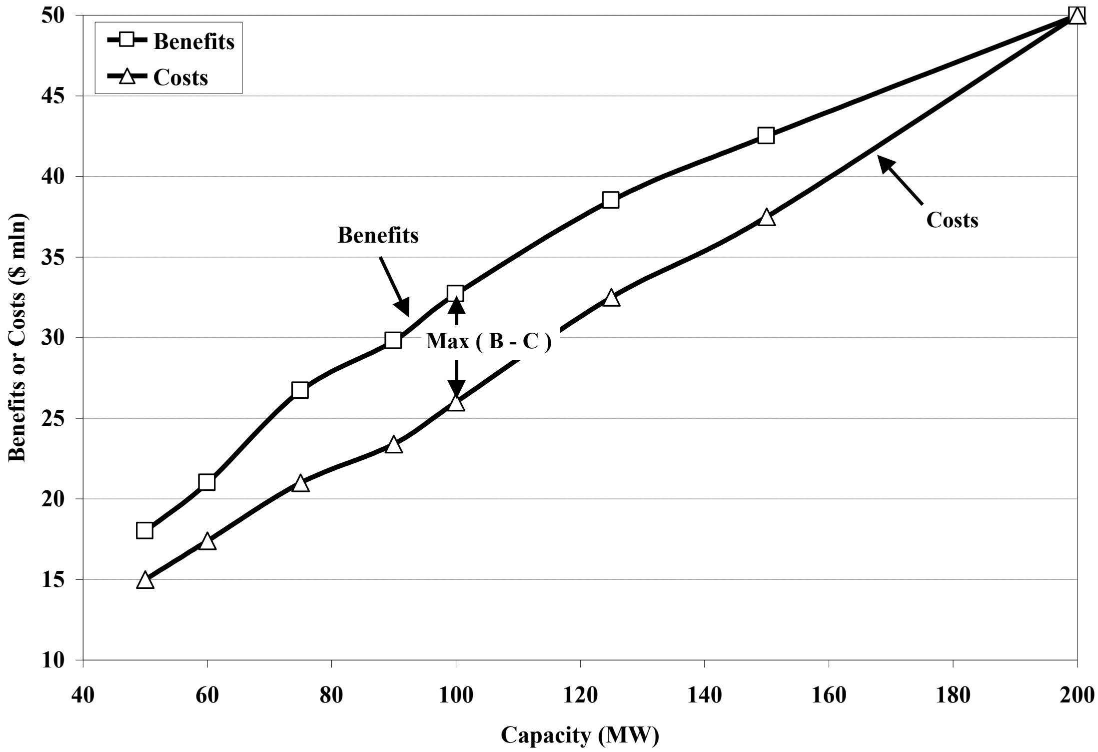

Figure 2.2. Project benefits and costs versus capacity.

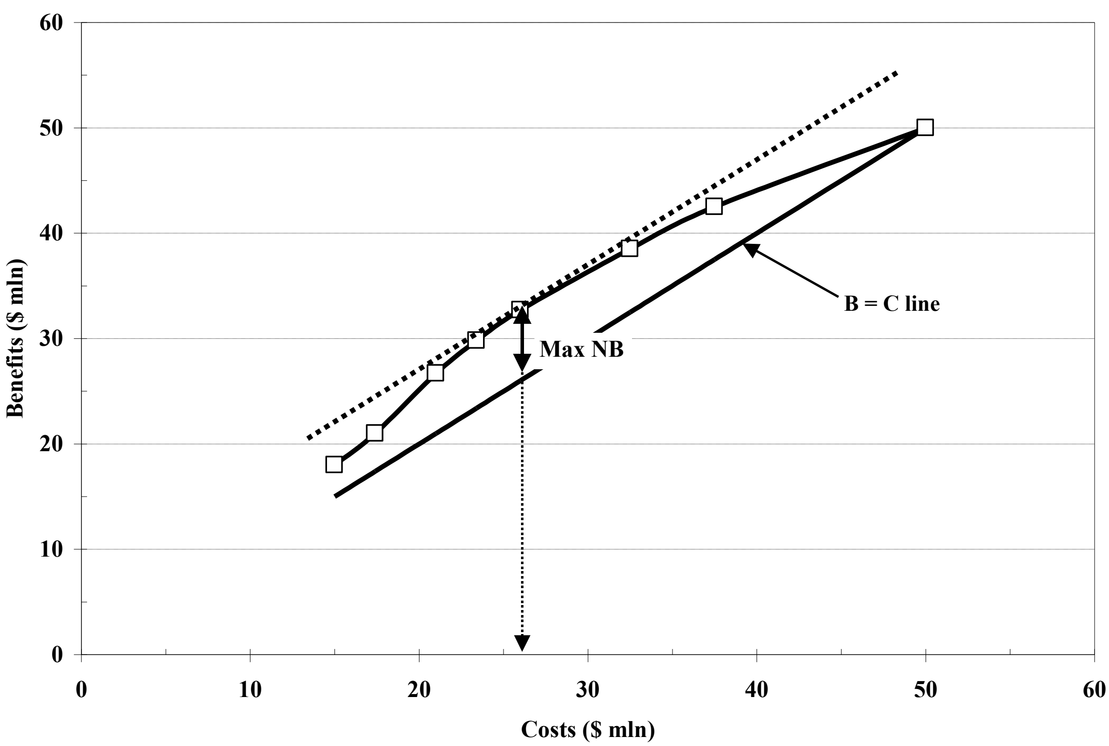

Figure 2.3. Project benefits versus costs.

Example 5. (after James and Lee, p. 33) The project with the highest benefit-cost ratio may not always be the preferred alternative. Consider a project whose benefits equal 3 units and whose costs equal 1 unit and which has an increment of investing an additional 4 units to increate benefits to 10 units. The smaller project has a benefit-cost ratio of 3, while the larger one has a ratio of 2. Because the incremental benefit-cost ratio is 1.75, the larger investment should be chosen even though it has a smaller individual benefit cost ratio.

Table 2.7. Incremental Cost-Benefit Analysis for Example 5

				Incremental			
	B	C	B/ C	ΔB	ΔC	$\Delta B/\Delta C$	Conclusion
A	3	1	3	3	1	3	A preferred
B	10	5	2	7	4	1.75	B preferred to A

Example 6. (after Thuesen, Fabrycky, and Thuesen, pp. 285-287 with correction) Suppose that four projects have been identified for providing recreational facilities for a Lower Colorado River Authority facility. The equivalent annual benefits, equivalent annual costs, and benefit cost ratios are given in Table 2.8. Inspection of the benefit-cost ratios might lead one to select Alternative B because the ratio is a maximum. Actually this choice is not correct. The correct alternative can be selected by applying the incremental benefit-cost method where the additional increment of investment is desirable if the incremental benefit realized exceeds the incremental outlay. The alternatives must be arranged in order of increasing outlay. Thus, the alternative with the lowest initial cost should be first, the alternative with the next lowest initial cost second, and so forth.

Table 2.8. Incremental Cost-Benefit Analysis for Example 6

	B (1000 \$)	C (1000 \$)	B/ C
A	182	91.5	1.99
B	167	79.5	2.10
C	115	88.5	1.30
D	95	50.0	1.90

Applying these rules to the alternatives indicates that Alternative A and not Alternative B ifs the most desirable alternative.

Table 2.9. Incremental Cost-Benefit Analysis for Example 6

				Incremental			
	B	C	B/ C	ΔB	ΔC	$\Delta B/\Delta C$	Conclusion
D	95	50.0	1.90	95	50	1.90	D preferred
B	167	79.5	2.10	72	29.5	2.44	B preferred to D
C	115	88.5	1.30	-52.0	9	-5.77	B preferred to C
A	182	91.5	1.99	15	12	1.25	A preferred to B

2.2 Demand for Water

2.2.1 Introduction

Consumers purchase goods produced by firms. They have *preferences* for some goods over others and they choose purchases from a set of feasible options. A *utility function* $u(x)$ is a numerical representation of consumer preferences. If one bundle of goods is preferred to another bundle, then it must have a higher utility. *Indifference curves* are the level sets of a utility function (see Figure 2.2.1.1).

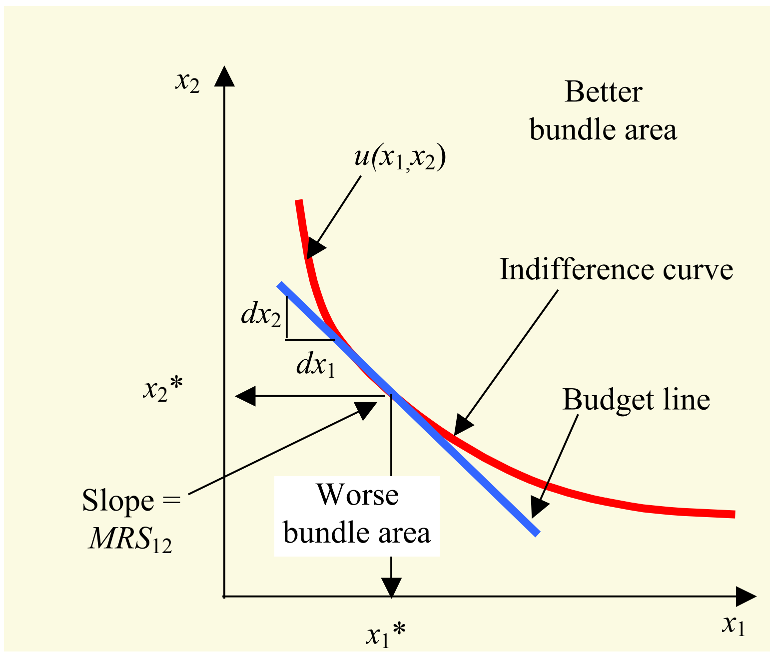

Figure 2.4. Indifference curve.

Consider the case when there are 2 goods to choose from, x_1 and x_2. If the consumer changes consumption by a small amount (dx_1, dx_2) but keeps utility constant, say at level u_o, then

$$du(x) = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0 \tag{2.2.1}$$

where

$$MU_i = \frac{\partial u}{\partial x_i} \qquad i = 1,2 \tag{2.2.2}$$

is the *marginal utility* of good i or the change in utility due to a small change in x_i. We can write

$$-\frac{dx_2}{dx_1} = \frac{MU_1}{MU_2} = MRS_{12} \tag{2.2.3}$$

where MRS_{ij} is the *marginal rate of substitution* of good i for good j, that is, the rate at which a consumer can substitute good i for good j.

2.2.2 Consumer's Problem

Consumers attempt to choose the best bundles of goods that they can afford. If there are K goods, whose quantities are represented by the vector $\boldsymbol{x} = (x_1, \cdots, x_K)$, available for consumption with unit prices $\boldsymbol{p} = (p_1, \cdots, p_K)$ and the total amount of money available to the consumer is m, then the consumer must make choices between goods according to a budgetary constraint

$$\boldsymbol{p}^{\mathrm{T}} \cdot \boldsymbol{x} = \sum_{k=1}^{\mathrm{K}} p_k x_k \le m \tag{2.2.4}$$

Consider the case of 2 goods. The budgetary constraint

$$p_1 x_1 + p_2 x_2 \le m \tag{2.2.5}$$

separates the decision space into two regions: (1) a region containing those combinations of goods whose purchase would exceed the budget; and (2) a region where those combinations that would not exceed the budget (See Figure 2.2.2.1). The slope of the budget line $(-p_i / p_j)$ is the rate at which the market will substitute good i for good j. Now, in the general case of K goods, the consumer is faced with the problem

$$\begin{aligned} &\text{Maximize } u(\boldsymbol{x}) \\ &\text{subject to} \\ &\qquad \boldsymbol{p} \cdot \boldsymbol{x} \le m \\ &\qquad \boldsymbol{x} \ge 0 \end{aligned} \tag{2.2.6}$$

That is, the consumer tries for maximize utility while satisfying the budget constraint. Now, assume that the budget constraint (Eq. 2.2.2.4) holds as an equality (all funds are expended or one of the goods is actually a savings account) and that the levels of consumption are all positive. Then we have the **classical programming problem** with a **Lagrangian** function

$$\begin{aligned} L(\boldsymbol{x}, \lambda) &= u(\boldsymbol{x}) + \lambda(m - \boldsymbol{p} \cdot \boldsymbol{x}) \\ &= u(\boldsymbol{x}) + \lambda\left(m - \sum_{k=1}^{K} p_k x_k\right) \end{aligned} \tag{2.2.7}$$

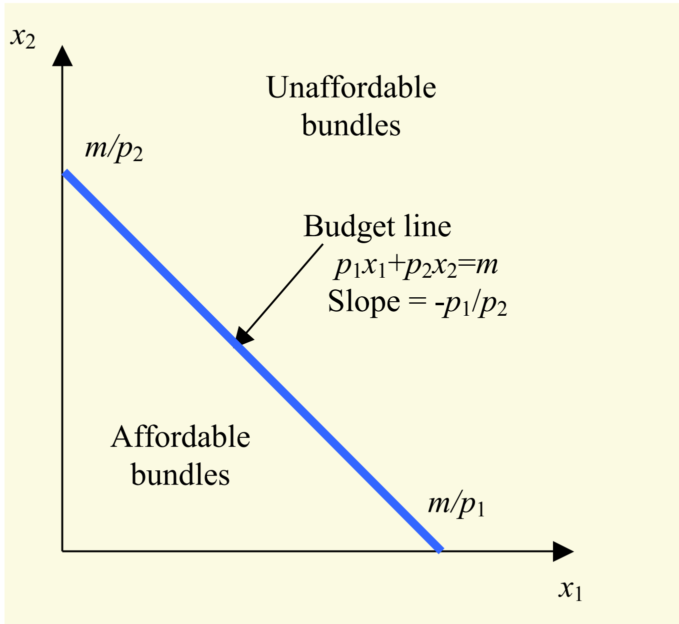

Figure 2.5. The consumer's budget set

The **first-order optimality conditions** for this problem are

$$\frac{\partial L}{\partial x_k} = 0 = \frac{\partial u}{\partial x_k} - \lambda p_k, \qquad k = 1,\ldots,K \tag{2.2.8}$$

$$\frac{\partial L}{\partial \lambda} = 0 = m - \sum_{k=1}^{K} p_k x_k \tag{2.2.9}$$

The first condition (Eq. 2.2.2.6) says that the ratio of the marginal utility to price is constant for all inputs

$$\frac{MU_k}{p_k} = \frac{\partial u / \partial x_k}{p_k} = \lambda \quad k = 1,\ldots,K \tag{2.2.10}$$

or

$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2} = \ldots = \frac{MU_K}{p_K} = \lambda \tag{2.2.11}$$

That is, a consumer chooses purchases of goods such that the ratio of marginal benefit (marginal utility) to marginal cost (price) is equal among all goods. This ratio, with units of utility/\$, is the value of the Lagrange multiplier (λ) which is also the ratio of the change in total utility for a change in income, or

$$\lambda = \frac{\partial u}{\partial m} \tag{2.2.12}$$

If we write Eq. 2.2.2.9 for two goods, say goods *i* and *j*, we have

$$\frac{MU_i}{MU_j} = \frac{p_i}{p_j} = MRS_{ij} \tag{2.2.13}$$

which says that the slope of the budget line will equal the slope of the indifference curve. or the ratio of the marginal utilities of any two goods equals the ratio of their prices.

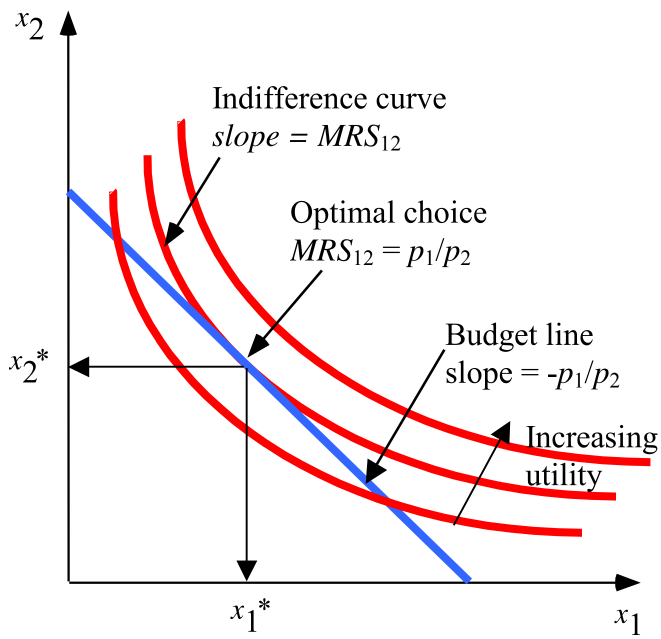

Figure 2.6. The consumer's problem and solution.

2.2.3 Demand

The optimal solution to the consumer's problem depends on income and prices so solving the problem (Eq. 2.2.2.3 and 2.2.2.4) results in an optimal level of consumption $\boldsymbol{x}^* = \boldsymbol{x}^*(\boldsymbol{p}, m)$ which is a function of the prices and the available income. This is the **demand function**. A typical demand function is shown in Figure 2.2.3.1. Often the **inverse demand function,** $\boldsymbol{p} = \boldsymbol{p}(\boldsymbol{x}^*, m)$, is used in analyses; this is simply the inverse of the demand function or price as a function of quantity and income. Market demand is the aggregation of all of the individual consumers' demands. Market demand depends on prices and the distribution of income in the economy.

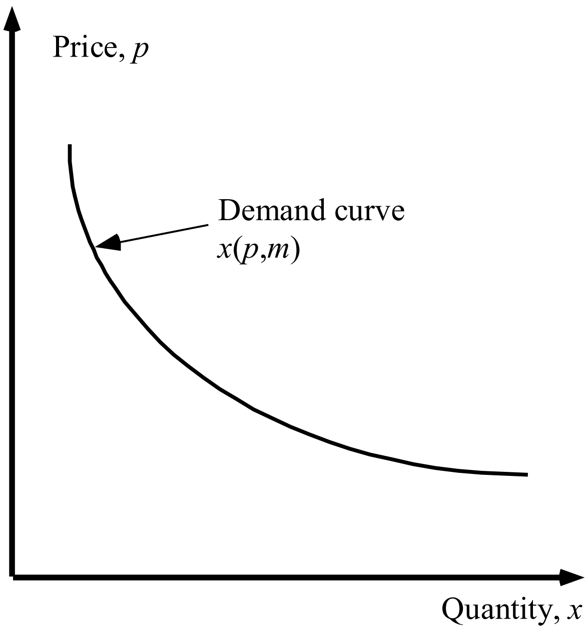

Figure 2.7. Typical demand curve.

2.2.4 Willingness-to-Pay

Demand is only real (or "effective") when it is accompanied by willingness to pay, in cash or kind, for the goods or services offered (Evans, 1992[1]). The value of a good to a person is what that person is willing, and able, to sacrifice for it (willingness-to-pay). How do we measure what a person is willing to pay for a good? Assume that a farmer has no irrigation water for production of a particular crop, but desires to purchase some water. If one unit of water became available, how much would the farmer be willing to pay to obtain that unit of water, rather than have no water at all? Suppose the farmer is willing to pay $38 for this first unit (see Figure 2.4.1) even though (s)he would prefer to pay less. Now, suppose that the farmer is willing to pay $26 for a second unit of water. Further, suppose that the farmer is willing to pay $17 for a third unit. According to the figure, at p* = $10 per unit, the farmer would purchase 4 units of water for a total cost of $40, but (s)he would have been willing to pay $93 for that water. Thus, the farmer receives a surplus of $53 (consumer surplus) when purchasing the 4 units of water.

Evans (1992) suggests three ways of determining willingness-to-pay from direct information in other, similar, situations and from survey information:

- *Indirect method,* involves analyzing what others in similar circumstances to the target population are already paying for services;
- *Direct method* (or contingent valuation method)*,* involves asking people to say what they would be prepared to pay in the future for improved services; and
- *Proxy measures*, e.g., use of case studies of water vending to provide indicators of willingness to pay.

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1 Evans, Phil, Paying the Piper: An overview of community financing of water and sanitation, Occasional Paper 18, IRC International Water and Sanitation Centre, The Hague, The Netherlands, April 1992

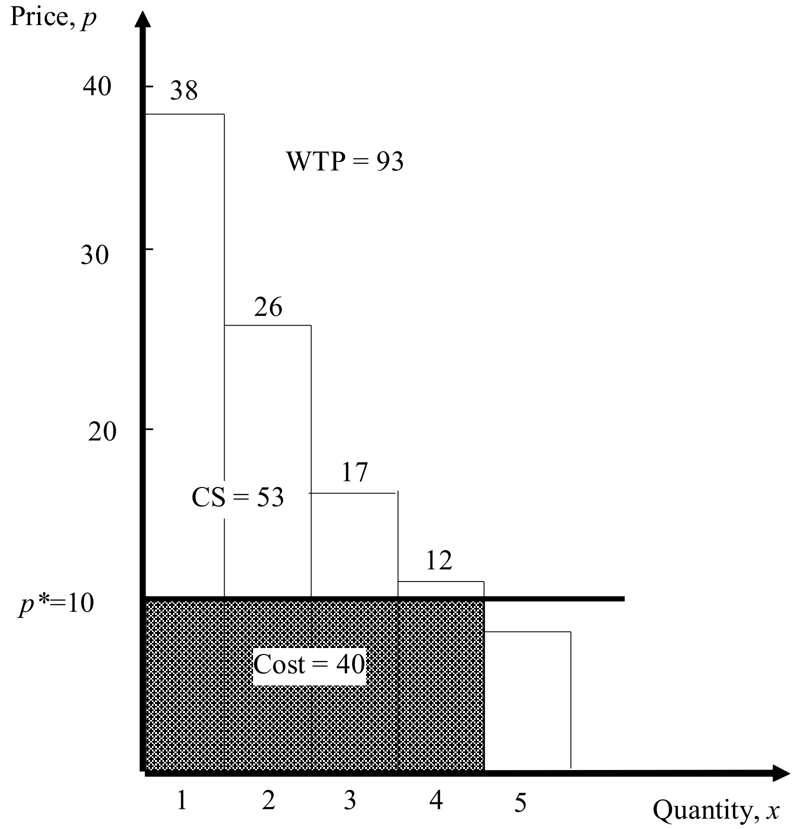


Figure 2.8. Willingness-to-pay for each additional unit of water.

If we assume that fractional amounts of a unit of a good can be purchased, then we obtain a continuous graph. Marginal willingness-to-pay is the height of the curve. Total willingness-to-pay is the sum of the heights of the rectangles between the origin and the particular consumption level, x, of interest. In the case of the continuous curve, willingness-to-pay is the area under the curve from the origin and the particular consumption level of interest and this represents the gross benefit of purchasing this amount of the good. The net-benefit from this purchase is the willingness-to-pay minus the cost or

$$NB = \int_{0}^{x^*} p(\eta, m)\,d\eta - p^* x^* \qquad (2.2.14)$$

which is termed the consumer's surplus.

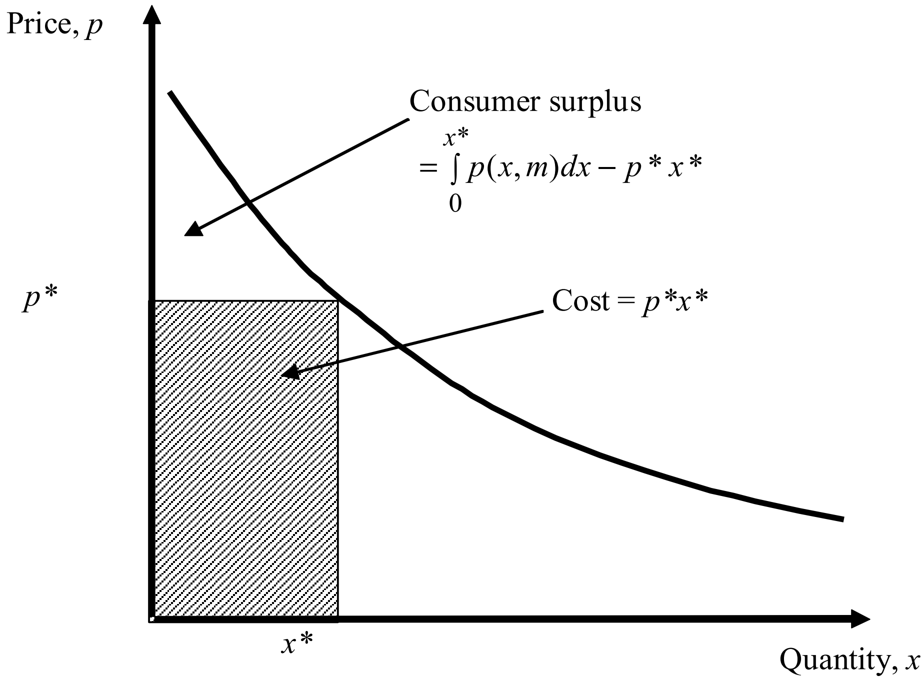


Figure 2.9. Willingness-to-pay curve.

Measuring benefit of water use in this manner requires that we can derive the demand curve for the water used for a particular purpose. For marketed commodities with available information on prices and quantities we can: (1) derive a demand curve, (2) quantify willingness-to-pay, and (3) use WTP to represent benefits. However, in many cases market prices may not exist, demands may not be revealed, and the change in benefits over time may be extremely uncertain. Examples include (1) the benefits of preserving space for recreation, and (2) the benefits derived from damages prevented due to pollution controls. If the physical damages of pollution can be identified and estimated, then a monetary value may be placed on them (for an example of applying this to the Aral Sea basin, see Anderson, 1997). Sometimes it is possible to survey people to determine their willingness-to-pay for different environmental assets such as environmental preservation, damage reductions, and lower risks. From these survey results we may be able to infer the valuation of the assets. Indeed, we may also be able to infer these values from related markets where values are observable.

The value of municipal water at its source minus any water utility costs is represented by the consumers' surplus. The area under the demand curve for an increment from x_1 to x_2 is (Gibbons, 1986)

$$Area = \frac{p x_2^{\chi}}{1-\beta}\left(\frac{x_2}{x_2^{\beta}} - \frac{x_1}{x_1^{\beta}}\right) \qquad \beta = \frac{1}{\varepsilon} \tag{2.2.15}$$

2.2.5 Elasticity (of demand)

The *price elasticity of demand* is a measure of how responsive consumers are to changes in price. The slope of the demand function $\boldsymbol{x}^* = \boldsymbol{x}^*(\boldsymbol{p}, m)$ is

$$slope = \frac{dx}{dp} \tag{2.2.16}$$

This quantity depends on the units used to describe the inputs and price. If we normalize this function, we obtain the price elasticity of demand

$$elasticity = \varepsilon = \frac{dx/x}{dp/p} \tag{2.2.17}$$

Consider the following example adapted from Merrett (1997). Table 2.7 shows the quantity of water demanded for different prices along with the price elasticity of water at various increments. Figure 2.10 plots the demand function for water and illustrates the ranges of elastic and inelastic behavior. Merrett (1997) proposes a cubic form for the demand function

$$p = ax^3 + bx^2 + cx + d \tag{2.2.18}$$

where a < 0, b > 0, c < 0, and d > 0. He points out, at low quantities, higher prices for water have little effect due to the intense need for the water. Similarly, at low quantities, higher prices for water have little effect due to the abundance of water. In the middle quantities, changes in price produce significant changes in the quantity of water demanded.

Price elasticity of municipal water demand was estimated (Gibbons, 1986) and in-house water use was found to be price-inelastic (= -0.23), while sprinkling use was found to be more elastic and differ between the Eastern US (= -1.6) and the Western US (= -0.7).

Table 2.7. Price Elasticity of Water (adapted from Merrett, 1997)

Quantity (m^3/month)	Price (per m^3)	ΔQ	ΔP	ε
700	6			
		300	-1	1.8
1000	5			
		500	-1	1.67
1500	4			
		500	-1	1
2000	3			
		500	-1	0.6
2500	2			
		300	-1	0.21
2800	1			

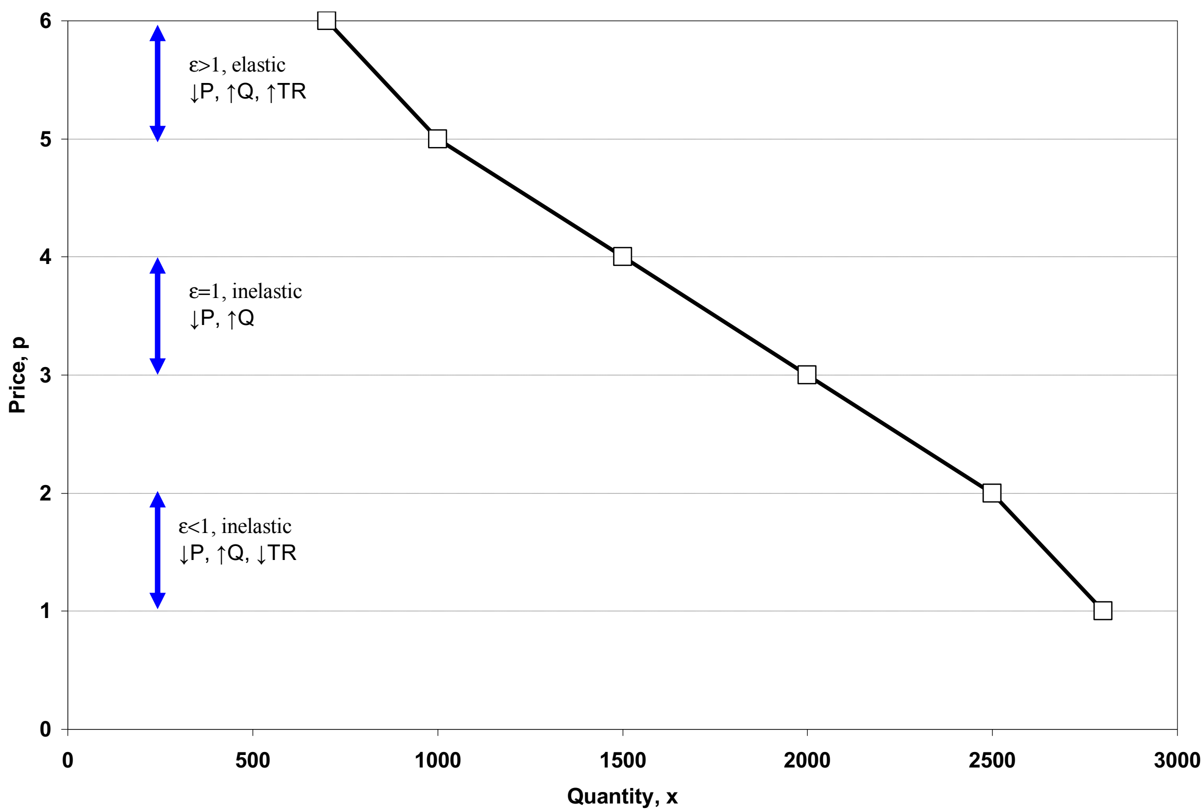


Figure 2.10. Demand function for water.

2.2.6 Water Values in the US

Table 2.8 shows data on the value of water for various uses within the United States [Frederrick et al. (1996).

Table 2.8. National Water Value by Use ($/af) [Frederrick et al. (1996)]

		Average	Median	Min	Max
Instream	Waste Disposal	3	1	0	12
	Recreation/F&W Habitat	48	5	0	2,642
	Navigation	146	10	0	483
	Hydropower	25	21	1	113
Offstream	Irrigation	75	40	0	1228
	Industrial	282	132	28	802
	Thermo Power	34	29	9	63
	Domestic	194	97	37	573

Table 2.9 shows data on the value of water for recreations and fish & wildlife uses within the United States and Table 2.2.6.3 shows the value of water use in irrigated agriculture.

Table 2.9. Water Values for Recreation/F&W Habitat ($/af) [Frederrick et al. (1996)]

	Average	Median	Min	Max
Fishing	34	5	0	158
Wildlife Refuge	24	6	1	44
Fishing & Whitewater	1042	1505	6	3
Whitewater	9	9	5	4
Shoreline Recreation	19	19	17	2

Table 2.10. Water Values by Crop ($/af) [Frederrick et al. (1996)]

	Average	Median
Alfalfa	51	44
Apples	151	151
Barley	33	39
Beans	58	58
Carrots	550	550
Corn	91	98
Cotton	114	103
Grain Sorgham	57	44
Hay	36	36
Hops	18	18
Lettuce	208	208
Melons	54	54
Onions	40	40
Pears	137	137
Potatoes	710	784
Rice	86	86
Safflower	53	58
Soybeans	121	127
Sugar Beets	121	119
Tomatoes	686	686
Vegetables	206	206
Wheat	51	47

2.3 Supply of Water

2.3.1 Introduction

Firms produce outputs from various combinations of inputs. The objective of a firm is to maximize profit subject to constraints imposed by technological capabilities. As long as inputs are costly, we can limit our consideration to those combinations of inputs that will produce the *maximum* output for a given level of inputs. This represents the boundary of the so-called production possibilities set and it is called the *production function*.

$$y = f(\boldsymbol{x}) \tag{2.3.1}$$

Level curves of the production function are called isoquants where $f(\boldsymbol{x}) = y_0$ and y_0 is a fized level of output. For a firm producing a single output (y) from two inputs (x_1 and x_2).

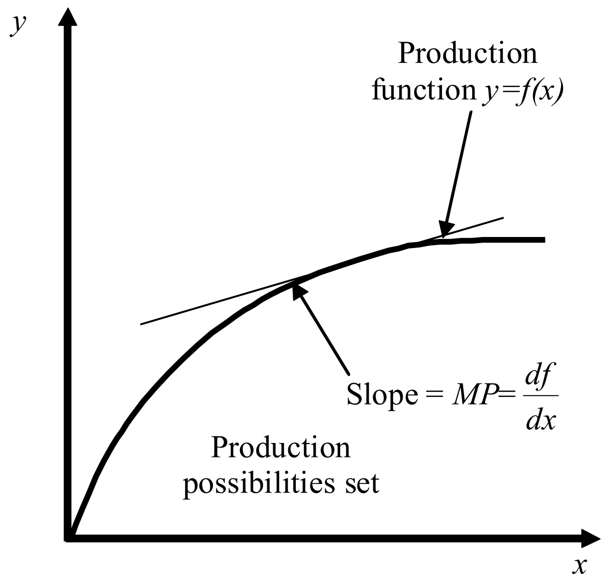


Figure 2.11. Production function.

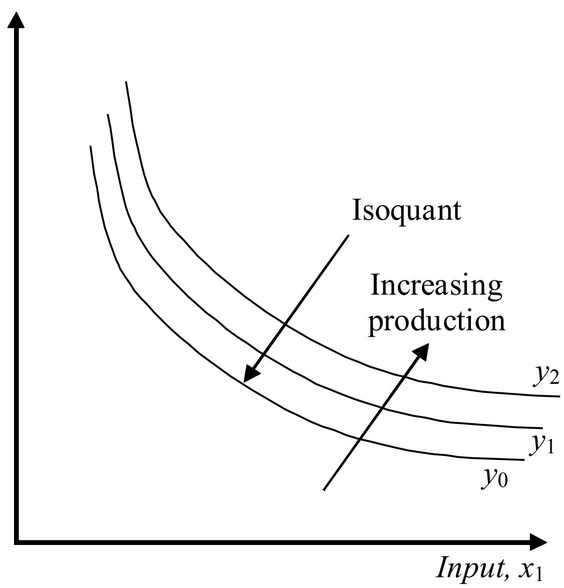


Figure 2.12. Isoquants

Suppose that a firm wants to increase the amount of one input and decrease the amount of another while maintaining production at a constant level of output $f(\boldsymbol{x}) = y_0$. So if production is to remain constant, we can write

$$dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x2} dx_2 = 0 \qquad (2.3.2)$$

where

$$\frac{\partial f}{\partial x_i} = \frac{\partial y}{\partial x_i} = MP_i, \qquad i = 1,2 \tag{2.3.3}$$

is the *marginal product* or the additional output available from using an additional unit of input x_i in the production process. Now, from Equations 2.3.1.1 and 2.3.1.2, we have

$$\frac{dx_2}{dx_1} = -\frac{MP_1}{MP_2} = TRS_{12} \tag{2.3.4}$$

where TRS_{12} is the *technical rate of substitution*. That is, the rate at which x_1 can replace x_2. TRS_{12} is the slope of the isoquant $f(x_1, x_2) = y_0$.

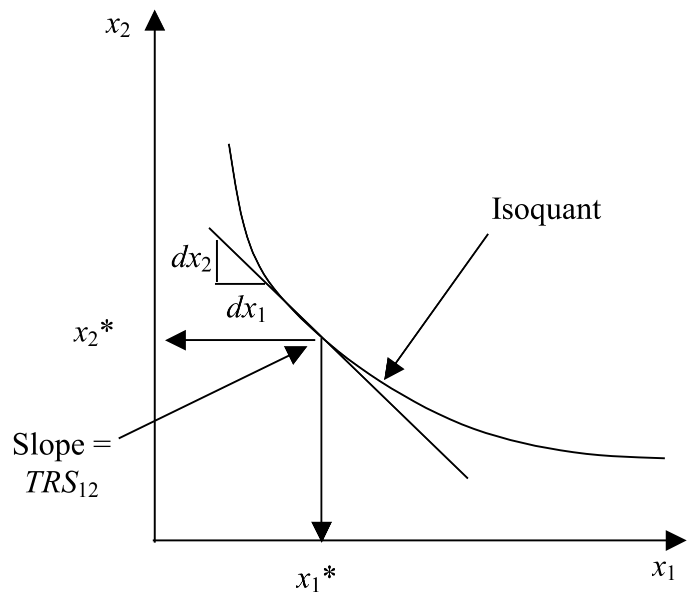


Figure 2.13. Technical rate of substitution.

Revenue R is the amount of money that a firm receives for selling an amount *y* of a product for a particular price *p*:

$$R = py \tag{2.3.5}$$

Marginal revenue is the change in revenue for a change in the output or the quantity sold

$$\frac{dR}{dy} = \frac{\partial R}{\partial y} + \frac{\partial R}{\partial p}\frac{dp}{dy} = p + y\frac{dp}{dy} \tag{2.3.6}$$

Example 5. Consider the case of a linear inverse demand function:

$$p(y) = a - by \tag{2.3.7}$$

Then, revenue is given by the quadratic function

$$R = py = ay - by^2 \tag{2.3.8}$$

and marginal revenue, the derivative of revenue with respect to output, is

$$\frac{dR}{dy} = a - 2by \tag{2.3.9}$$

Thus, the slope of the marginal revenue curve is twice as steep as that of the demand curve.

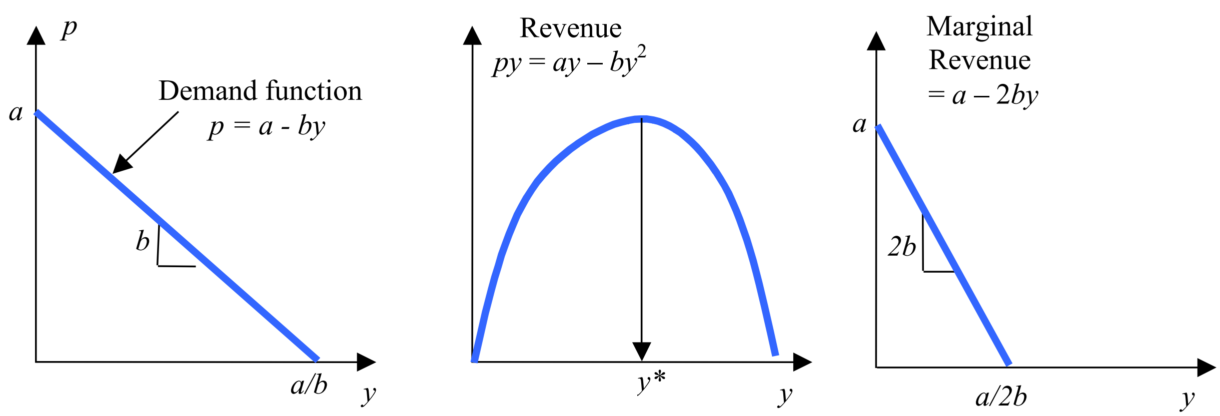


Figure 2.14. Marginal revenue and demand curves for a linear demand function.

Profit is the difference between the revenue a firm receives and the cost that it incurs

$$\pi(p, w) = py - \sum_{n=1}^{N} w_n x_n = pf(x_1, \ldots, x_n) - \sum_{n=1}^{N} w_n x_n \tag{2.3.10}$$

2.3.2 The Firm's Problem

The firm's problem is to maximize profit

$$\textit{maximize} \;\; \pi(p, \mathbf{w}) = pf(x_1, \ldots, x_n) - \sum_{n=1}^{N} w_n x_n \tag{2.3.11}$$

The **first-order optimality conditions** for this problem are

$$\frac{\partial \pi}{\partial x_n} = 0 \qquad \Rightarrow \qquad p\frac{\partial f}{\partial x_n} = w_n, \text{ or } \frac{\partial f}{\partial x_n} = \frac{w_n}{p} \quad n = 1,\ldots,N \tag{2.3.12}$$

This condition says that the value of the marginal product (price times marginal product $p\,\partial f/\partial x_n$) for input n must equal the price of that input (w_n). This tangency condition is illustrated in Figure 2.3.2.1.

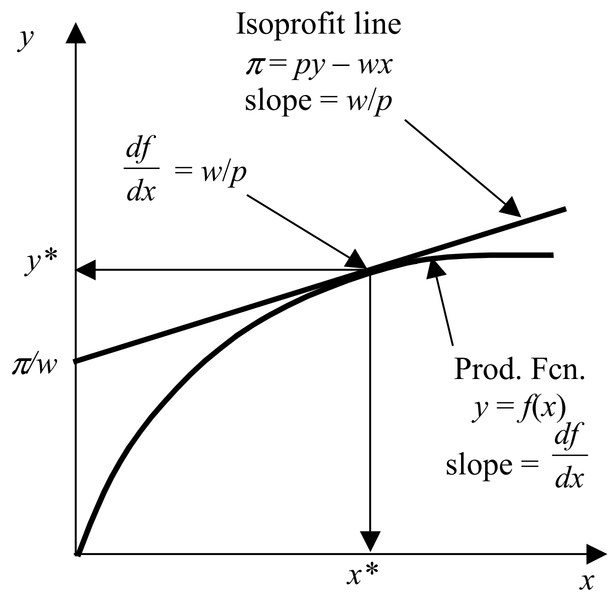


Figure 2.15. Profit maximization.

We can define a variant of the firm's problem, where a firm strives to minimize its costs while producing a specified level of output (y_0)

$$\begin{aligned} &\text{Minimize } \sum_{n=1}^{N} w_n x_n \\ &\text{subject to} \\ &\qquad f(x_1,\ldots,x_N) = y_0 \end{aligned} \tag{2.3.13}$$

The **Lagrangian** in this case is

$$L(\mathbf{x},\lambda) = \sum_{n=1}^{N} w_n x_n - \lambda[f(x_1,\ldots,x_N) - y_0] \tag{2.3.14}$$

The **first-order optimality conditions** are

$$\frac{\partial L}{\partial x_n} = w_n - \lambda\frac{\partial f}{\partial x_n} = 0 \qquad n = 1,\ldots,N \tag{2.3.15}$$

$$\frac{\partial L}{\partial \lambda} = f - y_0 = 0 \tag{2.3.16}$$

Writing the first condition for two products *i* and *j*, we have

$$\frac{w_i}{w_j} = \frac{MP_i}{MP_j} = -TRS_{ij} \tag{2.3.17}$$

That is, the technical rate of substitution equals the price ratio. The tangency condition for optimality in this case is illustrated in Figure 2.3.2.2.

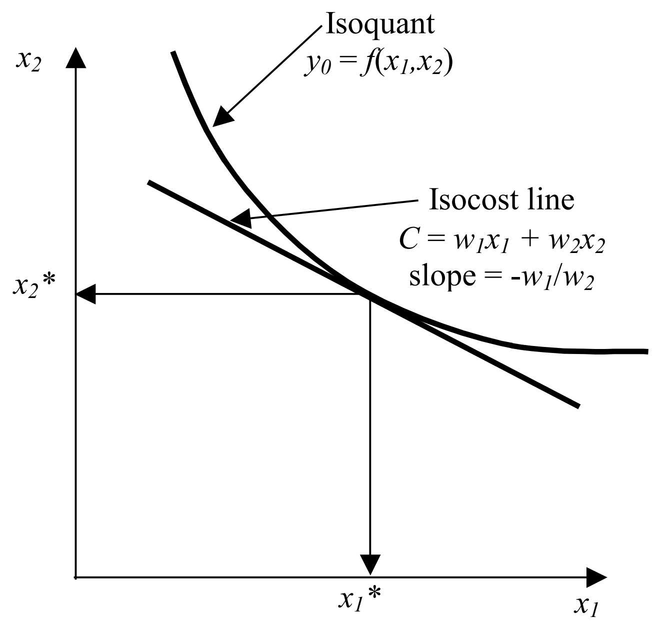


Figure 2.16. Cost minimization.

The total cost of producing at output level, *y*, is

$$TC(y) = \min\{\vec{w} \cdot \vec{x} : y = f(\vec{x})\} \tag{2.3.18}$$

The firm's total cost is comprised of *fixed* (*FC*) and *variable* (*VC*) *costs*

$$TC(y) = FC + VC(y) \tag{2.3.19}$$

The firm's *average cost* is the cost per unit to produce *y* units of output, or

$$AC = \frac{TC(y)}{y} \tag{2.3.20}$$

The firm's *marginal cost* is the cost of producing an additional unit of output

$$MC = \frac{dTC}{dy} = \frac{dVC}{dy} \tag{2.3.21}$$

Example 6. How much water should a water industry firm sell (produce) and at what price? The firm's problem can be defined as

$$\text{Maximize } py - TC(y) \tag{2.3.22}$$

The **first-order optimality conditions** are

$$MR(y) = \frac{dp}{dy}y + p = MC(y)$$
$$\textit{Marginal Revenue} = \textit{Marginal Cost} \tag{2.3.23}$$

An increase in output has two effects (1) adding *p* units to the benefits, and (2) causing the value placed on each unit of output to change by $p' = dp/dy$ (Dorfman, 1962). If the firm is **competitive**, then it has no market power, and $p' = dp/dy = 0$, and the price *p* is constant and fixed by the market. In this case, the first-order conditions are

$$p = MC \tag{2.3.24}$$

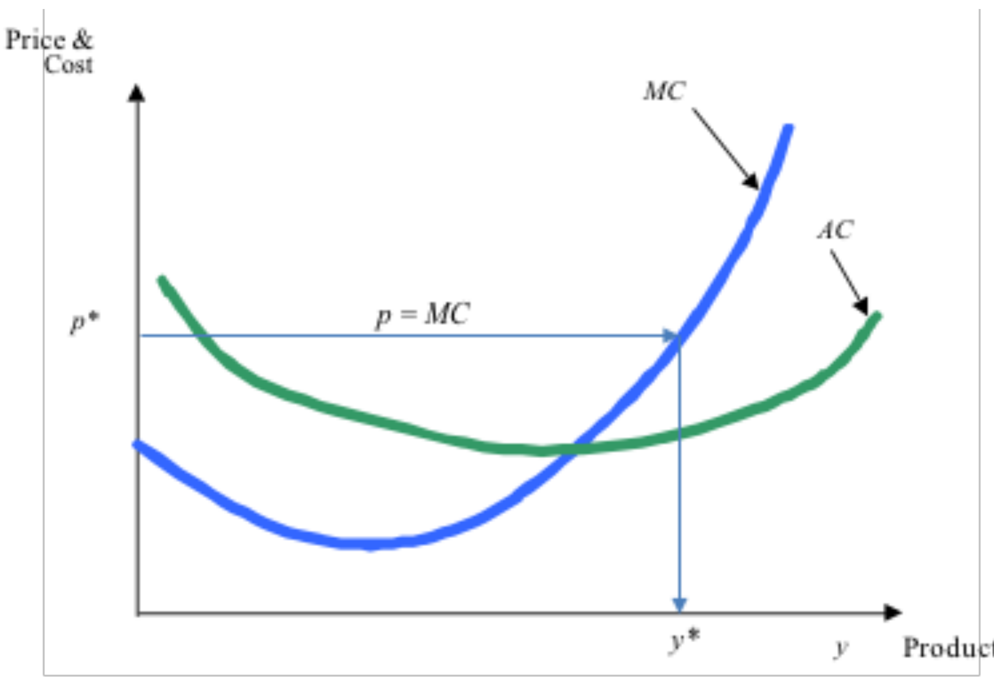


Figure 2.17. Average cost (AC), and Marginal cost (MC) curves.

However, if the firm is **monopolistic**, it is unlikely to take the output price as given, since the monopolistic firm recognizes its influence over market price. The firm is free to choose the price and level of output so as to maximize its profit. Since for a monopoly the price is not constant, but is a function of output, we have

$$MR = \frac{dp}{dy}y + p = MC \qquad (2.3.25)$$

If the monopolistic firm chooses to **maximize profit**, then its chosen price and level of output will be p_M and y_M in order to set $MR = MC$. However, the firm knows that consumers are willing to pay a price $p > MC$ (see Figure 2.3.2.4). Since the MR curve lies below the demand curve, the monopolistic firm will produce (y_M) which is less than the amount (y_C) which a competitive firm would produce. That is, the price will be higher and the output lower for the monopolistic firm. Government regulatory commissions often have substantial power over the prices charged by public utilities. Without regulation, the firm will charge the price p_M and produce y_M. By setting a maximum price of p_C, the commission can make the monopolist increase output, thus making price and output correspond more closely to what they would be if the industry were organized competitively. Commissions often set prices at the level at which it equals average cost.

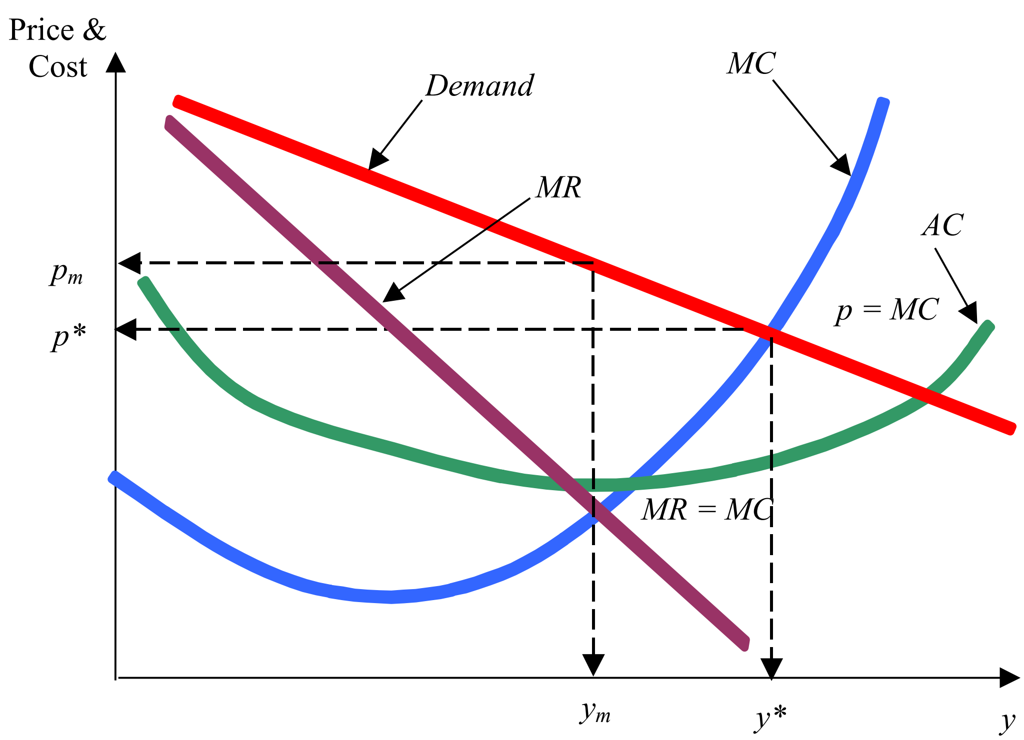


Figure 2.18. Optimal production level and price for a competitive firm and a monopolistic firm producing a good with a linear demand curve.

2.3.3 Crop Production Functions with Water

2.3.3.1 Introduction

The fundamental building block for the estimation of the demand for and value of water in the agricultural sector is a production function that relates crop production to the use of water and other inputs. An ideal crop-water production model should be flexible enough to address issues at the crop, farm, or basin levels. The production function should allow the assessment of policy-related problems, and results should be transferable between locations. In addition, the model should be simple to operate, requiring a small data set; easily adjustable to various farming conditions; and sufficiently comprehensive to allow the estimation of externality effects. In addition, the interaction between water quantity and quality and the water input/production output should be clearly defined (Dinar and Letey 1996).

Existing modeling approaches to crop-water relationships (for example, surveys by Hanks 1983 and Vaux and Pruitt 1983) address economic, engineering, and biological aspects of the production process. These surveys conclude that crop-water relationships are very complicated and that not all management issues have been fully addressed in one comprehensive model. In the following, the advantages and disadvantages of alternative production functions are summarized.

2.3.3.2 Types of Production Function Models

Four broad approaches to production functions can be identified:

1. Evapotranspiration and transpiration models:
2. Simulation models;
3. Estimated models; and
4. Hybrid models that combine aspects of the first three types.

The following overview on production functions related to water use draws heavily on Dinar and Letey (1996), chapters 2 and 3, for the first three types of models.

Evapotranspiration and Transpiration Models

Evapotranspiration models are physical models that predict crop yield under varying conditions of salinity levels, soil moisture conditions, and irrigation strategies. They assume a linear yield-evapotranspiration relationship and are usually site-specific and very data intensive (see also Hanks and Hill 1980).

A basic yield-seasonal evapotranspiration relationship is represented by:

$$Y / Y_{\max} = 1 - kc * (1 - E / E_{\max}) \qquad (2.3.26)$$

where

Y	=	actual yield (ton/ha)
Y_{max}	=	maximum dry matter yield (ton/ha)
kc	=	crop coefficient
E	=	actual evapotranspiration (mm)
E_{max}	=	maximum evapotranspiration (mm)

The parameter E can be estimated by

$$E = w + r + \Delta q - o - d \tag{2.3.27}$$

where

w	=	applied water (mm)
r	=	rainfall (mm)
Δq	=	change in soil water storage (mm)
o	=	runoff
d	=	drainage

Transpiration models use a similar approach but measurement of transpiration is more difficult because it is difficult to separate it from evaporation. Although evapotranspiration and transpiration models capture important aspects of crop-water relationships, they have limited ability to capture the impacts of non-water inputs, and are of limited use for policy analysis.

Simulation Models

Within the category of simulation models, Dinar and Letey (1996) distinguish between *holistic* simulation models, that simulate in detail the production process of one crop and *specific* models, that focus on one production input or the subsystems associated with a particular production input.

Detailed, data-intensive holistic models have been developed for most of the basic crops and a series of other agricultural production features (e.g., peanuts potatoes, maize, soybeans, and spring wheat). See also the CAMASE register, which currently includes more than 200 agro-ecosystem models or similar registers (CAMASE 1997). COTMOD, a model for cotton, for example, can be used to simulate the effects of various irrigation schedules, fertilizer application rates, and other management practices on cotton yield (Marani 1988). The relatively complicated data generation through field experiments and calibration procedures prevents the easy transferability of this model.

Dinar and Letey (1996) specify a model, in which annual applied water, irrigation water salinity, published coefficients relating crop sensitivity to salinity, the relationship between yield and evapotranspiration, and the maximum evapotranspiration for the area are the input parameters. Outputs include crop yield, amount of drainage water, and salinity of the drainage water. It is assumed that all nonwater-related inputs are applied at the optimum level. Water is the only limiting factor in the production process.

Estimated Production Function Models

Estimated production functions are more flexible than other model types. However, specification and estimation procedures must comply with plant-water relationships: (1) plant yield increases as water quantity increases beyond some minimum value; (2) yield possibly decreases in a zone of excessive water applications; (3) yields decrease as the initial level of soil salinity in the root zone or the salt concentration in the applied irrigation water increase beyond some minimum value; and (4) the final level of root zone soil salinity decreases with increasing irrigation quantities (except for possible increases, where relatively insufficient water quantities have been applied) (Dinar and Letey 1996). In order to meet these requirements, polynomial functions have been applied in many production functions. Dinar and Letey (1996) present the following quadratic polynomial form in the case of three production inputs:

$$Y / Y_{\max} = a_0 + a_1 w + a_2 s + a_3 u + a_4 w \cdot s + a_5 w \cdot u + a_6 s \cdot u + a_7 w^2 + a_8 s^2 + a_9 u^2 \tag{2.3.28}$$

where

Y	= yield
Y_{max}	= maximum potential yield
w	= water application to potential evapotranspiration,
s	= salinity of the irrigation water,
u	= irrigation uniformity,
a_i	= estimated coefficients (i=1,..9)

The quadratic form implies that an increase in the level of one of the decision variables results in a constant change in the level of the dependent variable up to a point. Any further increase results in an opposite response (positive-diminishing marginal-productivity zone on the production surface), followed by a zone of negative marginal productivity.

Hybrid Production Function Models

Hybrid models, which draw on the strengths of each production function approach, may offer considerable advantages to the three types of approaches taken individually. As noted above, each of the three basic methodologies for production functions have some weaknesses. Particularly limiting may be the data requirements for any given approach. It is likely that, for some relationships embodied in the model, available experimental and non-experimental data, especially on the interrelationships of water use, resource degradation, and production, may be inadequate. Several reasons can account for this. Non-experimental data (cross-section and time series data) collected by government agencies or targeted surveys rarely can adequately measure or control for water and important environmental variables (like water table depth and soil and water quality). Generation of this type of data can also be difficult, expensive, and often impractical, if not impossible, to achieve.

In many instances, however, data are not entirely absent. If data are relatively sparse, the available observations may not be adequate for statistical analysis but can be useful in calibrating generalized versions of simulation models. When important bio-physical and environmental variables in the study are inadequate or unavailable, simulation models can be used to generate pseudo-data. Pseudo-data are not true historical data, but rather are derived from process models replicating the real-world processes in computer experiments. Observations are generated by repeatedly solving the model for different initial values, and by parametrically varying input or output quantities and values. Simulation models are practical substitutes for complex biophysical experiments (or even non-experimental data), where it is often difficult to isolate the impacts of important policy, management, or environmental variables on output variables. In simulation models, the analyst can control institutional, technological and environmental factors, which is not possible with real-world experiments.

2.3.3.3 Example: Production of wheat in the Maipo basin, Chile

This example is adapted from *Rosegrant et al.* (2000). In agricultural production, water is allocated to crops according to their water requirements and economic profitability. Water demand can be determined in an optimization model based on empirical agronomic production functions for agriculture. The relationship between crop yield and seasonal applied nonsaline water provides values of crop yield under various water application, irrigation technology, and irrigation water salinity. The production function can be used directly used in an optimization model to calculate crop yields with varying water application, salt concentration, and irrigation technology. The crop yield (production) function is specified as follows:

$$y = y_{\max}[a_0 + a_1(x'/E_{\max}) + a_2 \ln(x'/E_{\max})] \tag{2.3.29}$$

where

$$\begin{aligned} a_0 &= b_0 + b_1 u + b_2 c \\ a_1 &= b_3 + b_4 u + b_5 c \\ a_2 &= b_6 + b_7 u + b_8 c \end{aligned} \tag{2.3.30}$$

and

y	crop yield (metric tons [mt]/ha),
y_{max}	maximum attainable yield (mt/ha)
a_0, a_1, a_2	coefficients,
$b_0 - b_8$	coefficients,
x'	infiltrated water (mm)
E_{max}	maximum evapotranspiration (mm)
c	salt concentration in water application (dS/m). Use the factor 1.14 to convert dS/m to g/L, and
u	Christiensen Uniformity Coefficient (CUC).

Uniformity (CUC) is used as a surrogate for both irrigation technology and irrigation management activities. The CUC value varies from approximately 50 for flood irrigation, to 70 for furrow irrigation, 80 for sprinklers, and 90 for drip irrigation, and also varies with management activities. By including explicit representation of technology, the choice of water application technology can be determined endogenously. The coefficients for the function as estimated by *Rosegrant et al.* (2000) are shown in Table 2.11. Using these coefficients in Table 2.12.

Table 2.11. Coefficients for the Production Function.

Coefficient	Wheat		
B1	0.284973	B5	-0.81096
B2	1.153264	B6	0.030845
B3	0.183139	B7	0.141539
B4	-0.05615	B8	1.181461
		B9	-0.03203

Table 2.12. Coefficients for the Production Function.

Coefficient	Wheat
a0	1.037233
a1	-0.35431
a2	0.937176

For the wheat produced in the Maipo basin in Chile, we have

E_{max} = 535.5 mm/m2/yr
y_{max} = 6 mt/ha

A typical crop yield function for wheat in the Maipo river basin is shown in Figure 2.3.3.1 and the data are shown in Table 2.3.3.3. In this figure and the table, the input, denoted x, is actually

$$x = x' * E_{\max} * 10{,}000 m^2 / ha \qquad (2.3.31)$$

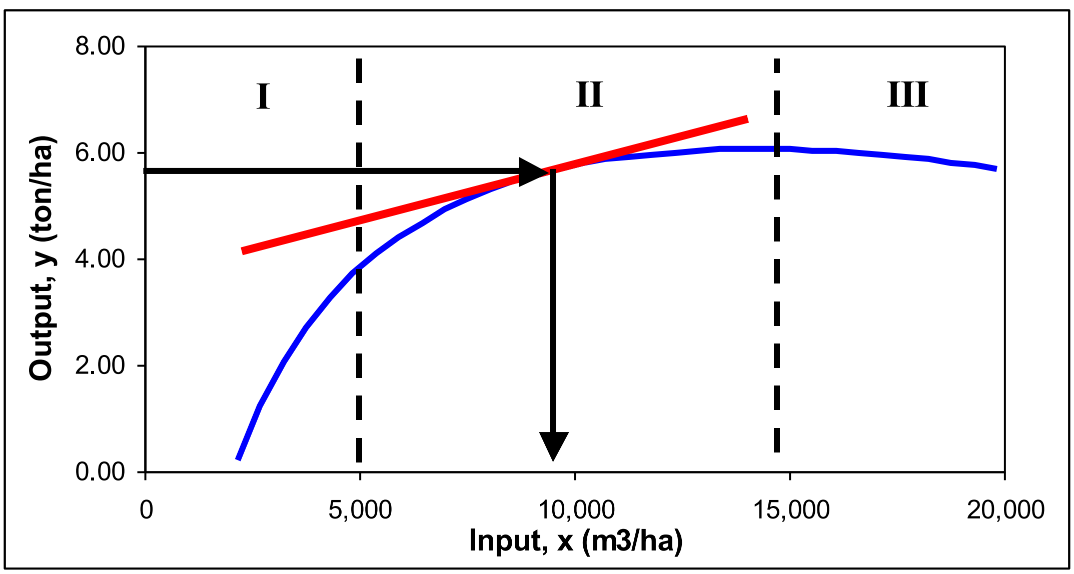


Figure 2.16. Production function for wheat as a function of applied water (CUC=0.7, salinity = 0.7).

In Fig. 2.16, it is evident that a certain amount of water must be applied to the crop before any production can result. Output increases at an increasing rate as the first few units of input are added; it continues to increase at a decreasing rate at higher input levels (Law of Diminishing Returns).

Average product is obtained by dividing the output by the input

$$AP = \frac{y}{x} \tag{2.3.32}$$

and it measures the efficiency of the input used in the production. *Marginal product* is the change in the output resulting from a unit increment in input, that is

$$MP = \frac{dy}{dx} = Y_{\max}\left(\frac{a_1}{E_{\max}} + \frac{a_2}{x}\right) \tag{2.3.33}$$

AP and *MP* are shown in Fig. 2.17.

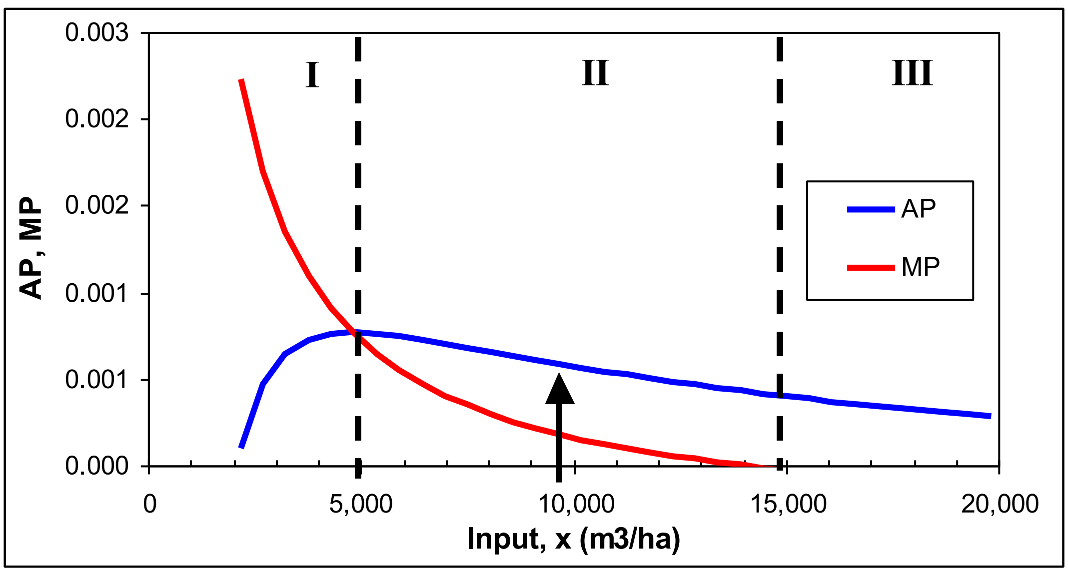


Figure 2.17. Average and marginal productivities versus input.

The production function can be broken into three regions, depending on the efficiency of resource use:

Region I *MP* > *AP*, not enough input is being used

If the product has value, input use should be increased until Region II is reached, since the physical efficiency of the input increases throughout Region I. It is not reasonable to cease using the input while efficiency is increasing.

Region II *MP* is decreasing and *MP* < *AP*, just enough input is being used

Region II defines the area of economic relevance and optimal input use must be in this range. The exact level of production and resource use depend on the input and output prices.

Region III $MP < 0$, too much input is being used

Even if the input is free, it will not be used in this stage, since maximum output occurs at the boundary of Region II and further inputs simply decrease output.

The costs of production for this example include: *Fixed costs*, *FC*, are \$100/ha and include ground preparation and other costs. *Variable cost*, *VC*, is computed by multiplying the input, x, by the unit price of the input, w (\$0.05/m^3 in this example)

$$VC = wx \tag{2.3.34}$$

The shape of the *VC* curve depends on the shape of the production function. *Total cost*, *TC*, is simply the sum of *FC* and *VC*.

Average costs (*AC*) follow in the same way that average production does, they are the costs divided by the amount of the output

$$AC = \frac{VC}{y} = \frac{wx}{y} \tag{2.3.35}$$

Average variable cost (*AC*) is inversely related to the average product, attaining a minimum when *AP* is at a maximum. When *AC* is decreasing, the efficiency of the input is increasing and efficiency is maximum when *AC* is minimum. *Marginal cost* (*MC*) is the change in the cost per unit of input

$$MC = \frac{dVC}{dy} = \frac{w}{\frac{dy}{dx}} \tag{2.3.36}$$

This is the slope of the cost curve cost and its value is the cost of producing an additional unit of output.

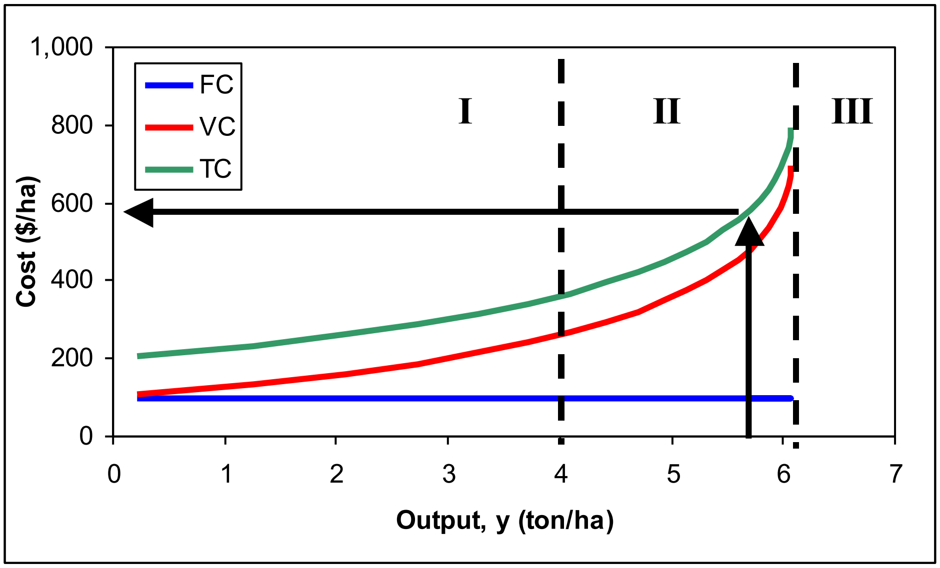


Figure 2.18. Fixed, variable and total costs versus output.

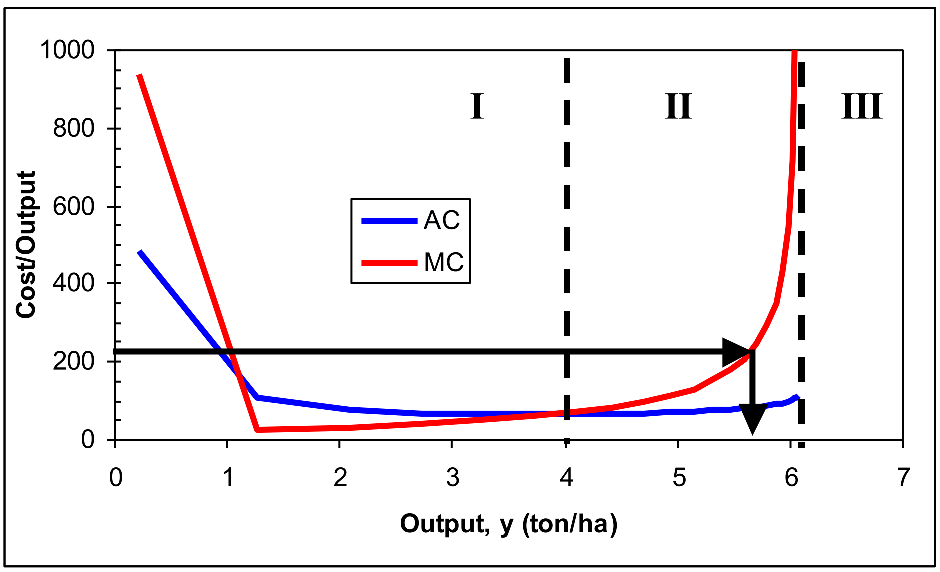


Figure 2.19. Average and marginal costs and the point where *p = MC*.

To determine the most profitable level of input, or the most profitable level of output, profit

$$\pi = py - wx \tag{2.3.37}$$

is maximized, where w is the price of the produced good (wheat in this example and p = \$230/mt). The problem is to find the level of x which maximizes this function. The derivative with respect to input must be set equal to zero

$$\frac{d\pi}{dx} = p\frac{dy}{dx} - w = 0 \tag{2.3.38}$$

We can rearrange this expression to yield

$$\frac{dy}{dx} = \frac{w}{p} \tag{2.3.39}$$

or

$$p = \frac{w}{\frac{dy}{dx}} \tag{2.3.40}$$

resulting in

$$p = MC \tag{2.3.41}$$

This condition is shown in Fig. 2.3.3.4.

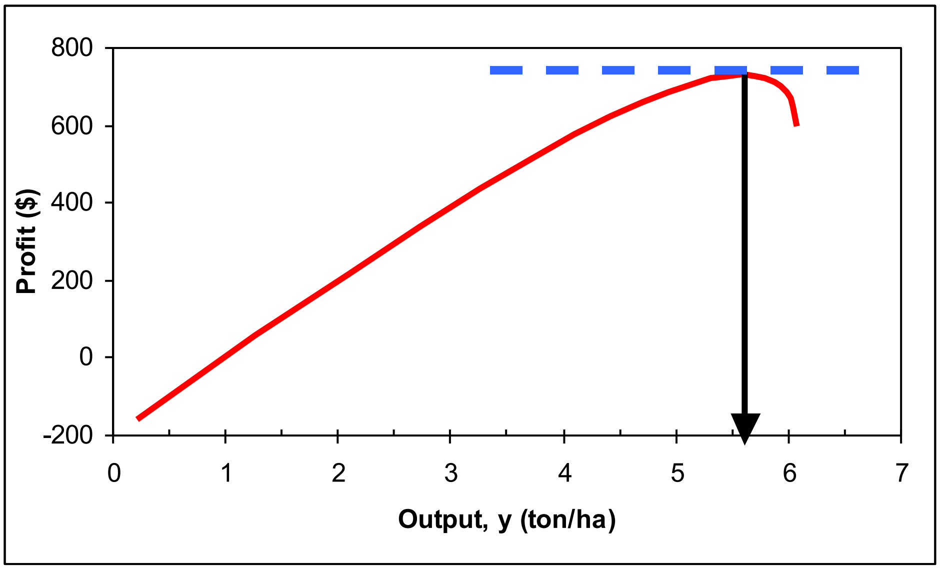


Figure 2.20. Profit versus output.

Table 2.13. Various production and cost data for wheat.

Input	Product	Ave. Prod.	Marg. Prod.	Fix. Cost	Var. Cost	Total Cost	Ave. FC	Ave. VC	Ave. TC	Marg. Cost	Total Val. Prod.	Profit
X	Y	AP=Y/X	MP	TFC	TVC	TC	AFC=FC/Y	AVC=AC/Y	ATC=TC/Y	MC	TVP= p_y*Y	π
(m3/ha)	(mt/ha)	(mt/m3)	(mt/m3)	($)	($)	($)	($/mt/ha)	($/mt/ha)	($/mt/ha)	($/mt/ha)	($/ha)	($/ha)
2142	0.22	1.030E-04	4.122E-04	100	107.1	207.1	453.1	485.3	938.3	938.3	50.8	-156.3
3213	2.08	6.460E-04	1.518E-03	100	160.7	260.7	48.2	77.4	125.6	32.9	477.4	216.7
4284	3.27	7.628E-04	1.005E-03	100	214.2	314.2	30.6	65.5	96.1	49.7	751.6	437.4
5355	4.10	7.652E-04	7.094E-04	100	267.8	367.8	24.4	65.3	89.7	70.5	942.4	574.7
6426	4.70	7.310E-04	5.167E-04	100	321.3	421.3	21.3	68.4	89.7	96.8	1080.4	659.1
7497	5.14	6.855E-04	3.812E-04	100	374.9	474.9	19.5	72.9	92.4	131.2	1182.0	707.2
8568	5.46	6.378E-04	2.807E-04	100	428.4	528.4	18.3	78.4	96.7	178.1	1256.9	728.5
9639	5.70	5.916E-04	2.032E-04	100	482.0	582.0	17.5	84.5	102.1	246.0	1311.5	729.5
10710	5.87	5.480E-04	1.416E-04	100	535.5	635.5	17.0	91.2	108.3	353.0	1349.9	714.4
11781	5.98	5.076E-04	9.151E-05	100	589.1	689.1	16.7	98.5	115.2	546.4	1375.4	686.4
12852	6.04	4.703E-04	4.992E-05	100	642.6	742.6	16.5	106.3	122.9	1001.6	1390.2	647.6
13923	6.07	4.359E-04	1.486E-05	100	696.2	796.2	16.5	114.7	131.2	3365.0	1395.9	599.7
14994	6.06	4.042E-04	-1.510E-05	100	749.7	849.7	16.5	123.7	140.2	-3311.3	1394.0	544.3
16065	6.02	3.749E-04	-4.100E-05	100	803.3	903.3	16.6	133.4	150.0	-1219.7	1385.4	482.1
17136	5.96	3.479E-04	-6.360E-05	100	856.8	956.8	16.8	143.7	160.5	-786.1	1371.1	414.3
18207	5.88	3.228E-04	-8.351E-05	100	910.4	1010.4	17.0	154.9	171.9	-598.7	1351.7	341.3
19278	5.77	2.995E-04	-1.012E-04	100	963.9	1063.9	17.3	167.0	184.3	-494.2	1327.8	263.9

2.3.4 Opportunity Cost

Private firms operating in a market value productive resources as the cost to procure them in the market. We need a somewhat broader concept of cost here. The question we ask ourselves is: "What could have been produced with these productive inputs had they not been used in the current alternative?" This is the opportunity cost of using the inputs in the current alternative being considered. It is the maximum value of the other outputs we could have produced had we not used the resources to produce the item in question (Field, 1994). Opportunity costs include the out-of-pocket expense that the private firm operating in a competitive market considers, but they are broader than this.

Example: A manufacturing process may produce waste products that are discharged to a nearby stream. Downstream these production residuals produce environmental damage, which are the real opportunity costs of the production process, even though they do not show up as costs in a profit-and loss statement.

2.3.5 Average Cost Pricing

The demand curve (*D-D*) of the consumers of a water utility and the utility's Average Cost (*AC*) and Marginal Cost (*MC*) curves are given in Figure 2.3.5.1. Recall that the *MC* is less than the *AC* where the latter is declining and greater than *AC* where the latter is rising. If a single price is charged so as to "cover" costs, while clearing the market, that price can only be equal to OT, since at a price OT, the quantity OA would be demanded, the production of which involves an average cost of OT. At this solution, zero profits are earned; price equals unit cost. But this is not the solution that corresponds to the best use of society's resources. To see this, consider the units of output between OB and OA. For each of these units the marginal cost--the additional cost of producing the unit considered--is greater than the amount anyone is willing to pay for the extra unit supplied---the consumers' marginal value in use (*D-D*, the marginal willingness to pay). The quantity OB is demanded at price OU, and, if any larger quantity is to be taken by consumers, the price will have to be reduced below OU. But the marginal cost is higher than OU throughout the range being considered, which means that there are alternative uses of the resources entering into this *MC* which consumers value more highly than what those resources can produce in the use considered here. The solution for best use of resources is to produce just up to the point where the *MC* begins to exceed the price that consumers are willing to pay for the additional unit produced; that is, the correct output is OB at the marginal-cost price OU. (Hirshleifer et al., 1960)

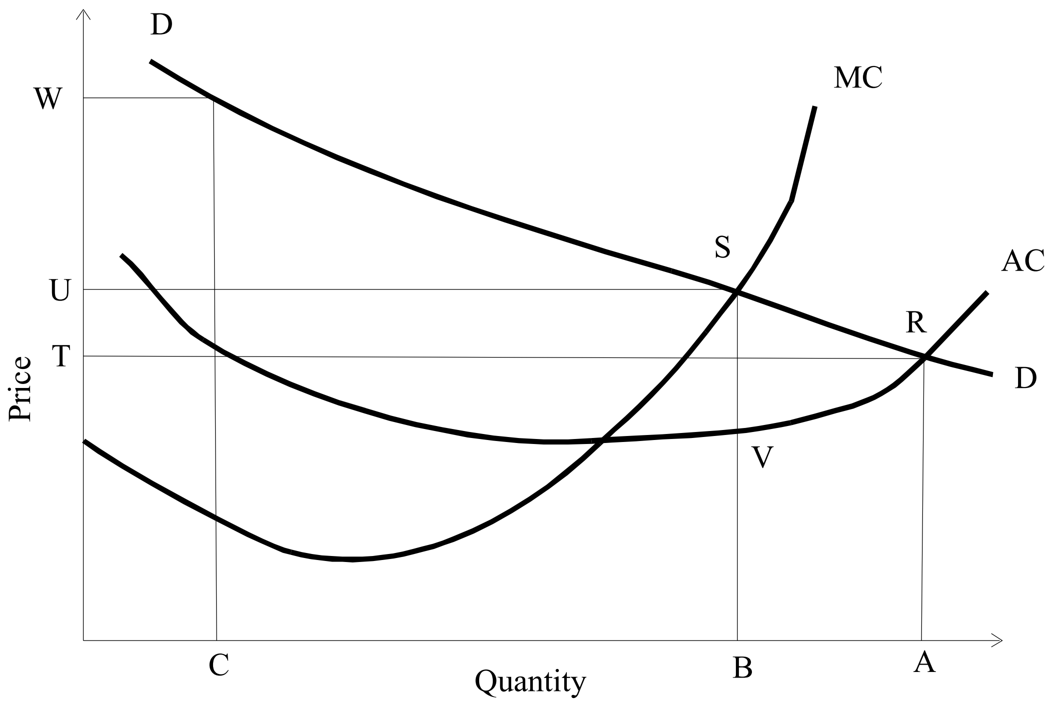


Figure 2.21. Average cost (AC), and Marginal cost (MC) curves. The optimal level of production occurs where the demand intersects the MC curve.

2.3.6 Criteria for Decision Making

Efficiency can be defined as an allocation of resources where the net benefits from the use of those resources is maximized. Net benefits are the excess of benefits over costs. How do we measure benefits and costs?

Environmental goods and services have costs associated with them even when they are produced without human input. Opportunity costs are the net benefits foregone because resources providing services can no longer be used in the next most beneficial use. For example, consider a river. Possible uses include (1) white-water rafting, and (2) hydroelectric power production. Constructing a dam for the purpose of power production would flood rapids that are used for the purpose of rafting. The opportunity cost of saving the river for rafting is the net benefit of the power production that is foregone. The marginal opportunity cost curve is the supply curve for a good in a competitive market. The total cost equals the area under the marginal cost curve. The net-benefits are the area under the demand curve above the supply (marginal cost) curve. The net-benefits will be maximized when the output level and price are set at the point where the marginal benefit and marginal cost curves intersect.

This section is largely adapted from Hutchens and Mann2. Figures 2.3.6.1 – 2.3.6.4 illustrate the roles of demand and supply functions, in the theoretical derivation of consumers' and producers' surpluses, and show the proportioning of each as a result of market interaction. Figure 2.3.6.1 presents a demand curve and the total utility derived by consumers in the consumption of quantity x_0. The negative slope of the demand curve is derived from the definition of demand, which states that for any commodity that can be purchased in a market, the quantity demanded in a given period of time varies inversely with the price, other things equal. The demand curve consists of the locus of points of marginal utility associated with each incremental unit of a commodity consumed. Consequently, total utility is the integral represented by the area under the demand curve.

The area under the demand curve within the points O, p_1, A, and x_0 represents the maximum amount consumers would be willing to pay for the consumption of x_0 units of the commodity rather than go without it. This maximum willingness to pay reflects the total utility or benefit to the consumer. However, resources were expended to produce that output and the value of those expended resources must be deducted from the total benefit to determine the net benefit. Figure 2.3.6.2 illustrates the cost of resources (factors of production) required to produce x_0. The supply curve represents the locus of marginal cost associated with producing each increment of commodity x. The integral of that function, represented by the area under the supply curve delineated by points O, p_2, A, and x_0, is the total value of the resources required to produce x_0. This cost represents the minimum amount that the producer will accept for x_0 units and, therefore, the minimum amount that the consumer must pay.

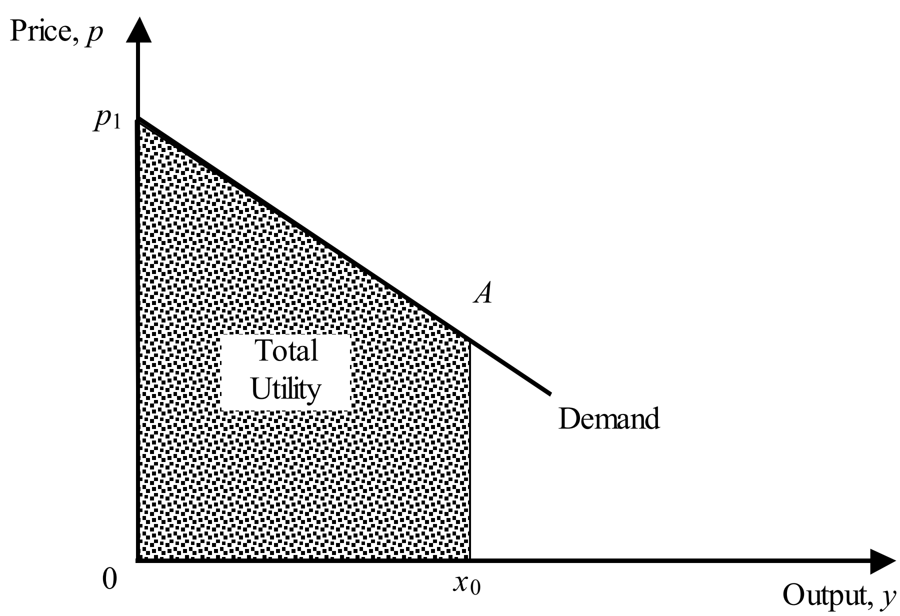


Figure 2.22. Net-benefits for a linear demand function and constant marginal costs.

2 Hutchens, A.O., and P.C. Mann , Review of Water Pricing Policies, Institutions and Practices in Central Asia, Environmental Policy and Technology Project, US Agency for International Development, Almaty, Kazakhstan, 1998.

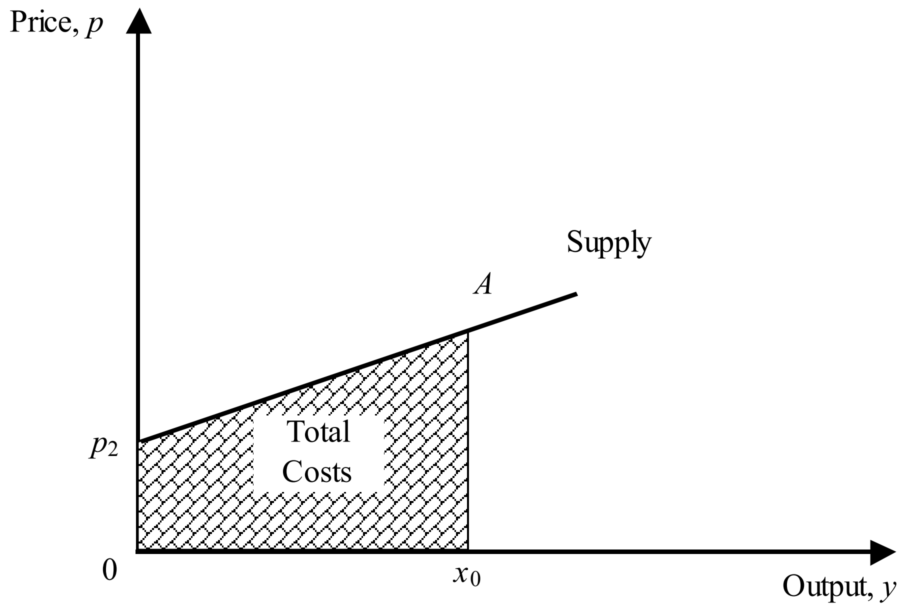


Figure 2.23. Marginal Opportunity Cost Curve for a constant marginal costs.

The total utility or benefit minus the total factor cost yields the total surplus net of resource costs delineated by p_2, p_2, A. This, then, represents the difference between the maximum the consumer would be willing to pay rather than go without and the minimum he must pay in order to cover costs of production. It can also be viewed as the total net benefit to society.

The crucial issue of how this surplus or net benefit is shared or proportioned between producers and consumers is determined by the interaction of supply and demand in the market to determine the market price. The area O, p_0, A, x_0, represents the amount that the consumer actually pays and, also, the amount that the producer actually receives. Therefore, the price line p_0, A, divides the total surplus into the amount the consumer would have been willing to pay, but did not have to, (consumers' surplus) and the amount in excess of what the producer would have been willing to accept, but was able to realize more (producers' surplus). Total costs shared in proportion to producers' and consumers' surpluses will be shared in proportion to benefits received, which satisfies the economic equity criterion**.** For most commodities, this would be automatically taken care of if there were an open competitive market; however, water and the necessary infrastructure to control it tend to exhibit some common property inflexibility and irreversibility characteristics that hinder a purely competitive market reallocation of water.

Output from the Egyptian Agricultural Sector Model (EASM) was used to derive estimates of consumers' and producers' surpluses under both financial and economic (free market) prices (Huchens and Mann, 1998). A run of EASM89 model derived the following estimates of producers' and consumers' surpluses:

	Base Case (1986)		Free Market (EASM89)	
Consumers' surplus	10067	55	6767	32
Producers' surplus	8236	45	14662	68
Total surplus	18303	100%	21449	100%

This shows that under 1986 financial price conditions, i.e., actual price controls and subsidies, consumers realized 55% and producers 45% of the "surplus value" in the agricultural sector. Under free market conditions, i.e., elimination of price, area and procurement controls, consumption subsidies, input subsidies and trade barriers, the proportions were estimated to be 32% consumers' surplus and 68% producers' surplus. The point to be realized from this is that, under free market conditions where farmers are not restricted by production quotas and administratively set prices, they will realize financial benefits that will enable him to pay for water services.

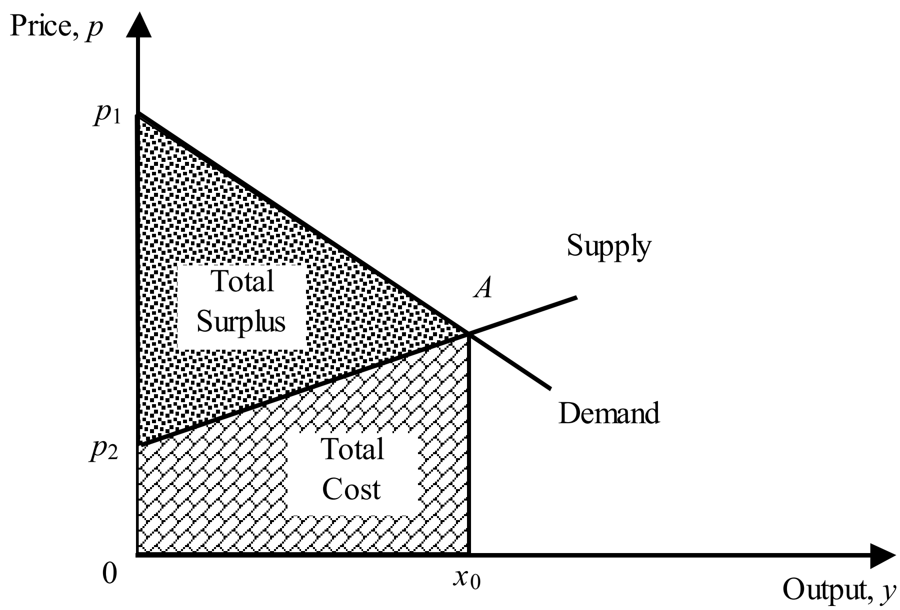


Figure 2.24. Maximum net-benefits for linear demand and constant marginal costs.

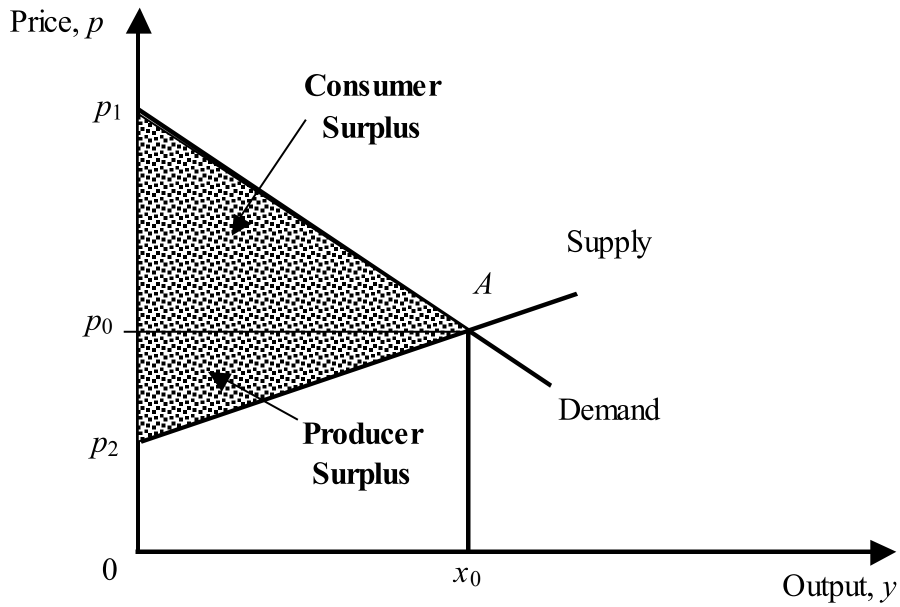


Figure 2.25. Maximum net-benefits for linear demand and constant marginal costs.

2.3.7 Externalities

The exclusivity of property rights is often violated, e.g., when a decision maker does not bear all of the consequences of a decision. Consider an example where a factory is producing a product and discharging waste to a nearby river. A hotel downstream of the factory uses the river for recreation. If there are different owners for the factory and the hotel, then an efficient use of the water in the river is not likely to occur. That is, the factory owner may not bear the cost of reducing business at the hotel as a result of the production decisions and resulting effluent discharge. The factory is likely to discharge too much effluent for a socially optimal solution.

An externality exists whenever the welfare of some agent (firm or consumer) depends on its own activities and the activities of some other (external) agent as well. In the above example, the additional cost to the hotel as a result of the factory discharge is an externality.

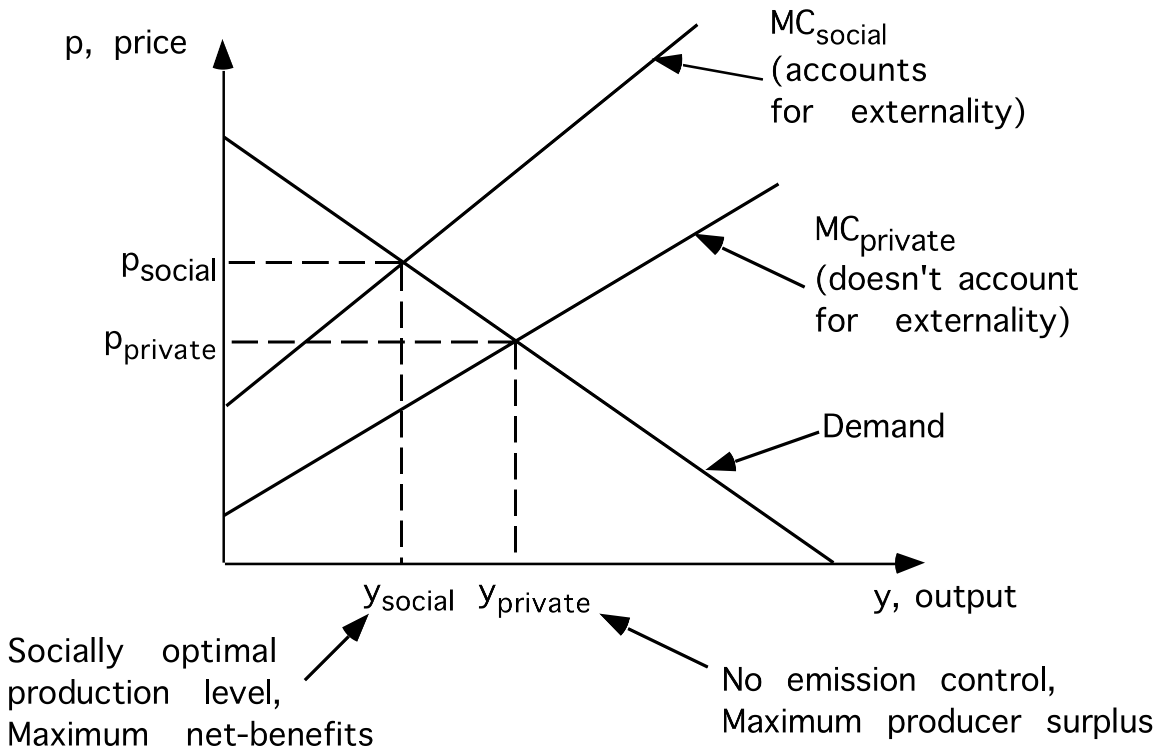


Figure 2.26. Effect of externality on production.

2.3.8 Production of Multiple Outputs

Previously, we have dealt with the production of a single output from multiple inputs. Suppose one input, x (water, say), can be used to produce two products, y_1 (irrigation, say) and y_2 (recreation, say) and that all other inputs to produce the outputs are fixed. So, the resource manager must decide how much input to allocate to the production of each output. If input x is unlimited, then the answer is found from equating the price of the input to the value of the marginal product of the input in production. When the input is limited, then the optimum amount of input can not be used in production of each output.

Production possibility curves (product transformation curves) represent the combinations of products that can be produced with a given set of inputs. Each point on the curve represents combinations of outputs produced using equal amounts of the input. A production possibility curve can be derived from two production functions (see Figure 2.27 and Table 2.14). The production functions use the same input x (water). Suppose that 10,000 m^3/ha of water are available. By using all 10,000 units of input on y_1, we can produce 5.8 mt/ha of wheat, or if all 10,000 units are used in y_2, 7.5 mt/ha of corn can be produced. We can consider many combinations between these two extremes. These combinations represent some of the production possibilities for 10,000 units of input.

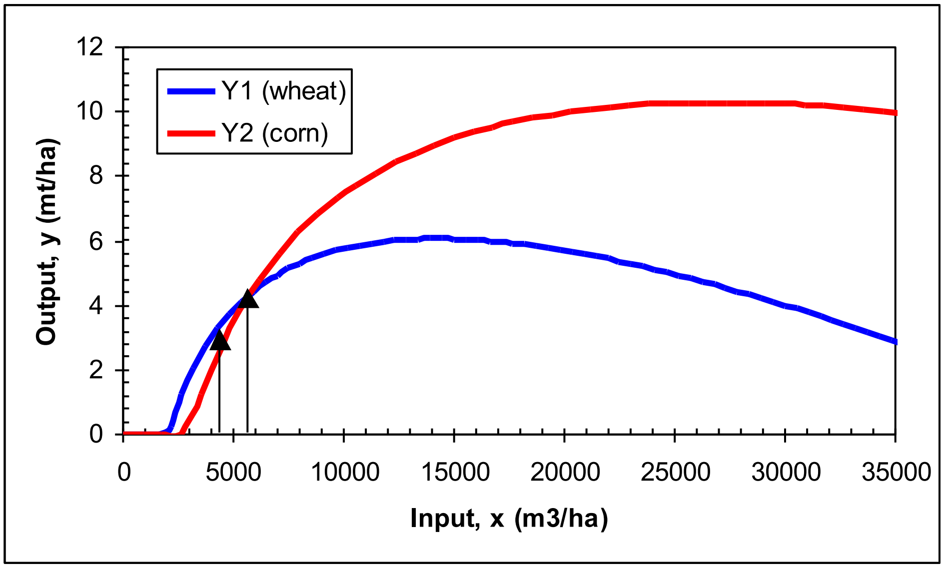


Figure 2.27. Production functions for wheat and corn.

Table 2.14. Production Functions for Wheat and Corn.

Production functions **for wheat and corn**				**Production Possibilities for *x* = 10000 m3/ha**	
Water x (m3/ha)	Wheat y_1 (mt/ha)	Water x (m3/ha)	Corn y_2 (mt/ha)	Wheat y_1 (mt/ha)	Corn y_2 (mt/ha)
0	0.00	0	0.00	5.8	0.0
536	0.00	883	0.00	5.6	0.0
1071	0.00	1765	0.00	5.3	0.0
1607	0.00	2648	0.00	4.9	0.5
2142	0.22	3530	1.21	4.5	2.0
3213	2.08	5295	3.87	3.8	3.5
4284	3.27	7060	5.60	3.0	4.6
5355	4.10	8825	6.83	1.8	5.5
6426	4.70	10590	7.74	0.2	6.3
7497	5.14	12355	8.43	0.0	6.9
8568	5.46	14120	8.96	0.0	7.5
9639	5.70	15885	9.37		
10710	5.87	17650	9.68		
11781	5.98	19415	9.91		
12852	6.04	21180	10.08		
13923	6.07	22945	10.19		
14994	6.06	24710	10.26		
16065	6.02	26475	10.28		
17136	5.96	28240	10.27		
18207	5.88	30005	10.23		
19278	5.77	31770	10.17		

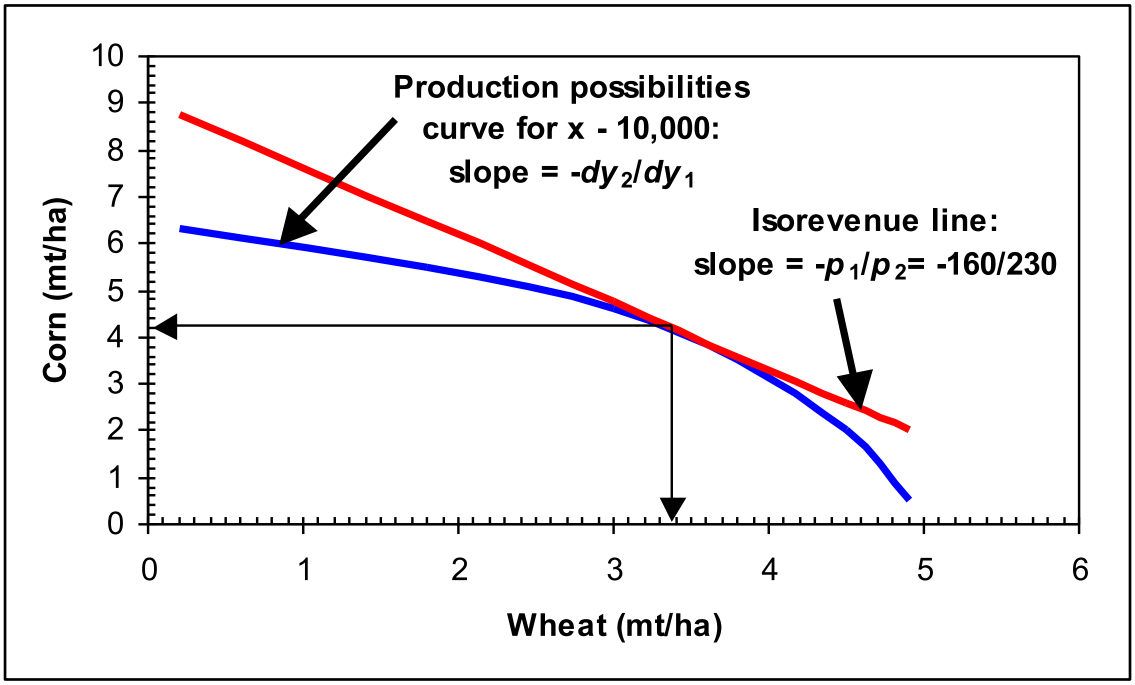


Figure 2.28. Production possibilities curve for wheat and corn with input of 10,000 m^3/ha of water.

Now, let's consider the problem of the farmer in this case. The decision to be made is how to allocate the scarce resources, $\boldsymbol{x} = (x_1,\ldots,x_J)$, to the production of various combinations of outputs, $\boldsymbol{y} = (y_1,\ldots,y_I)$. The objective is to maximize the farm income while satisfying the constraints of the production function. In general, we can write this problem as (*Willis and Finney*, 2000)

$$\begin{aligned} &\underset{\boldsymbol{x}}{\text{maximize}}\; U(\boldsymbol{y},\boldsymbol{x}) \\ &\quad\text{subject to} \\ &\qquad f(\boldsymbol{y},\boldsymbol{x}) = 0 \end{aligned} \tag{2.3.42}$$

Form the Lagrangian function

$$L(\boldsymbol{y},\boldsymbol{x},\lambda) = U(\boldsymbol{y},\boldsymbol{x}) - \lambda f(\boldsymbol{y},\boldsymbol{x}) \tag{2.3.43}$$

The optimality conditions are

$$
\begin{aligned}
\frac{\partial L}{\partial x_j} &= \frac{\partial U}{\partial x_j} - \lambda \frac{\partial f}{\partial x_j} = 0, \qquad j = 1,\ldots,J \\
\frac{\partial L}{\partial y_i} &= \frac{\partial U}{\partial y_i} - \lambda \frac{\partial f}{\partial y_i} = 0, \qquad i = 1,\ldots,I \\
\frac{\partial L}{\partial \lambda} &= f(\boldsymbol{y}, \boldsymbol{x}) = 0
\end{aligned}
\tag{2.3.44}
$$

Now, if the second of these equations is written for two products *i* and *k*, we have

$$
\begin{aligned}
\frac{\partial U}{\partial y_i} - \lambda \frac{\partial f}{\partial y_i} = 0 \\
\frac{\partial U}{\partial y_k} - \lambda \frac{\partial f}{\partial y_k} = 0
\end{aligned}
\tag{2.3.45}
$$

or

$$
\frac{\partial U / \partial y_i}{\partial U / \partial y_k} = \frac{\partial f / \partial y_i}{\partial f / \partial y_k}
\tag{2.3.46}
$$

but, from the total derivative of the production function, we have

$$
\frac{\partial f}{\partial y_i} dy_i + \frac{\partial f}{\partial y_k} dy_k = 0
\tag{2.3.47}
$$

or

$$
\frac{\partial f / \partial y_i}{\partial f / \partial y_k} = -\frac{dy_k}{dy_i}
\tag{2.3.48}
$$

so

$$
\frac{\partial U / \partial y_i}{\partial U / \partial y_k} = -\frac{dy_k}{dy_i} = MRS_{i,k}
\tag{2.3.49}
$$

where $MRS_{i,k}$ is the marginal rate of substitution of product *i* for product *k* which is the slope of the production possibilities curve, and $\partial U / \partial y_i$ is the marginal benefit from producing an additional unit of product *i*, that it, its price p_i. Total revenue is the value of the output produced:

$$
TR = p_1 y_1 + p_2 y_2
\tag{2.3.50}
$$

For various given values of total revenue (*TR*) this relationship is a straight line (*isorevenue line*). The distance of the isorevenue line from the origin is determined by the value of *TR*. As *TR* increases the line moves away from the origin. The slop of the isorevenue line is determined by the prices of the outputs.

Total costs are constant for all combinations of outputs on the production possibility curve. Profits will be maximized if the output combinations with the maximum *TR* is selected. This will be achieved at the point where the slope of the isorevenue line and the production possibility curve coincide.

Consider the case where

$$\begin{aligned} &\text{Maximize } p_1 y_1 + p_2 y_2 + FC \\ &\text{subject to} \\ &\qquad f(x, y_1, y_2) = 0 \end{aligned} \tag{2.3.51}$$

The optimality conditions are

$$\begin{aligned} \frac{\partial L}{\partial x} &= p_1 \frac{\partial y_1}{\partial x} + p_2 \frac{\partial y_2}{\partial x} - \lambda \frac{\partial f}{\partial x} = 0 \\ \frac{\partial L}{\partial y_1} &= p_1 - \lambda \frac{\partial f}{\partial y_1} = 0 \\ \frac{\partial L}{\partial y_2} &= p_2 - \lambda \frac{\partial f}{\partial y_2} = 0 \\ \frac{\partial L}{\partial \lambda} &= f(x, y_1, y_2) = 0 \end{aligned} \tag{2.3.52}$$

Now, from the second and third equations, we have

$$\frac{p_1}{p_2} = \frac{\partial f / \partial y_1}{\partial f / \partial y_2} \tag{2.3.53}$$

but, from the total derivative of the production function, we have

$$\frac{\partial f}{\partial y_1} dy_1 + \frac{\partial f}{\partial y_2} dy_2 = 0 \tag{2.3.54}$$

or

$$\frac{\partial f / \partial y_1}{\partial f / \partial y_2} = -\frac{dy_2}{dy_1} \tag{2.3.55}$$

so

$$\frac{p_1}{p_2} = -\frac{dy_2}{dy_1} = MRS_{1,2} \tag{2.3.56}$$

Thus, we see from these results that the point of optimal production is where a line with slope equal to the ratio of the prices of the outputs is equal to the slope of the production possibilities curve. This is illustrated in Figure 3.8.2 for our example of wheat and corn production. The prices are p_1 = \$230/mt (wheat) and p_2 = \$160/mt (corn). The resulting outputs are 3.3 mt of wheat (which uses 4284 m^3/ha of water) and 4.2 mt/ha of corn (which uses 5606 m^3/ha of water). The resulting total revenue is \$1450/ha.

2.4 Water Rights and Markets

2.4.1 Introduction

As the costs of water supply development increase it is increasingly important that supplies be allocated more efficiently than in the past. Systems of water allocation with nontransferable water rights can lead to rigid, inflexible, and inefficient allocations of water (Gibbons, 1986; Howe et al., 1986). Several advantages of private ownership and market exchange over bureaucratic control and allocation of water often exist. Markets have been established which have been successful in transferring water from low-valued to higher-valued uses over time (Rosegrant and Binswanger, 1994), where value is defined as the maximum amount a user would be willing to pay for the use of water (Gibbons, 1986). However, the establishment of water markets are often inhibited by the presence of externalities (third-party effects) such as increased pollution or changes in return flows. These effects must be accounted for in deciding a water right transfer and losing parties must be compensated for these effects (Howe, 1996).

Water is a necessary and scarce resource for the sustenance of human society and culture. However, the allocation of water to beneficial uses is a difficult problem, quite different from other resource allocation problems commonly considered in economics (Tregarthen, 1983). The quantity of water available in a river basin will fluctuate year-to-year. The interactions between surface water and groundwater are complex and not well known in many basins. Economies of scale exist in developed storage and distribution systems, encouraging the development of large systems that may be inflexible and non-robust over the long run. Externalities often exist and may require public sector intervention. The consumptive use of water in many cases may lead to the degradation of in-stream flow values and the loss of those benefits.

Several policy options exist that may lead to increases in water use efficiency while reducing environmental degradation and releasing water for increasing demands in other economic sectors (Rosegrant and Binswanger, 1994):

1. **Technological solutions:** (a) construction of new water resource systems, and (b) rehabilitation and modernization of existing systems, e.g., canal and drainage lining, and field drainage in irrigation systems.

2. **Reform of public management of water resource systems:** (a) modification of water distribution methods, (b) implementation of water pricing policies, and (c) reform of water management bureaucracies.

3. **Communal water resource system management** which involve water users more directly in both the process of system management and improvement.

4. **Establishment of tradable property rights** in water and development of markets in these rights. Market allocation of resources may be efficient given well-defined and nonattenuated (completely specified, exclusive, transferable, and enforceable) initial property rights allocation and low transaction costs.

Options 1 - 3 have been widely used by international lending institutions and national governments. Option 4 is somewhat new and is explored below.

2.4.2 Water Rights

Water law differs from country to country and within the US it varies from state to state. Traditionally, water law has been based on common law, but more recently it has shifted a bit toward legislative law. Most common law is based on protecting the quantity, not the quality of water. Common law is comprised of traditional legal aspects laid down by court decisions and it is based on precedent set by older cases. Common law can be overturned as the needs of society change over time. In the US, common law derives its legitimacy from the constitutions (US and States). Legislative law, on the other hand, is comprised of statutory law, where the legislature passes laws that regulate water, and administrative law, where the legislature may enable administrative bodies to write rules and regulations that have the power of law.

A well defined system of water rights is a necessary condition for the development of water markets. Water rights can be vested with individual citizens or with the government (local, state or national). Water rights are essentially a bundle of entitlements defining a water right owner's:

1. rights;
2. privileges; and
3. limitations

for the use of the water. Water rights are generally treated as real property with the right holder having a usufructuary right to make use of the water but not a right to physical possession of the water (Hirshleifer, et al., 1960). These rights must be well defined and exclusive to the person or entity owning them. Such a system of water rights must completely specify the (Howe, et al., 1986):

1. quantity of water that may be diverted;
2. quantity of water that may be consumed;
3. timing of the water delivery;
4. quality of the delivered water;

5. place of diversion; and
6. place of application.

Changes in any of these characteristics will likely affect other water users in the basin or system.

An efficient water rights structure should include:

- **Universality:** Rights must be privately owned, and entitlements must be completely specified.

- **Exclusivity:** Benefits and costs accrued as a result of owning and using the rights accrue to the owner (and only the owner). This property of water rights is often violated, e.g., when a user does not bear all of the costs of a water allocation, creating an externality that is not paid for by the user. An externality may exist whenever the welfare of some user depends on its own activities and the activities of some other user(s) as well.

- **Transferability:** The transfer of rights from one user to another must be entirely voluntary. The inability to transfer the rights would prevent the owner of the right from recognizing the true opportunity cost of the water, i.e., the value that another person may place on it.

- **Enforceability:** Right owners must be secure from involuntary seizure of their rights or encroachment on their rights.

Usually, systems for water rights (implicit or explicit) fall into one of several categories (Howe, 1996):

- **Non-tradable permits for water from undeveloped (natural) supplies**
  Non-tradable permits or rights are typically specified by laws or regulations, they are for specific or defined periods and they are not tradable. Problems associated with this type of arrangement include the fact that this method of water allocation does not consider the economic efficiency or equity of the use, and allocations may be inflexible and unresponsive to changes in social values.
- **Contracts for water from developed supplies**
  Developed supplies usually provide storage and distribution facilities and water is allocated to customers by contracts (as opposed to non-tradable permits where water is distributed by water right). Contracted water supply is usually for a specific use. Problems associated with this type of arrangement include the fact that the economic efficiency or equity of the water use is not considered.

In many cases, water demand is estimated from the projected requirement or need for water, that is, the farmer's ability to put water to use (Gardner, 1983). This method of demand estimation results in a maximization of physical yield rather than profit or social benefits. This can lead to the development of new water supplies rather than using economic incentives and market mechanisms to allocate water to its best uses. Systems of this type are common in the states of California (Gardner, 1983) and Texas (TWDB, 1997).

In other cases, a fixed cost per hectare of crop is charged for water. This violates economic principles since the price is not related to the quantity of water applied or used, and there is little incentive to conserve water. This type of system is often justified by the difficulty and expense of determining how much water is delivered to a farm.

2.4.2.1 Riparian water rights systems

Under riparian water rights systems, the owner of land bordering a stream or lake has the right to take water for use on the land. The right to use the water exists solely because of the relation of the land to the water and resides in the ownership of the land. The first riparian user acquires no priority over those who may use the stream at a later date; the rights of upstream and downstream users are viewed as being coequal (Hirshleifer et al., 1960).

Under riparian systems, the owners of lands bordering water bodies may have "reasonable use" of the waters, provided that the water is returned undiminished in quality or quantity (Howe et al., 1986). That is, the withdrawal must be reasonable with respect to the requirements of the other riparians. The determination of what constitutes reasonable use is left to the courts. A riparian right subject to the reasonable use doctrine has no guaranty to a definite quantity of water. Under the riparian system, the transfer of water rights between competing uses by a market system is severely hampered (Hirshleifer et al., 1960).

Riparian rights are most appropriate for humid regions. Where water is truly scarce and/or where water quality problems are important, the riparian doctrine simply doesn't work (Howe, 1996).

2.4.2.2 Appropriative water rights systems

The doctrine of appropriation gives no preference to the use of water by riparian landowners. Appropriative (or prior) systems tend to exist in areas of water scarcity where users are located away from water bodies. Scarcity means that each succeeding appropriation results in fewer or less valuable resources available for other users. Scarce resources come to be appropriated in their natural state according to the principles of priority of right and beneficial use (Cuzan, 1983).

The earliest water right on a given watercourse has preference over later users, "first in time means first in right." Once the appropriation is granted, it becomes senior to subsequent appropriations. In times of shortage senior or older rights have precedence over junior or newer rights. That is, senior rights have first call on available water. Appropriative rights are a right to use, not a right to own, and the beneficial use of the water is required. Beneficial use has been described as use of water in a useful industry or to supply a well-recognized want (Tregarthen, 1983). In many cases, the owner of an appropriation may lose the right as a result of failure to put it to beneficial use.

The two rules of appropriative water rights, priority and beneficial use, result in the separation of rights to water from the rights to land. Persons can mobilize capital to build water supply works and transport water to wherever it is most productively used (Cuzan, 1983). Appropriative rights may be a system in which rights are clearly defined and transferable subject to the stipulation of "no injury" (Tregarthen, 1983). It is the severability of appropriative rights that causes them to be transferable.

Often under appropriative rights systems water is owned by the public and appropriators are granted the right to use the water but ownership of the resource remains with the state (Cuzan, 1983). Often this state expropriation of water rights leads to a system of controls which makes it difficult for water to be transferred privately through sales. These systems can generate pressure for monumental water schemes by governmental agencies which subsidize low-value water uses.

Economic efficiency requires that the marginal value of water used be equal in each use, net of transport costs, and assuming that marginal values include both private and social benefits and costs (Gardner, 1983; Howe et al., 1986). Assuming that water is homogeneous, i.e., no quality variations, water prices should vary among users only by the cost of moving it from one user to another (Hirshleifer, et al., 1960).

Two main types appropriative rights systems are common: priority rights and proportional rights systems.

Priority rights

Priority rights operate on the doctrine of "first in time, first in right." If the flow in a river is sufficient to provide only x% of the water appropriated, then a call for water from the senior water rights holders can shut off diversions to the lowest (100 - x)% priority rights holders. Senior water rights holders have less risk than junior rights holders, but senior rights holders may place a lower value on the last unit of water than junior rights holders. In this case, a trade should occur, the senior rights holder selling water rights to junior rights holders, thus reducing the risk to the junior appropriators (Tregarthen, 1983). Priority rights allow different degrees of water supply reliability to be purchased, but the heterogeneous nature of the rights makes it difficult to organize markets.

Proportional rights

A proportional rights system shares available water among users according to a set of percentages determined by the number of rights owned, e.g., if a user owns 10 rights out of 100, then the owner is entitled to 10% of available water. Proportional rights systems require the purchase of more shares to reach any given level of assurance of water supply. The homogeneity of proportional rights makes it much easier to create markets than under the priority system.

2.4.3. Water Markets

2.4.3.1 Right to divert or consume

The right to transfer water may not be the amount of water appropriated to the use, but the "duty of water" at the point of use (Tregarthen, 1983). This concept limits the transfer of water rights, based on the consumptive use by the seller and the prospective consumptive use of the buyer. In many situations it is desirable to protect downstream users from a loss of water due to an upstream water rights trade. When rights are transferred, the use of the water may change, and with it the amount of water that is consumed. The result may be a change in the amount of water available to downstream users. Often it is desirable to limit transfers so that the amount of water consumed is not changed.

Consider the following example adapted from Gisser and Johnson (1983) where there are three users along a river and flow into the system is 1000 units per time period (see Fig. 2.4.3.1.1). User 1 diverts S_1 = 1000 units and has a return flow coefficient (R_1) of 0.5, that is, User 1 consumes C_1 = 500 units of water. Downstream, User 2 diverts S_2 = 500 units and has a return flow coefficient (R_2) of 0.5, and User 3 diverts S_3 = 250 units and has a return flow coefficient (R_3) of 0.5. The total diversion is 1,750 units of water, a greater amount than the initial streamflow.

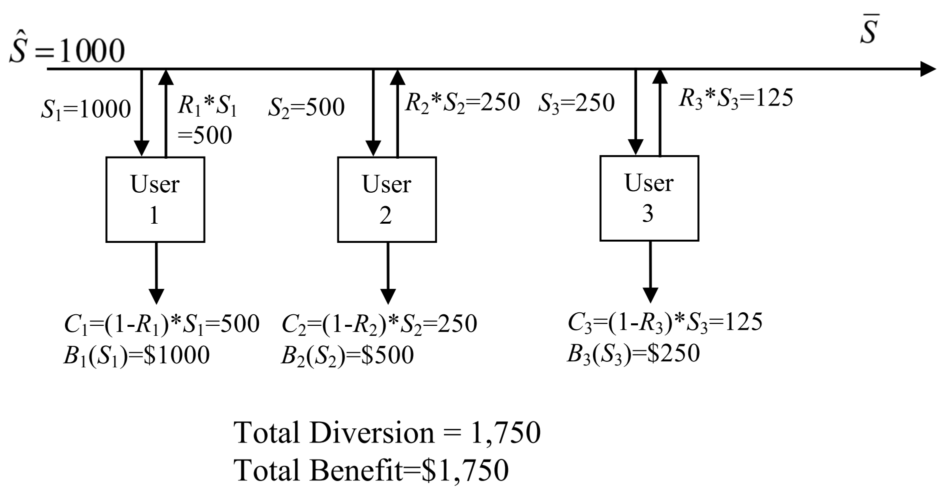


Figure 2.29. River system for water allocation.

Now consider that User 1 decides to sell his/her entire diverted amount to another user outside the basin for $1.1 per unit. The net result is that User 1 is no better off than before, but Users 2 and 3 have been left without any water to divert and there is an overall net loss for the basin of $650

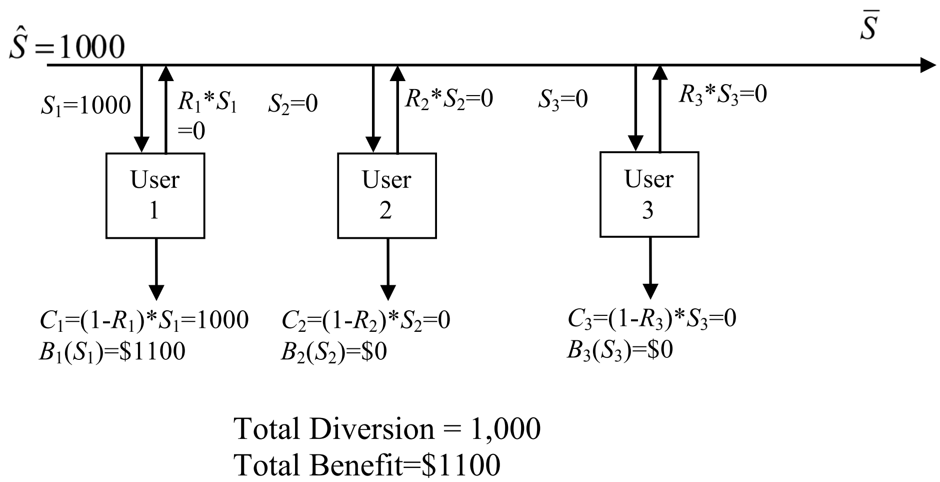


Figure 2.30. River system for water allocation without consideration of consumptive use.

An appropriator (user) may own a right to divert a given quantity of water but the user can only transfer this right according to the amount of water consumed. Determining the consumptive use of water and the amount of water that returns to the river or canal can be difficult and costly. Consider again the above example, but now User 1 decides to only sell the amount of his/her previous consumptive use (500 units) for a price of $1.1 per unit. In this case the downstream users continue to receive their water and may divert as before.

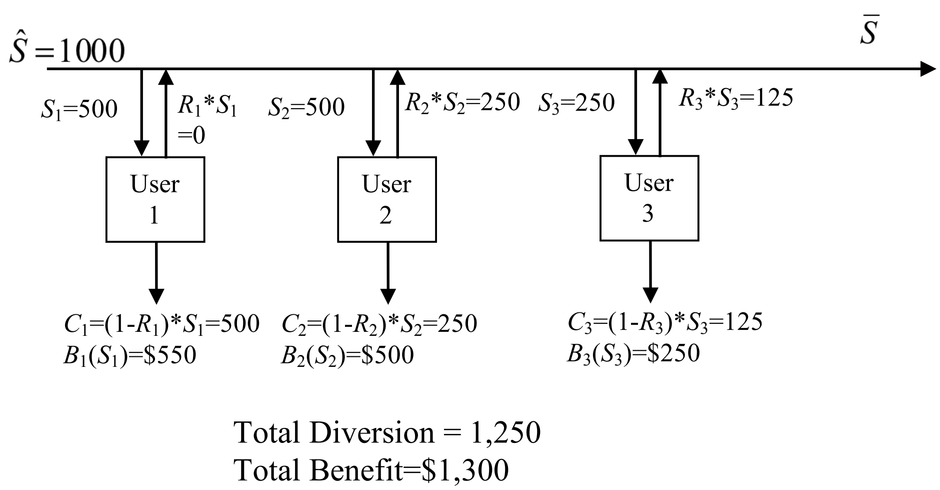


Figure 2.31. River system for water allocation with consideration of consumptive use.

Several authors have suggested that it is better to define water rights according to the consumptive use system rather than the diversion rights system. Using the consumptive rights system the ownership of the right is clear, transfers do not require litigation, the incentive to conserve water exists, and this conservation leads to water that can be sold to downstream users

(Tregarthen, 1983). However, others have suggested that the diversionary rights system is preferable (Rosegrant et al. 1995).

The definition of the tradable portion of a water right depends on the method of handling return flows. In California, the tradable portion is limited to consumptive use (consumptive right) with protection of third-party rights to return flows. This method increases transaction costs because of the difficulty in measuring consumptive use and return flows. Consumptive use is defined to be the actual evapotranspiration of crops plus any water lost to deep percolation. Thus, the water available to trade includes water that would have been consumptively used and water that would be irretrievably lost to beneficial use. In Chile and Mexico, rights are proportional to streamflow (diversionary right) and rights to return flow are retained by the water authority. Return flows are made available to users at no charge, but no rights are assigned to these flows. Changes in return flows due to water rights trades are not actionable. This method has been demonstrated to reduce transaction costs. So the tradable water is the full diversion right which is proportional to stream flow.

What is the most appropriate method in developing countries? The transaction costs of enforcing consumptive rights increase but they protect third-parties against adverse impacts from water trades. If the lost benefits from not trading exceed the costs of adverse impact from lost return flows, then diversionary rights system is preferred. In general, the diversionary rights system will be preferred in developing countries so as to prevent high transaction costs, thus preventing the development of markets.

2.4.3.2 Tradable water rights markets

Tradable water rights are rights to use water that can be transferred all or in part, separately from the transfer of land (Rosegrant et al., 1996). Tradable water rights may be permanent, long-term, or even short-term. Tradable water rights markets may be capable of allocating water more effectively than other more restrictive and centrally controlled systems. Markets can operate most efficiently when the commodity being allocated is homogeneous (Howe et al., 1986). Heterogeneity of uses leads to difficulty in organizing a market, transmitting information to users, and matching sellers and buyers. Rights to water resources already exist in most countries (a) by custom, or (b) by law and regulation. Establishing tradable rights is a matter of reforming existing systems.

Characteristics of markets

Several desirable characteristics for water allocation mechanisms (regional, river basin or irrigation district level) have been described by Howe (Howe et al., 1986; Howe, 1996):

- **Flexibility over time**

  Water can be shifted from use to use and place to place as climate, demographics, and economic conditions change over time. Short-term (responding to climatic factors) and long-term (responding to demographic and economic factors) flexibility is necessary. It is important to note that not all water must be subject to reallocation, only a tradable margin must exist within each water-using area that is subject to low cost reallocation and this volume can be a relatively small part of the regional supply (Howe et al., 1986). Flexibility allows equating the marginal values in the water's various uses (Howe et al., 1986; Gisser and Johnson, 1983).

- **Security of tenure for established users**

  Water users must be assured of continued use or they will not invest in and maintain the water resource system. This encourages long-term investments that generate positive net benefits. In a market system, no one can be forced to sell.

- **Real opportunity costs of water**

  Valuation of water at its opportunity cost, the maximum value of outputs that could have been produced had inputs not been used to produce the item in question (Field, 1994), provides incentives for users to shift from inefficient water uses and methods to more highly valued, less water-intensive uses and methods. Opportunity cost pricing can be implemented through the establishment of tradable water rights and development of markets in these rights (Rosegrant et al., 1996).

  Water is often a scarce input to production and it is frequently priced well below its value in use (Gardner, 1983). Historically, the price of water, at most, has reflected the costs of its capture and distribution. The control of low priced water can provide access to enormous profits in many cases. A perpetual contract for the supply of water at a fixed price may fail to reflect changing opportunity costs involved in continued use. Water is one input to agricultural production, other inputs include land, capital, energy, chemicals, and labor. If production is to be profitable, all inputs used must be valued at least at their opportunity costs. A price for water established in a market and the ability to sell water (transfer or trade water rights) recognizes the real opportunity costs of the water in the use being considered. This prevents the acceptance of water uses that are less valuable than alternative uses.

  It is desirable to maximize the scope of a water rights market so that transactions take place over as wide a geographical area and among as wide a variety of users as possible, subject to transaction cost limitations.

- **Differentiated risk-bearing**

  While old methods are familiar even if they are outmoded, new methods may increase uncertainty, even while they promise advantages. Predictability of the outcome of the transfer process is necessary to ensure that long-term investments that generate positive net benefits are encouraged.

- **Fairness to participants**

Water users should not impose uncompensated costs (externalities) on other parties. Externalities occur whenever withdrawal, consumption, or quality changes by one user affect other water users. Parties giving up water should be compensated and those injured by changes in allocations should be compensated. Market transactions should guarantee fairness since no person will sell if they will not be made better off.

- **Protection of public values**

  Some values may be of little concern to individual water users and they may not be adequately reflected in the market exchange and these must be protected by social oversight, e.g., water quality and instream flows (Howe, 1996). Protection of public values will ensure that allocations will achieve the highest aggregate benefit level.

Benefits of tradable water rights markets

The benefits from establishing tradable water rights markets include (Rosegrant and Binswanger, 1994; Rosegrant et al., 1996):

- **Empowerment of water users** by requiring their consent to any reallocation of water and compensation for any water transferred;

- **Security of tenure** of water rights to the water users, which encourages investment in system efficiency improvements;

- Induces users to consider the **full opportunity cost** of water, including its value in alternative uses, providing incentives to efficiently use water and gain additional income through the sale of saved water;

- Provide incentives for users to **take account of external costs** imposed by their water use, reducing resource and environmental degradation;

- **Formalizes existing rights** to water; and

- **Provides maximum flexibility** in responding to changes in crop prices and water values.

Constraints of tradable water rights markets

Constraints to establishing tradable water rights markets leading to high transaction costs include (Rosegrant and Binswanger, 1994; Rosegrant et al., 1996):

- The unique physical, technological and economic **characteristics of water resources** systems pose problems;

- The **variable nature of water flow** makes achieving necessary certainty; and

- **Return flows** from water use can generate environmental degradation. Multiple reuse of water creates the likelihood of significant externalities imposed on third parties.

Policy considerations of tradable water rights markets

Policy considerations in developing tradable water rights markets include (Rosegrant et al., 1996):

- Definition of a method of initial allocation of water rights. This can be based on, among other things, historic water use (Chile and Mexico), fully appropriated existing rights (California);
- Type of rights, prior or proportional appropriative rights: Prior rights (California), Proportional (Chile and Mexico);
- Consumptive use or diversionary treatment of return flows;
- Indirect economic effects;
- Environmental protection;
- Water user associations;
- Infrastructure;
- Public and private institutions; and
- Regulations: Excessive regulation leads to high transaction costs, inadequate regulation leads to third-party costs or environmental degradation

2.4.3.3 Modeling tradable water rights markets

There are two fundamental strategies for dealing with water scarcity in river basins, supply management and demand management; the former involves activities to locate, develop, and exploit new sources of water, and the latter addresses the incentives and mechanisms that promote water conservation and efficient use of water (Rosegrant et al., 2000). Markets in tradable water rights can reduce information costs; increase farmer acceptance and participation; empower water users; and provide security and incentives for investments and for internalizing the external costs of water uses. Market allocation can provide flexibility in response to water demands, permitting the selling and purchasing of water across sectors, across districts, and across time by opening opportunities for exchange where they are needed. The outcomes of the exchange process reflect the water scarcity condition in the area with water flowing to the uses where its marginal value is highest (Rosegrant and Binswanger 1994; Rosegrant 1997). Markets also provide the foundation for water leasing and option contracts, which can quickly mitigate acute, short-term urban water shortages while maintaining the agricultural production base (Michelsen and Young 1993).

Water trading in a basin is constrained by the hydrologic balance in the river basin network; water may be traded taking account of physical and technical constraints of the various users, reflecting their relative profitability in trading prices; water trades reflect water scarcity in the basin that is influenced by both basin inflows and the water use plans of the users (Rosegrant et al., 2000).

The price that a water user would be willing to pay to acquire additional water must be determined for each user. This can be achieved by determining a shadow price – water withdrawal relationship can determined for each user. For this, a model must be run with varying water rights for each user as inputs and shadow prices or marginal values as output derived from the water balance equations (each user has a water balance equation in the model). If necessary, these shadow prices can be averaged over all uses for each user to obtain one shadow price for each water supply level for user. The Figure below shows the result of this for the problem of Exercise 1 below.

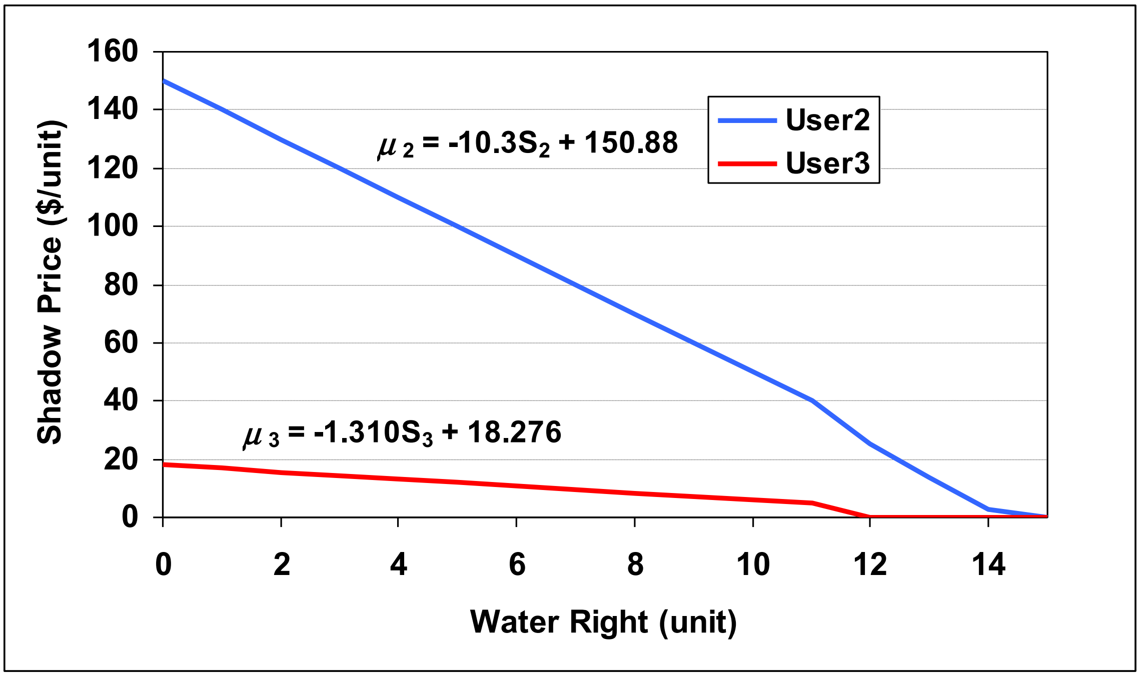


Figure 2.32. Shadow price for water users.

In the model of water trading, the objective is to maximize the combined benefits of all the users

$$\text{Maximize } B = \sum_i B_i = \sum_i (a_i S_i - b_i S_i^2 + \sum_j x_{i,j} \cdot wtp_j - \sum_j x_{j,i} \cdot wtp_i) \qquad (2.4.\ 1)$$

where

B	Total benefit to all water users;
B_i	Benefit to User i, a quadratic benefit function is assumed here with coefficients a_i and b_i;
S_i	Water withdrawal by user i;
wtp_i	Water trading price for user i;
$x_{i,j}$	Water sold by user i to user j;

User i has access to water from water right or purchase

$$S_i \leq wright_i - \sum_j x_{i,j} + \sum_j x_{j,i} \tag{2.4.2}$$

where *wright* is water allocated to a user under prescribed water rights. Each user has shadow price for water which is a linear function of the amount of water demanded

$$wtp_i = am_i + bm_i S_i \tag{2.4.3}$$

No user is allowed to sell more water than their water right

$$\sum_j x_{i,j} \leq wright_i \tag{2.4.4}$$

Trades are unidirectional, that is, if a user buys water from another user, then they can not sell water to the same user

$$x_{i,j} * x_{j,i} = 0 \tag{2.4.5}$$

2.5 Exercises

1. You are working with the manager of an irrigation facility who is interested in installing a more efficient pumping system. The proposed system costs $15,000 and you project that it will reduce the annual utility costs by $2,000. After five years, you expect to upgrade the system for $4,000. This upgrade is expected to further reduce utility costs by $1,000 annually. The annual effective interest rate is 7% and the life of the system, after upgrade is 50 years. What is the Present Value of the investment in the system?

2. You have a small excavation firm and wish to purchase a small backhoe. Based on your research, you need to have $54,000 to purchase one used. If your cost of capital is 0.50%/month and you want to recover your capital (on a Present Worth basis) in 20 months, how much profit must this backhoe generate each month.

3. (after North, 1985, Exercise 5.8) A flood control district can construct a number of alternative control works to alleviate the flood pattern in that area. These alternatives include dam A, dam B, and a levee system C. The levee system can be built alone or in combination with dam A or B. Both dams can not be built together but either one can function alone. The lofe of each dam is 80 years and the life of the levee system is 60 years. The cost of capital is 6 percent. Information on total investment, operation and maintenance costs, and average annual flood damage is given below. What form of flood control would be the most economical?

Table. Flood Control Project data

Project	Total Investment (million $)	Annual Operation and Maintainence (thous. $)	Average Annual Flood Damages (million $)
Dam A	6.2	93	1.10
Dam B	5.3	89	1.40
Levee C	6.7	110	0.80
Do nothing	0	0	2.15

4. (after Mays & Chung, 1992, Exercise 2.2.2) Four alternative projects can be used for developing a water supply for a community for the next 40 years. Use the incremental benefit-cost method to compare and select an alternative. Use a 6% interest rate.

Years	Project A	Project B	Project C	Project D
Construction cost ($)				
0	40,000,000	30,000,000	20,000,000	10,000,000
10				10,000,000
20		10,000,000	20,000,000	10,000,000
30				10,000,000
Operation and Maintenance Cost ($)				
0-10	100,000	110,000	120,000	120,000
10-20	120,000	110,000	130,000	120,000
20-30	140,000	120,000	140,000	130,000
30-40	160,000	140,000	150,000	130,000

5. (after James and Lee, 1971, Problem 2.6) The three alternatives described below are available for supplying a community water supply for the next 50 years when all economic lives as well as the period of analysis terminates.

Construction cost	***Project A***	***Project B***	***Project C***
Year 0	$20,000,000	$10,000,000	$15,000,000
Year 20	0	10,000,000	12,000,000
Year 35	0	10,000,000	0
O&M cost			
Year 1-20	70,000	40,000	60,000
Year 21-35	80,000	70,000	80,000
Year 36-50	90,000	90,000	90,000

Using $2,500,000 in benefits each year for each project, and a 4.5% discount rate where applicable, compare the projects using:

a. The present-worth method

The present-worth method selects the project with the largest present worth of the discounted sum of benefits minus the costs over its life

$$P_w = \sum_{t=1}^{T} \frac{B_t - C_t}{(1+i)^t}$$

where C_t is the cost and B_t is the benefit in year t, T is the period of analysis, and i is the discount rate.

b. The rate-of-return method

The rate-of-return is the discount rate at which the present worth as defined above equals zero as found by trial and error.

c. The benefit-cost ratio method

The benefit-cost ratio PW_b/PW_c is the present worth of the benefits PW_b divided by the present worth of the costs PW_c. Annual values can be used with out altering the ratio.

$$PW_b = \sum_{t=1}^{T} \frac{B_t}{(1+i)^t} \quad \text{and} \quad PW_c = \sum_{t=1}^{T} \frac{C_t}{(1+i)^t}$$

d. The annual-cost method

The annual-cost method converts all benefits and costs into equivalent uniform annual figures.

6. (after Thuesen et al., 1977, Problem 10.20) The federal government is planning a hydroelectric project for a river basin. In additionto the production of electric power, this project will provide flood control, irrigation, and recreation benefits. The estimated benefits and costs that are expected to be derived from the three alternatives under consideration are listed below:

Construction cost	***Project A***	***Project B***	***Project C***
Initial cost	\$25,000,000	\$35,000,000	\$50,000,000
Annual benefits and costs			
Power sales	\$1,000,000	\$1,200,000	\$1,800,000
Flood control savings	250,000	350,000	500,000
Irrigation benefits	350,000	450,000	600,000
Recreation benefits	100,000	200,000	350,000
O&M costs	200,000	250,000	350,000

The interest rate is 5% and the life of each of the projects is estimated to be 50 years.

a. Using the incremental benefit-cost method, determine which project should be selected.

b. Calculate the benefit cost ratio for each alternative. Is the best alternative selected if the alternative with the maximum benefit cost ratio is chosen?

c. If the interest rate is 8%, what alternative would be chosen?

7. (After D. P. Loucks, Course Notes, Engineering Economics, Cornell University.) Three mutually exclusive water resources projects, *A, B,* and *C,* are under consideration. Each project has a fixed initial cost (FC_A, FC_B, and FC_C). Their (unequal) useful lives are L_A, L_B, and L_C, and during each year y of those lives they generate annual benefits of B_{Ay}, B_{By}, and B_{Cy}, and costs of C_{Ay}, C_{By}, and C_{Cy}. Assume that an appropriate interest rate, r, has already been determined for this analysis. Show how you would calculate each project's equivalent end-of-year annual benefits and costs, and based on these, their benefit-cost ratios.

8. Find the optimal levels of two goods purchased by a consumer with a utility function

$$u(x_1, x_2) = x_1^{1.5} x_2$$

and a budget constraint

$$3x_1 + 4x_2 = 100$$

9. Agricultural production is described by the quadratic equation

$$q_i = a_i + b_i x_i + c_i x_i^2$$

where q_i is the yield in hectares of crop i, a_i, b_i, and c_i are parameters of the production functionfor crop i, and x_i is the amount of water (m^3) applied to crop i. The unit cost of water is w; the unit market price of each crop is p_i. Develop a model, based on the theory of the firm, to determine the optimal water allocation to each crop. What is the demand function for water assuming the production function is a concave function of x_i (i.e., $c_i < 0$).

10. (After Mays and Tung, 1992, Problem 2.4.1; 2.4.4; 2.4.5)

(1) For the production process in the following Table, determine and plot the total, average, and marginal product curves for nitrogen fertilizer given that water is fixed at x_1 = 7 inches/acre.

(2) Determine and plot the cost curves for the production process. Assume that water is fixed at 7 inches per acre. Use input prices of $2.50 per pound of nitrogen fertilizer and $10 per acre-inch of water. Plot average fixed cost (AFC), average variable cost (AVC), average total cost (ATC), and marginal cost (MC) on one plot.

(3) Determine the profit for various levels of output for the production process. Assume that

corn sells for \$1.49 per bushel, and input prices are the same as in Part (2) and that irrigation water is fixed at 7 inches per acre. How much corn should be produced? How much nitrogen fertilizer is used in this production. What it the value of total product that maximizes profit?.

Table. Production Schedule of the Relationship Between Irrigation Water, Fertilizer & Yield (bushels per acre) of corn.

					x_1*									
x_2**	0	1	2	3	4	5	6	**7**	8	9	10	11	12	13
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	1.8	5.0	9.0	13.2	17.0	19.8	**21.0**	20.0	16.2	9.0	0.0	0.0	0.0
20	0.0	5.0	12.8	22.2	32.0	41.0	48.0	**51.8**	51.2	45.0	32.0	11.0	0.0	0.0
30	0.0	9.0	22.2	37.8	54.0	69.0	81.0	**88.2**	88.0	81.0	63.0	33.0	0.0	0.0
40	0.0	13.2	32.0	54.0	76.8	98.0	115.2	**126.0**	128.0	118.8	96.0	57.2	0.0	0.0
50	0.0	17.0	41.0	69.0	98.0	125.0	147.0	**161.0**	164.0	153.0	125.0	77.0	6.0	0.0
60	0.0	19.8	48.0	81.0	115.2	147.0	172.8	**189.0**	192.0	178.2	144.0	85.8	0.0	0.0
70	0.0	21.0	51.8	88.2	126.0	161.0	189.0	**205.8**	207.2	189.0	147.0	77.0	0.0	0.0
80	0.0	20.0	51.2	88.0	128.0	164.0	192.0	**207.2**	204.8	180.0	128.0	44.0	0.0	0.0
90	0.0	16.2	45.0	81.0	118.8	153.0	178.2	**189.0**	180.0	145.8	81.0	0.0	0.0	0.0
100	0.0	9.0	32.0	63.0	96.0	125.0	144.0	**147.0**	128.0	81.0	0.0	0.0	0.0	0.0
110	0.0	0.0	11.0	33.0	57.2	77.0	85.8	**77.0**	44.0	0.0	0.0	0.0	0.0	0.0
120	0.0	0.0	0.0	0.0	0.0	6.0	0.0	**0.0**	0.0	0.0	0.0	0.0	0.0	0.0
130	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**0.0**	0.0	0.0	0.0	0.0	0.0	0.0

* x_1 = inches of water per acre
** x_2 = pounds of nitrogen fertilizer per acre

11. (After Loucks et al., Problem 4.2). Assume that a farmer's demand for water q is a linear function of the price p, i.e., $q(p) = a-bp$, where a, $b > 0$.

(1) Calculate the farmer's willingness-to-pay for a quantity of water q.

(2) If the cost of delivering a quantity of water q is $C(q) = cq$, $c > 0$, how much water should a public agency supply to maximize willingness-to-pay minus cost?

(3) If the local water district is owned and operated by a private firm whose objective is to maximize profit, how much water would they supply and how much would they earn?

(4) The farmer's consumer surplus is their willingness-to-pay minus what they must pay for the resource. Compare the farmer's consumer surplus in the two cases.

(5) Does the farmer lose more than the private firm gains by moving from the social optimum to the point that maximizes the firm's profit?

(6) Illustrate these relationships with a graph showing the demand curve and the unit cost c of water. Label the firm's profits and the farmer's consumer surplus?

12. Given the production functions for wheat and corn in the Maipo basin of Chile shown in the following Table, determine a Production Possibility curve if x = 15,000 m3/ha of water is available. If the prices are p_1 = \$230/mt (wheat) and p_2 = \$160/mt (corn), find the point of optimal production. What are the resulting outputs and amounts of water used for wheat and corn. What is the resulting total revenue?

Table. Production functions for wheat and corn.

Production functions

for wheat and corn

Water x (m3/ha)	Wheat y_1 (mt/ha)	Water x (m3/ha)	Corn y_2 (mt/ha)
0	0.00	0	0.00
536	0.00	883	0.00
1071	0.00	1765	0.00
1607	0.00	2648	0.00
2142	0.22	3530	1.21
3213	2.08	5295	3.87
4284	3.27	7060	5.60
5355	4.10	8825	6.83
6426	4.70	10590	7.74
7497	5.14	12355	8.43
8568	5.46	14120	8.96
9639	5.70	15885	9.37
10710	5.87	17650	9.68
11781	5.98	19415	9.91
12852	6.04	21180	10.08
13923	6.07	22945	10.19
14994	6.06	24710	10.26
16065	6.02	26475	10.28
17136	5.96	28240	10.27
18207	5.88	30005	10.23
19278	5.77	31770	10.17

13. (After Willis and Finney, 2000, Example Problem 4-8) Water quality pollution is an example of an externality, i.e., a "harmful effect on one or more individuals that emenates from the action of a different person or firm" (Samuelson, 1973). Downstream water users, for example, will have to treat water prior to use because of upstream firms don't consider the externalities in their decision making process.

A regional water management authority proposes to reduce point source wastewater discharges by imposing an effluent tax on each unit of waste discharged. These effluent charges are a method of internalizing the externalities created by the discharges. Investigate how the effluent charge affects the optimum production levels for a firm discharging waste. Assume:

1. the tax, τ, is expressed in \$ per unit of output of the firm, q, and
2. the pollution generated by the firm, P, is a linear function of the production level, $P=\delta q$.

a. Find the first order optimality conditions for the firm production.

b. What is the effect of the tax on marginal revenue and marginal cost?

b.1 Plot a diagram of Price versus Quantity showing marginal revenue with and without the tax (make whatever assumptions you need to to develop the graph).

b.2 What is the difference between the point of intersection of the marginal cost curve and the marginal revenue curve with and without the tax?

b.3 What is the difference in the production level with and without the tax?

c. What is the effect on the level of pollution produced with and without the tax?

14. A regional water management authority must pay an effluent tax on waste discharged. Consider

q	= firm's output (units)
$p(q) = \omega - \eta q$	= price of the firm's output ($/unit)
$C(q) = \gamma q^2$	= firm's cost function ($)
$D(q) = \alpha + \beta q$	= firm's pollution level (kg)
$t = p_c$	= pollution tax ($/kg of waste produced)

$\gamma, \alpha, \beta, p_c, \omega, and\ \eta$ are just constants, but you and I do not know their values.

Part A: Suppose the firm is maximizing profits and price is given as p = *constant* (that is, $p(q) = \omega = p, and\ \eta = 0$).

(1) If the tax is zero (t = 0), what level of output (q) will the firm select?

(2) If the tax is not zero ($t = p_c$), what level of output (q) will the firm select?

Part B: Suppose that the firm is socially conscious and wishes to maximize the benefits of the consumers and the inverse demand function (marginal willingness-to-pay) for its product is given by $p(q) = \omega - \eta q$. Be sure to continue considering the cost function in your model.

(1) If the tax is zero (t=0), what level of output (q) will the firm select?

(2) If the tax is not zero ($t = p_c$), what level of output (q) will the firm select?

15. The inverse demand curve for a depletable, nonrecyclable resource in year t is

$$p_t(q_t) = a - bq_t \quad t = 1, 2, \ldots, T$$

where q_t is the amount of resource demanded in year t, p_t is the price of the resource in year t, and $a,b > 0$ are constants. The marginal cost of extracting a unit of resource in any year is a constant (= c). The total amount of resource available (= Q) is less than the amount needed to satisfy demand over a T year planning horizon.

(a) Determine the first order optimality conditions for resource extraction if the objective of the extraction is to balance the current and future uses of the resource by maximizing the present value of net benefits derived from the use of the resource over the T years. Assume that the discount rate is i.

(b) Calculate numerical values for the optimal extraction rates ($= q_t$, $t = 1,2, ..., T$) if
$T = 2$, $a = 8$, $b = 0.4$, $c = 2$, $Q = 20$, $i = 0.10$.

16. Part A. Assume that stream flow ($\hat{S}$) is 26 million m^3 per unit of time and there is an interstate agreement ($\bar{S}$) calling for 14.5 million m^3 Also, initially there are two users on the river diverting S_2 and S_3 million m^3. The benefits to each user are:

User 2: $B_2(S_2) = a_2 S_2 + b_2 S_2^2 = 150 S_2 - 5 S_2^2$
User 3: $B_3(S_3) = a_3 S_3 + b_3 S_3^2 = 18 S_3 - 0.6 S_3^2$

In addition, both users have the same return flow coefficient, $R_2 = R_3 = 0.5$. Write an optimization model to determine an efficient allocation that results in maximizing the value of water use in the basin and respects the interstate compact. What are the water allocations and the benefits to each user?

Part B. Now, assume that an additional user wants to divert S_1 million m^3 of water from the river. User 1's benefit function is identical to User 2's benefit, i.e.,

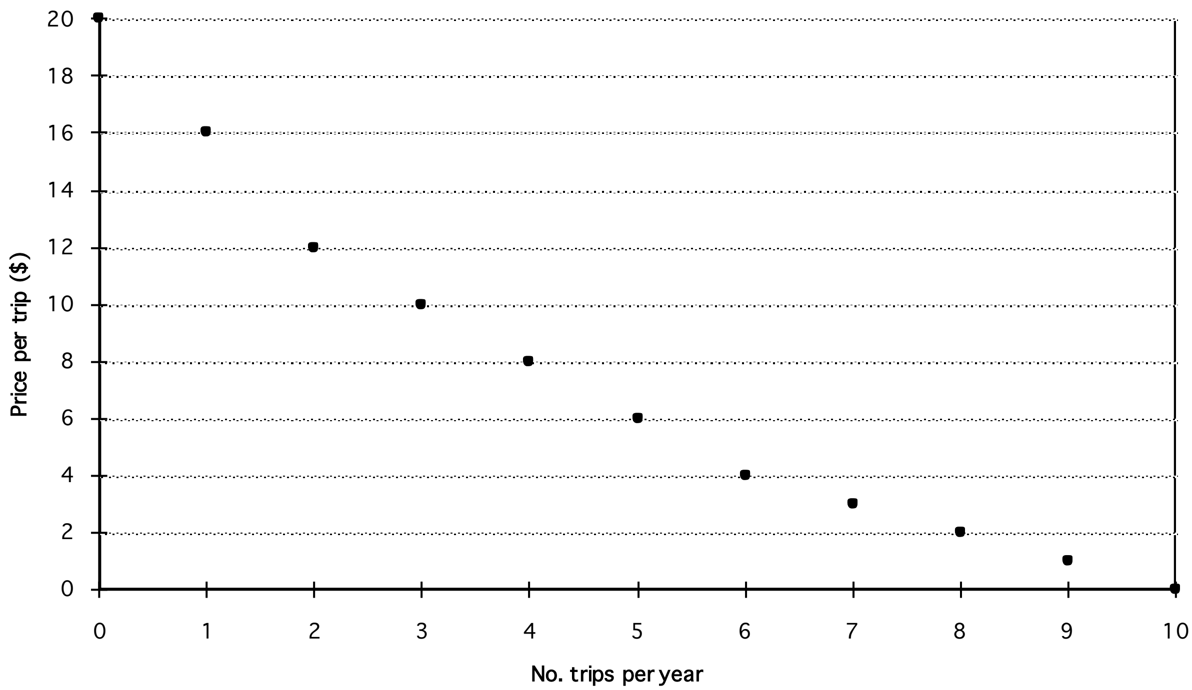
User 1: $B_1(S_1) = a_1 S_1 + b_1 S_1^2 = 150 S_1 - 5 S_1^2$

User 1 also has a return coefficient of $R_1 = 0.5$. Modify your optimization model to determine a new efficient allocation that results in maximizing the value of water use for all three users in the basin and respects the interstate compact. What are the water allocations and the benefits to each user?

Part C. Assume that the solution to Part A represents the initial water rights of Users 2 and 3, using the results obtained in Part B, what is the minimum payment that User 1 should pay to Users 2 and 3 in order to divert water from the river? What are the resulting net benefits to each of the three users?

Part D. Assume that the system described in Part B is modeled as a water market where users 1, 2, and 3 have water right allocations, 0.0, 4.0, and 7.5, respectively. Develop a model which will determine the optimal use of water by each user, assuming that the users are free to trade their water rights according to the model structure described in the text above.

17. Lewis and Clark Lake is a large reservoir in South Dakota created on the Missouri River by the Gavins Point Dam. It is located in an area where there are few natural bodies of water, and it has become very popular as a recreational area. Suppose that 10,000 families are potential users of the lake for recreational purposes and that each family's demand curve for recreational trips to the lake is as follows:



(1) If an ordinance were passed which limited each family to no more than 5 trips per year to the lake, what is the loss (in money terms) to each family?

(2) If an ordinance were passed which allowed a family to use the lake for recreational purposes only if it purchased a permit for $75 a year, would it be worthwhile for each family to buy a permit, if it could not use the lake without the permit (and it could use the lake as much as it liked with one)?

(3) How much is the consumer's surplus from each family's utilization of the lake if there is a charge of $8 for each trip to the lake?

18. (after North, 1985) **Part A.** Consider the following set of data regarding the production of lettuce

Acre-inch water	Production of lettuce	Price of lettuce/head
0	186	50
1	698	25
2	1185	20
3	1648	18
4	2085	17
5	2496	16
6	2883	15
7	3245	14
38	3582	13
9	3895	12
10	4184	10
11	4442	8
12	4679	6
13	4891	4

Determine and plot a schedule for:

(1) Physical production (total, average, and marginal)

(2) Cost functions (total, average, and marginal for both input and output)

(3) Revenue functions (total, average, and marginal for both a competitive market price of 15 cents per head of lettuce and for the industry demand schedule given in column 3 below)

Part B. What are the firm equilibrium positions for both the competitive and monopolistic price structures, demonstrating total revenues, total costs, and net revenues.

Part C. What are the optimum levels of production and resource use under both pricing structures when water costs \$40/acre-inch and fixed costs are \$160.

19. (After Linsley et al., 1079) The average annual damage from floods in a river basin is estimated to be \$400,000. Estimates have been made for several alternate proposals for flood mitigation works: channel improvements (25 yr life), two mutually exclusive dams (A and B, 100 yr lives), and various combinations of these. The table below shows the first cost, estimated annual damages, and the annual OM&R disbursements for each alternative, and the sum of the annual damages and annual costs.

Project	1st Cost $	Annual Damages, $	Annual OM&R Costs, $
Do nothing	0	400,000	0
I. Channel Improvement	500,000	250,000	100,000
II. Dam A	3,000,000	190,000	60,000
III. Dam B	4,000,000	125,000	80,000
IV. Dam A with Channel Improvements	3,500,000	100,000	160,000
V. Dam B with Channel Improvements	4,500,000	60,000	180,000

a. Compare the projects using an interest rate of 6 percent.
b. Compare the conclusions of the economic analysis with the interest rate of 3 percent used in Sec. 13-9 of Linsley and Franzini, Water Resources Engineering, with the 6 percent rates used in your solution to Part (1). What generalizations can you make regarding the influence of the interest rate on such studies?
c. Comment on the statement, "I would select the plan with the highest benefit cost ratio of all the plans."
d. Comment on the statement, "I would select the plan with the highest possible benefits for which benefits are greater than costs."

20. Assume that stream flow ($\hat{S}$) is 26 million m^3 per unit of time and there is an interstate agreement ($\bar{S}$) calling for 14.5 million m^3 Also, initially there are two users on the river diverting S_2 and S_3 million m^3. The benefits to each user are:

User 2: $B_2(S_2) = a_2S_2 + b_2S_2^2 = 150S_2 - 5S_2^2$
User 3: $B_3(S_3) = a_3S_3 + b_3S_3^2 = 18S_3 - 0.6S_3^2$

In addition, both users have the same return flow coefficient, $R_2 = R_3 = 0.5$. Write an optimization model to determine an efficient allocation that results in maximizing the value of water use in the basin and respects the interstate compact. What are the water allocations and the benefits to each user?

21. Now, assume that an additional user wants to divert S_1 million m^3 of water from the river. User 1's benefit function is identical to User 2's benefit, i.e.,

User 1: $B_1(S_1) = a_1S_1 + b_1S_1^2 = 150S_1 - 5S_1^2$

User 1 also has a return coefficient of $R_1 = 0.5$. Modify your optimization model to determine a new efficient allocation that results in maximizing the value of water use for all three users in the basin and respects the interstate compact. What are the water allocations and the benefits to each user?

22. Assume that the solution to Part (1) represents the initial water rights of Users 2 and 3, using the results obtained in Part (2), what is the minimum payment that User 1 should pay to Users 2 and 3 in order to divert water from the river? What are the resulting net benefits to each of the three users?

23. Assume that the system described in Problem (2) is modeled as a water market where users 1, 2, and 3 have water right allocations, 0.0, 4.0, and 7.5, respectively. Develop a model which will determine the optimal use of water by each user, assuming that the users are free to trade their water rights according to the model structure described in Section 3.2.5 above.

24. A flood control district can construct a number of alternative control works to alleviate a flood problem in the area. These alternatives include dam A, dam B, and a levee system C. The levee system can be built alone or in combinations with dam A or dam B. Both dams can not be built together but either one can function alone. The life of each dam is 80 years and the life of the levee system is 60 years. The cost of capital (discount rate) is 6 percent. Information total investment costs, operating and maintenance costs, and average annual flood damages is given below. What form of flood control is preferred?

Project	Life (years)	Total investment ($1000)	Annual operation and maintenance ($1000)	Average annual flood damages ($1000)
No control at all	-	0	0	$2,150
Dam A	80	$6,200	$93	$1,100
Dam B	80	$5,300	$89	$1,400
Levees C	60	$6,700	$110	$750

25. (after Grant et al., 1976, p. 138) Just before a creek has its outlet into a salt water bay, it goes through an urban area. Because there have been occasional floods that have caused damage to property in this area, a flood control project has been proposed. Estimates have been made for two alternative designs, one involving channel improvements (CI) and the other involving a dam and reservoir (D & R). Economic analysis is to be based on an estimated 50-year project life assuming zero terminal salvage value and using a discount rate of 6%.

The expected value of the annual cost due to flood damages is $480,000 with a continuation of the present condition of no flood control (NFC). Alternative CI will reduce this figure to $105,000; alternative D&R will reduce it to $55,000.

Alternative CI has an estimated first cost of $2,900,000 and estimated annual maintenance costs of $35,000. Alternative D&R has an estimated first cost of $5,300,000, and estimated annual operation and maintenance costs of $40,000. Alternative D&R also has two types of adverse consequences related to the conservation of natural resources. These are to be treated in the economic analysis as disbenefits (negative benefits). The dam will cause damage to anadromous fisheries; this is priced at $28,000 per year. The reservoir will cause a loss of land for agricultural purposes including grazing and crop raising; this is priced at $10,000 per year.

Using Benefit – Cost analysis, which alternative is preferred?