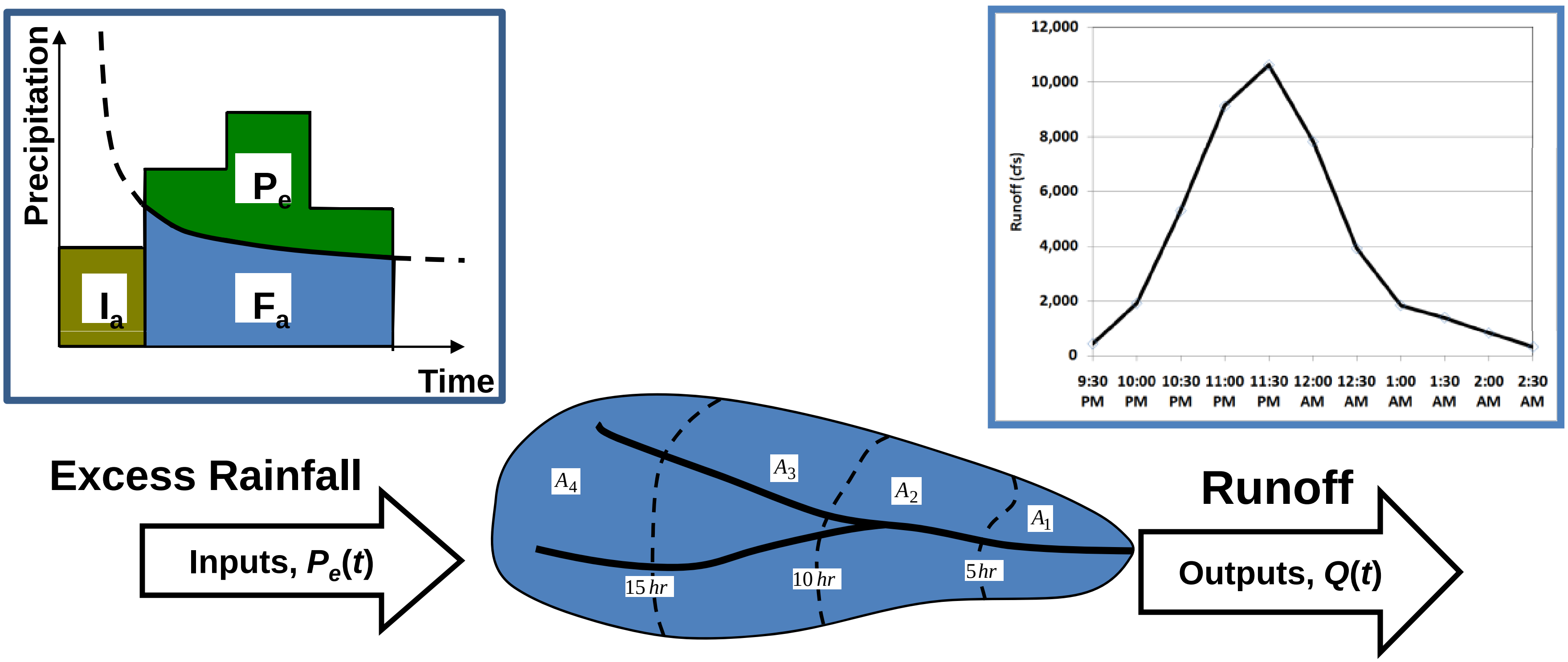
CE 374 K – Hydrology

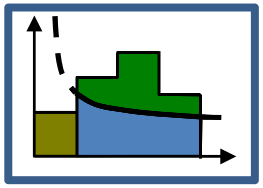
Unit Hydrograph

Daene C. McKinney

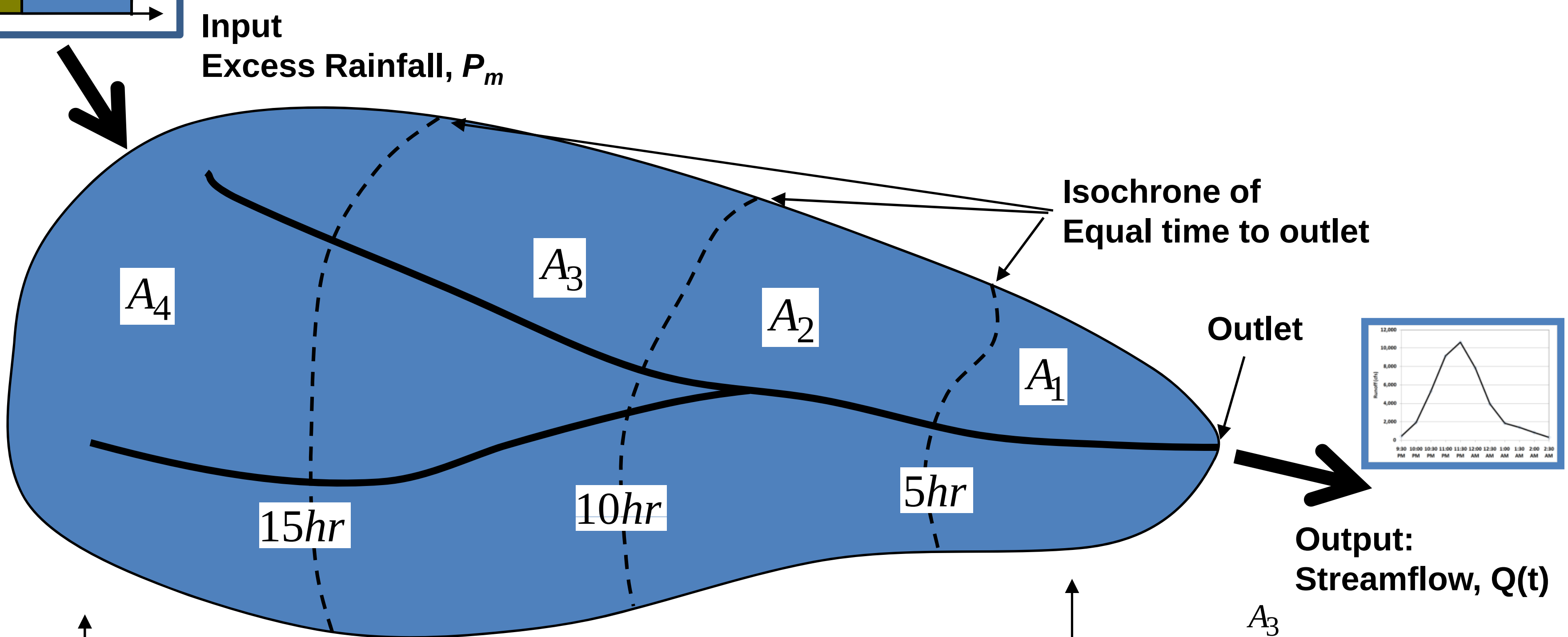
System Response

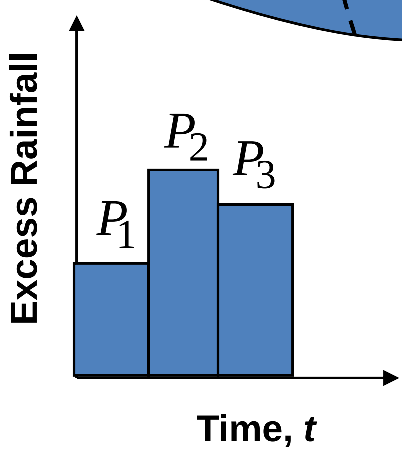


System = Watershed



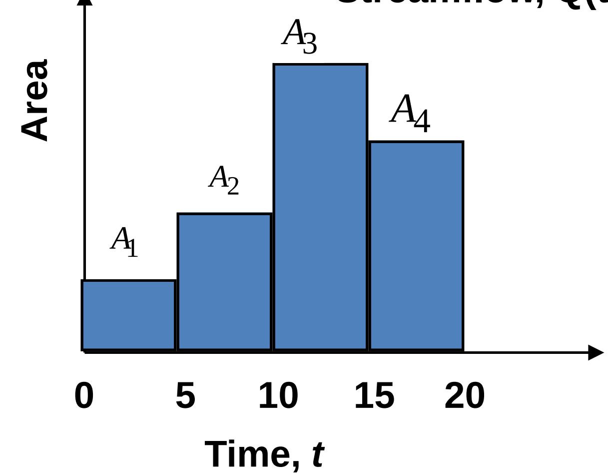
Watershed as a System





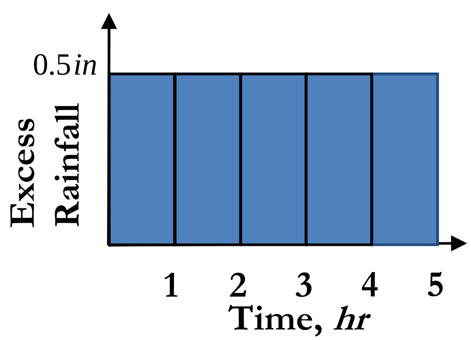
Suppose Transfer Function is catchment area

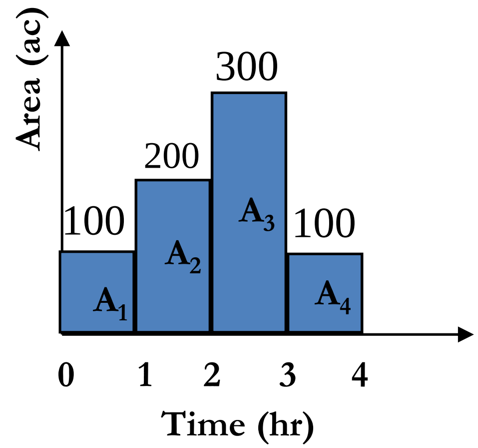
Runoff from storm increment R_1 falling on A_2 reaches outlet at same time as runoff from R_2 on A_1, etc.



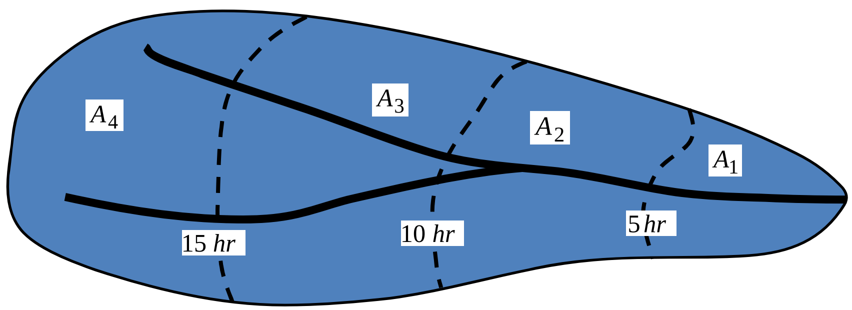
Example

- Rainfall of 0.5 in/hr falls uniformly for 5 hrs.
- Derive a hydrograph for flow at outlet
- Response function is approximately the area $U_i \approx A_i$

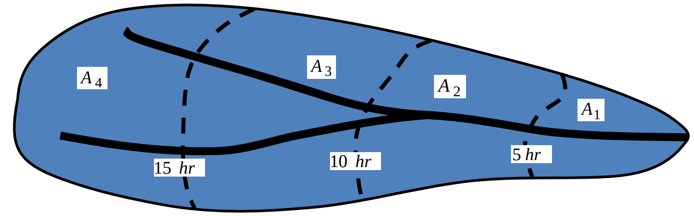




$$
\begin{aligned}
Q_1 &= P_1U_1 \\
Q_2 &= P_2U_1 + P_1U_2 \\
Q_3 &= P_3U_1 + P_2U_2 + P_1U_3 \\
Q_4 &= P_4U_1 + P_3U_2 + P_2U_3 + P_1U_4 \\
Q_5 &= P_5U_1 + P_4U_2 + P_3U_3 + P_2U_4 \\
Q_6 &= P_5U_2 + P_4U_3 + P_3U_4 \\
Q_7 &= P_5U_3 + P_4U_4 \\
Q_8 &= P_5U_4
\end{aligned}
$$

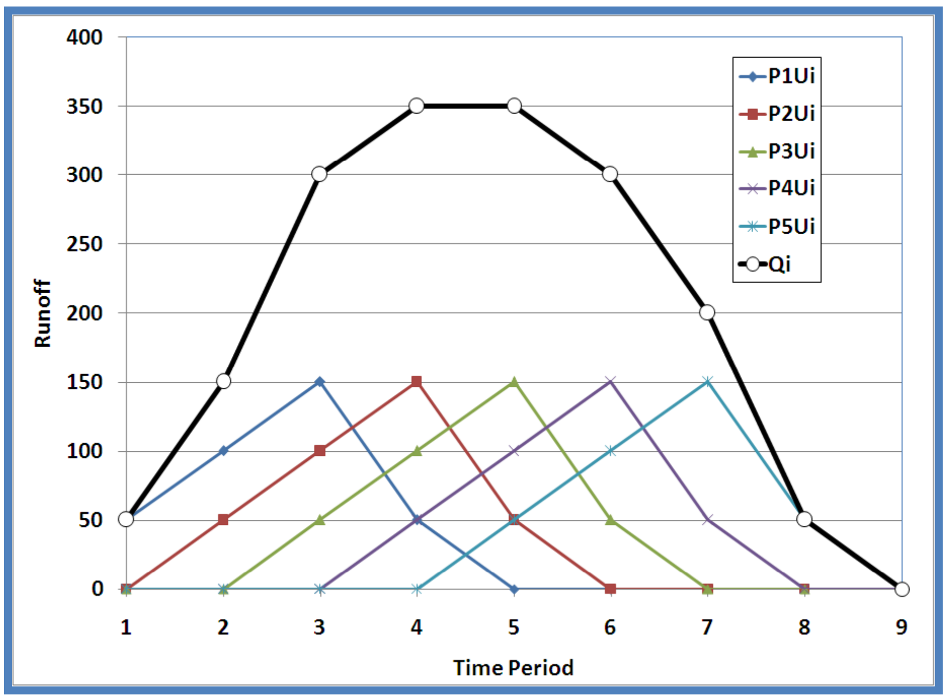


Example (Cont.)



$$
\begin{aligned}
Q_1 &= 0.5*100 && && && &= 50\\
Q_2 &= 0.5*100 + && 0.5*200 && && &= 150\\
Q_3 &= 0.5*100 + && 0.5*200 + && 0.5*300 && &= 300\\
Q_4 &= 0.5*100 + && 0.5*200 + && 0.5*300 + && 0.5*100 &= 350\\
Q_5 &= 0.5*100 + && 0.5*200 + && 0.5*300 + && 0.5*100 &= 350\\
Q_6 &= && 0.5*200 + && 0.5*300 + && 0.5*100 &= 300\\
Q_7 &= && && 0.5*300 + && 0.5*100 &= 200\\
Q_8 &= && && && 0.5*100 &= 50
\end{aligned}
$$

i	Pi	Ui	P1Ui	P2Ui	P3Ui	P4Ui	P5Ui	Qi
1	0.5	100	50	0	0	0	0	50
2	0.5	200	100	50	0	0	0	150
3	0.5	300	150	100	50	0	0	300
4	0.5	100	50	150	100	50	0	350
5	0.5		0	50	150	100	50	350
6			0	0	50	150	100	300
7			0	0	0	50	150	200
8			0	0	0	0	50	50
9			0	0	0	0	0	0



But, runoff is NOT just a function of area!

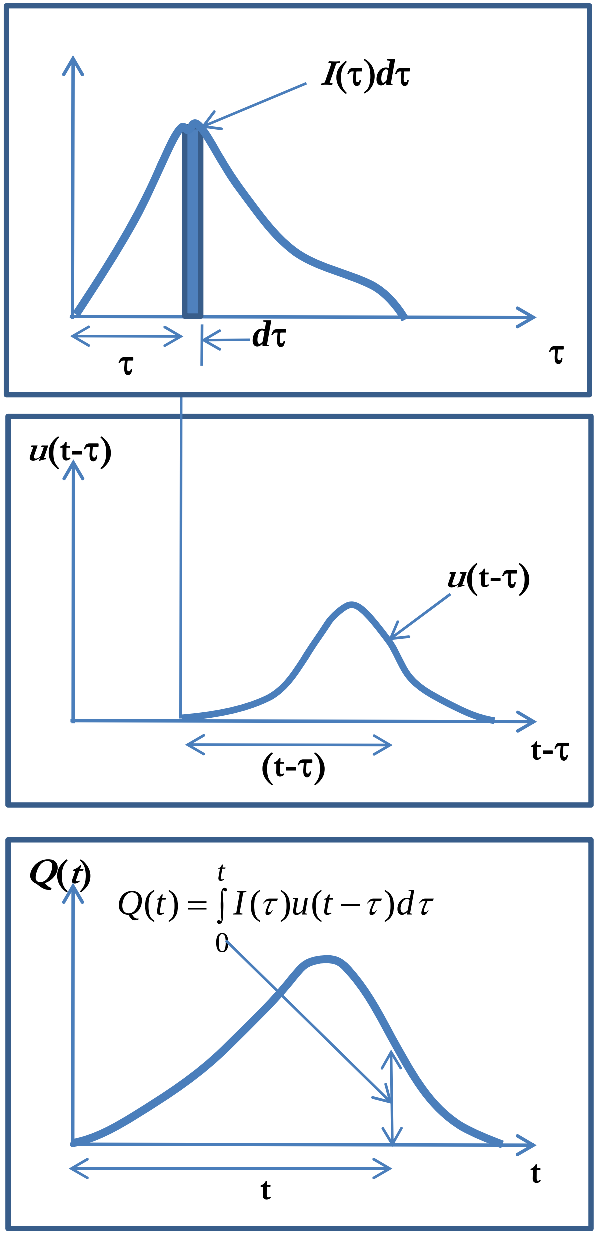
It depends on storage in the watershed, among other things.

Generalize to <u>Unit Hydrograph</u>

Unit Response Function: Continuous

- Suppose at time τ, a unit of excess rainfall is distributed uniformly over the catchment instantaneously (in time *d*τ). What's the response?
- Call this the Unit Response Function.
- To account for the time lag between the rainfall and the runoff, use the notation *u*(t-τ).
- Runoff, *Q*(*t*), at any time, *t*, for any excess rainfall, *I*(t) can be computed with a convolution integral:

$$Q(t)=\int_0^t I(\tau)u(t-\tau)d\tau$$



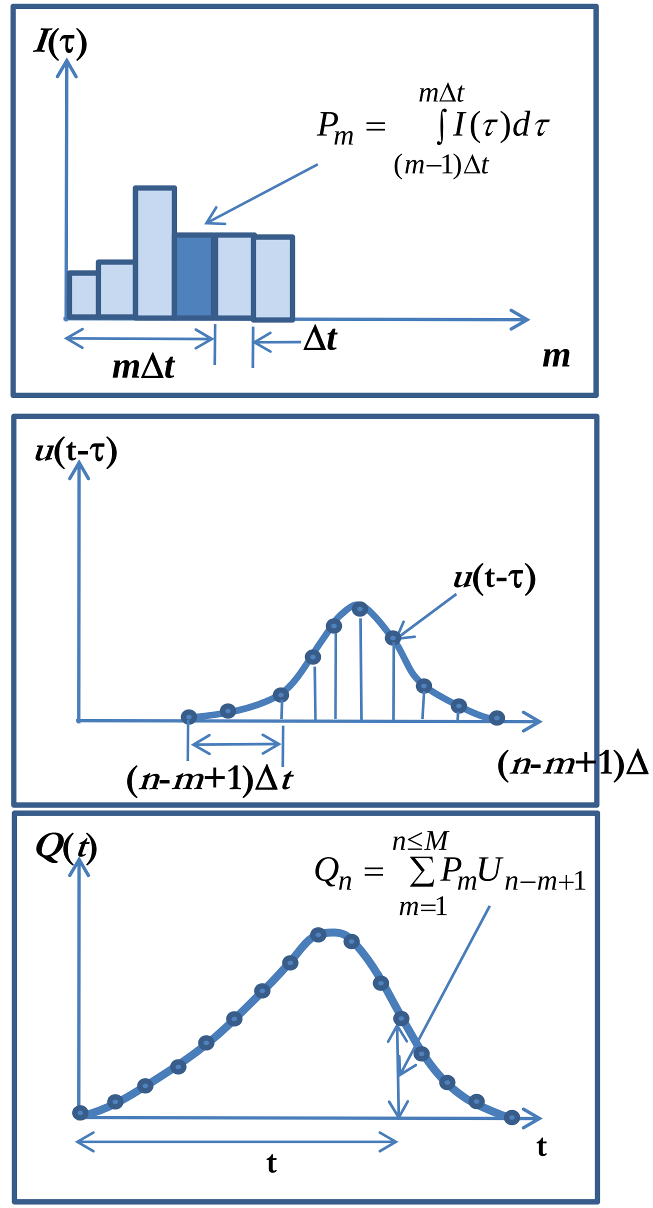
Unit Response Function: Discrete

- Numerically, runoff can be computed with a discrete form of the convolution integral and a discrete unit response function (called a **unit hydrograph**), using averal excess rainfall for time intervals Δt:

$$Q_n = \sum_{m=1}^{n<M} P_m U_{n-m+1}$$

  - Q_n = runoff hydrograph ordinate at time nΔt;
  - P_m = excess rainfall depth in time interval m;
  - U_{n-m+1} = Δt-unit response function ordinate n-m+1;
  - M = number of periods of excess precipitation.

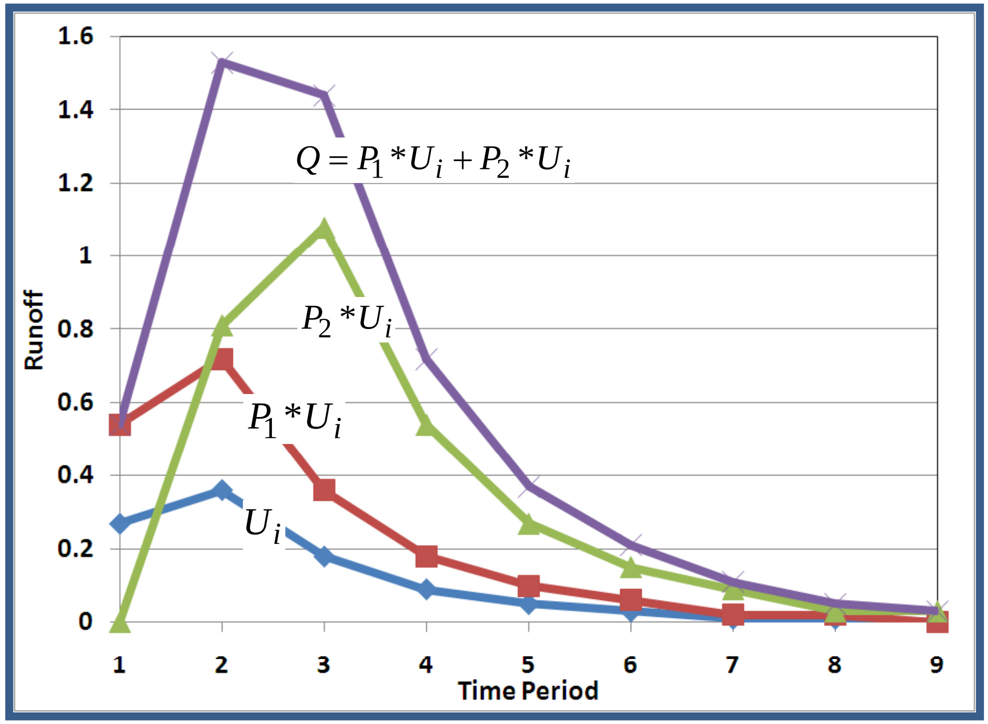
"…the basin outflow resulting from one unit of direct runoff generated uniformly over the drainage area at a uniform rainfall rate during a specified period of rainfall duration."
Sherman 1932



Example (7.2.2)

- A system has the following unit pulse response function (U_{n-m+1}).
- Find the output if the input is 2 units in the first time interval and 3 units in the second time interval

i	Pi	Ui	P1Ui	P2Ui	Qi
1	2	0.27	0.54	0.00	0.54
2	3	0.36	0.72	0.81	1.53
3		0.18	0.36	1.08	1.44
4		0.09	0.18	0.54	0.72
5		0.05	0.10	0.27	0.37
6		0.03	0.06	0.15	0.21
7		0.01	0.02	0.09	0.11
8		0.01	0.02	0.03	0.05
9			0.00	0.03	0.03
		1.00	2.00	3.00	5.00



Goal: Model process by which rainfall is transformed into streamflow