CE 374 K – Hydrology

Evaporation

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Evaporation

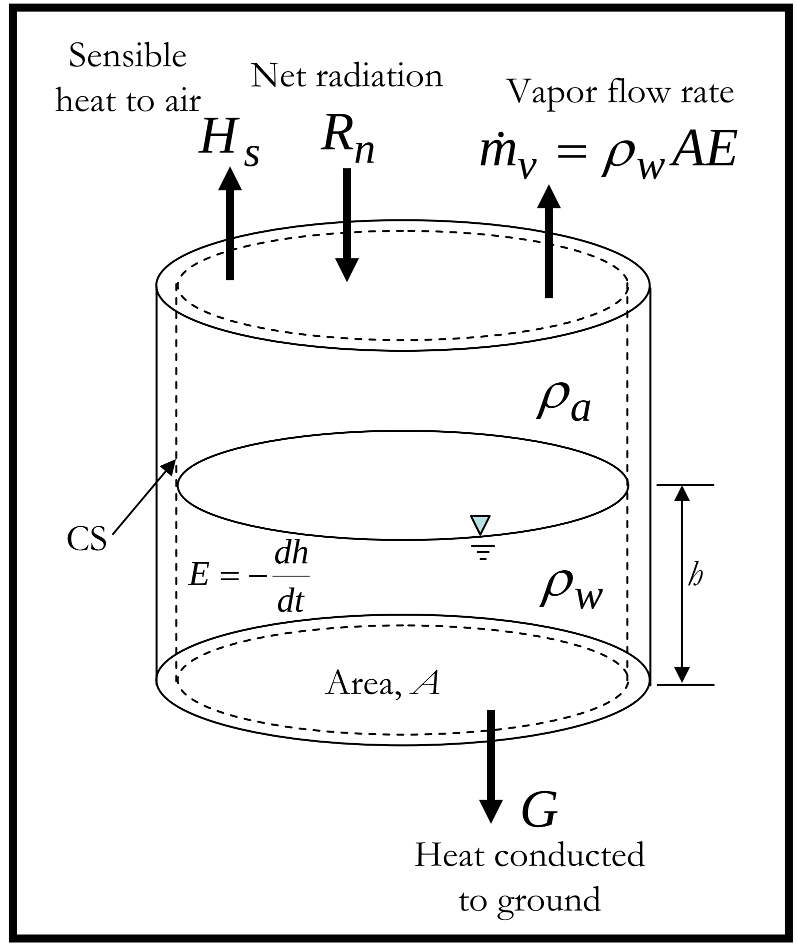
- **Terminology**
  - **Evaporation**: liquid water passes directly to the vapor phase
  - **Transpiration**: liquid water passes from liquid to vapor through plant metabolism
  - **Sublimation**: water passes directly from the solid phase to the vapor phase

Factors Influencing Evaporation

- Energy supply for vaporization (latent heat)
  - Solar radiation
- Transport of vapor away from evaporative surface
  - Wind velocity over surface
  - Specific humidity gradient above surface
- Vegetated surfaces
  - Supply of moisture to the surface
  - Evapotranspiration (ET)
    - Potential Evapotranspiration (PET) – moisture supply is not limited

Evaporation from a Water Surface

- National Weather Service Class A type
- Installed on a wooden platform in a grassy location
- Filled with water to within 2.5 inches of the top
- Evaporation rate is measured by manual readings or with an analog output evaporation gauge

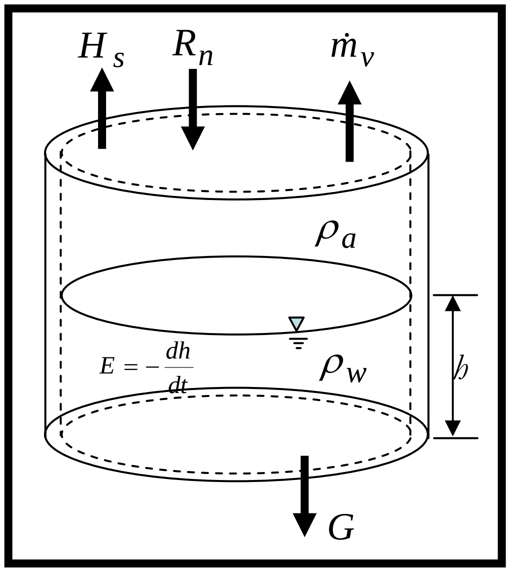


Methods of Estimating Evaporation

- Energy method
- Aerodynamic method
- Combined method

Energy Method

Continuity of Liquid Phase



$$-\dot{m}_v = \frac{d}{dt}\iiint_{CV}\rho_w d\forall + \iint_{CS}\rho_w \mathbf{V}\cdot d\mathbf{A}$$

$$= \rho_w A \frac{dh}{dt} \qquad E = -\frac{dh}{dt} \qquad = 0$$

No flow of liquid water through CS

$$\dot{m}_v = \rho_w A E$$

Energy Method (2)

Vapor Phase - Continuity

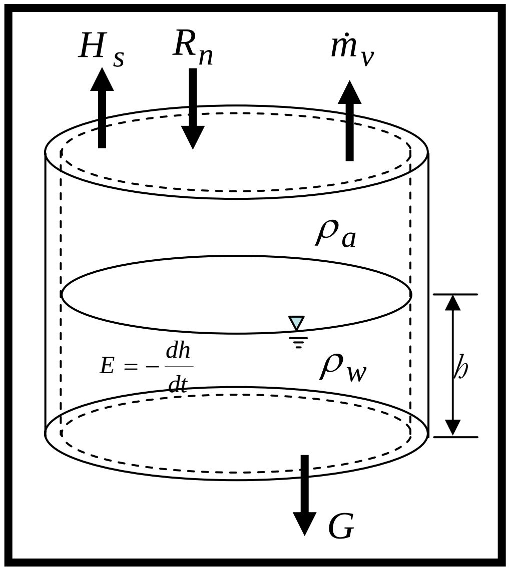
$$\dot{m}_v = \frac{d}{dt}\iiint_{CV} q_v \rho_a \, d\forall + \iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

$$= 0$$

$$\dot{m}_v = \iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

Steady flow of air over water

$$= \rho_w A E$$

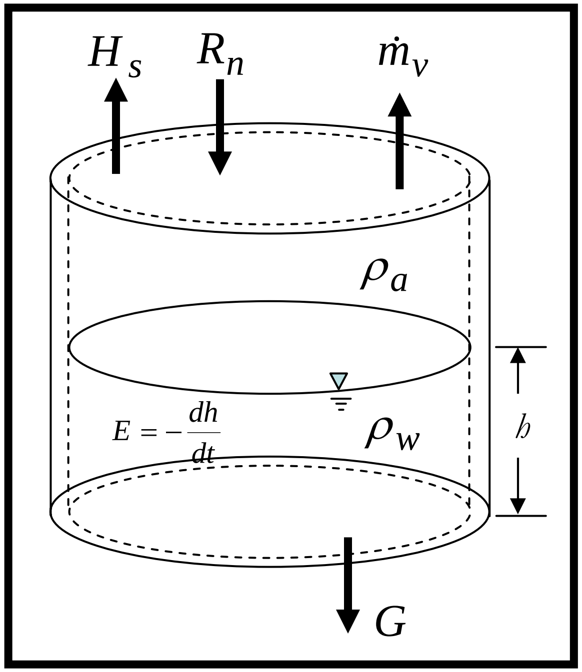


$$\rho_w A E = \iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

$$E = \frac{1}{\rho_w A}\iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

Energy Method (3)

Energy Eq.



$$\frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt}\iiint_{CV}(e_u + V^2/2 + gz)\rho d\forall + \iint_{CS}(e_u + V^2/2 + gz)\rho\vec{V}\cdot d\vec{A}$$

$$= 0$$

$$\approx 0; \qquad V = 0, h \approx const.$$

$$\frac{dH}{dt} = \frac{d}{dt}\iiint_{CV} e_u \rho_w d\forall$$

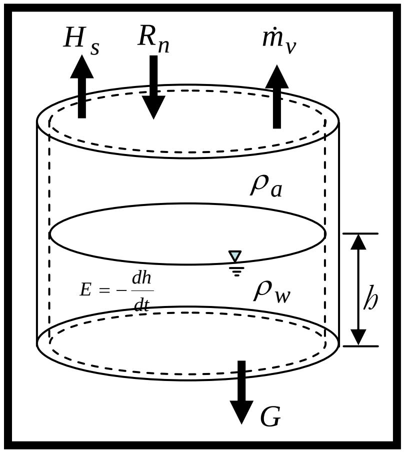
$$= R_n - H_s - G$$

$$\boxed{\frac{dH}{dt} = R_n - H_s - G}$$

Energy Method (4)

Energy Eq. for Water in CV

$$\frac{dH}{dt} = R_n - H_s - G$$



Assume:

1. Constant temp of water in CV
2. Change of heat is change in internal energy of evaporated water $\frac{dH}{dt} = l_v \dot{m}_v$

$$l_v \dot{m}_v = R_n - H_s - G$$

Recall: $\dot{m} = \rho_w A E$

$$E = \frac{1}{l_v \rho_w A}(R_n - H_s - G)$$

$$E_r = \frac{R_n}{l_v \rho_w}$$

Neglecting sensible and ground heat fluxes

Aerodynamic Method

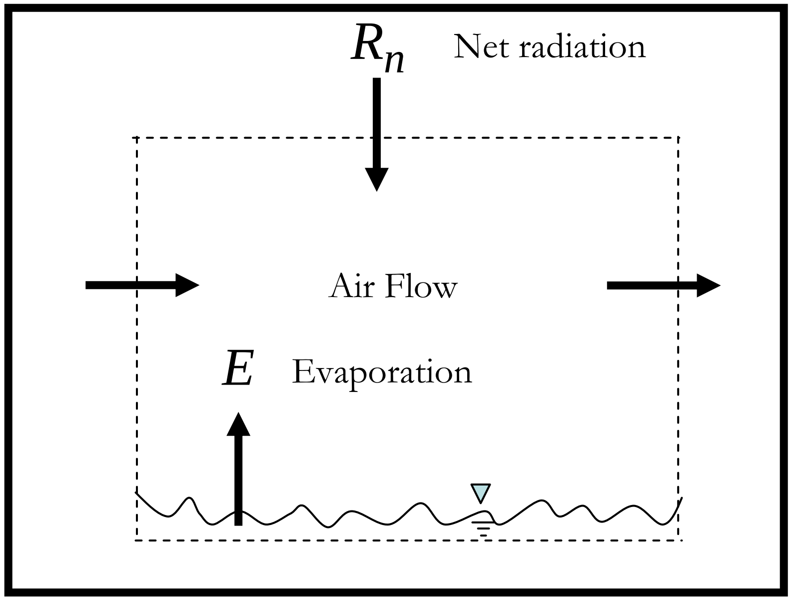
- Include transport of vapor away from water surface as function of:
  - Humidity gradient above surface
  - Wind speed across surface
- Upward vapor flux

$$\dot{m} = -\rho_a K_w \frac{dq_v}{dz} = -\rho_a K_w \frac{q_{v_2} - q_{v_1}}{z_2 - z_1}$$

- Upward momentum flux

$$\tau = \rho_a K_m \frac{du}{dz} = \rho_a K_m \frac{u_2 - u_1}{z_2 - z_1}$$

$$\frac{\dot{m}}{\tau} = \frac{K_w\left(q_{v_1} - q_{v_2}\right)}{K_m\left(u_2 - u_1\right)}$$



Aerodynamic Method (2)

$$\dot{m} = \tau \frac{K_w\left(q_{v_1} - q_{v_2}\right)}{K_m\left(u_2 - u_1\right)}$$

- Log-velocity profile

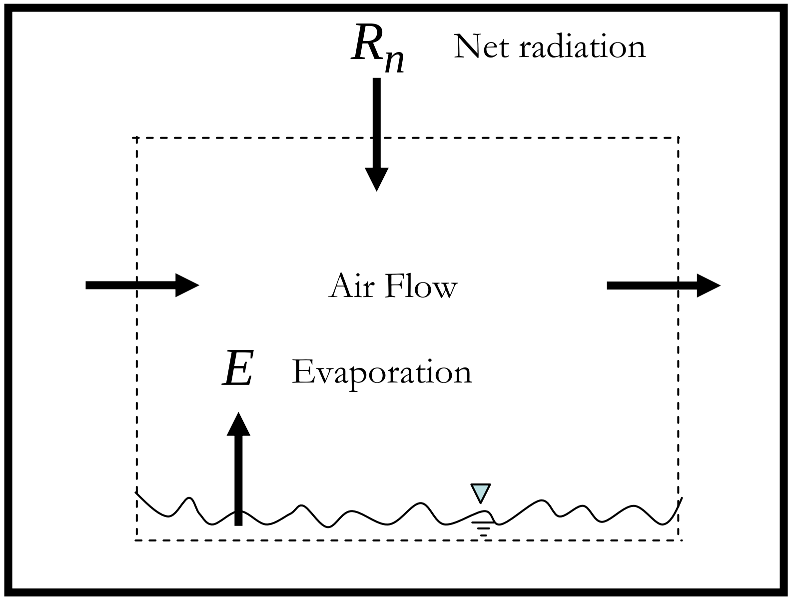
$$\frac{u}{\sqrt{\tau/\rho_a}} = \frac{1}{k}\ln\left(\frac{Z}{Z_o}\right)$$

- Momentum flux

$$\tau = \rho_a \left[\frac{k\left(u_2 - u_1\right)}{\ln\left(Z_2/Z_1\right)}\right]^2$$

$$\dot{m} = \frac{K_w k^2 \rho_a \left(q_{v_1} - q_{v_2}\right)\left(u_2 - u_1\right)}{K_m\left[\ln\left(Z_2/Z_1\right)\right]^2}$$

Thornthwaite-Holzman Equation



Too many variables!

Often only know q_v and u at 1 elevation

Aerodynamic Method (3)

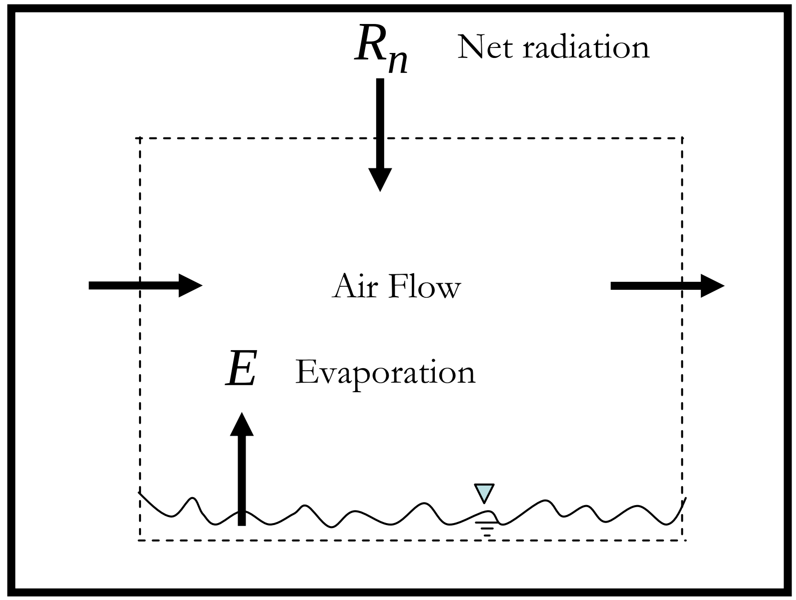
- Simplify

$$E_a = B(e_s - e)$$

$$B = \frac{0.622k^2 \rho_a u_2}{P\rho_w [\ln(Z_2/Z_o)]^2}$$

$$e = \text{vapor pressure}$$

$$e_s = \text{sat. vapor pressure}$$



Combined Method

- Evaporation could be calculated by
  - Aerodynamic method: when energy supply is not limiting
  - Energy method: when vapor transport is not limiting
- Normally, both are limiting, so use a combination method

Combined Method (Cont.)

- Combining
  - Energy balance
  - Aerodynamic Methods

$$E_r = \frac{R_n}{l_v \rho_w}$$

$$E_a = B(e_s - e)$$

- Combined Method

$$E = \frac{\Delta}{\Delta + \gamma} E_r + \frac{\gamma}{\Delta + \gamma} E_a$$

$$\gamma = \frac{C_p K_h p}{0.622 l_v K_w} \qquad \Delta = \frac{4098 e_s}{(237.3 + T)^2}$$

  - Well suited to small areas with detailed data
    - Net Radiation
    - Air Temperature
    - Humidity
    - Wind Speed
    - Air Pressure

$$E = 1.3 \frac{\Delta}{\Delta + \gamma} E_r$$

Priestly & Taylor

Example

- Use Combo Method to find Evaporation

z = 2 m

P = 101.3 kPa

u = 3 m/s

R_n = 200 W/m^2

T = 25 degC

R_h = 40%

$$l_v = 2.501x10^6 - 2370T$$

$$= (2500 - 2.36 * 25)x10^3 = 2441\,\mathrm{kJ/kg}$$

$$E_r = \frac{R_n}{l_v \rho_w} = \frac{200}{2441x10^3 * 997}$$

$$= 7.10\,\mathrm{mm/day}$$

Example (Cont.)

- Use Combo Method to find Evaporation

z = 2 m

P = 101.3 kPa

u = 3 m/s

R_n = 200 W/m^2

T = 25 degC

R_h = 40%

$$e_s = 3167\,\text{Pa}$$

$$e = R_h * e_s = 0.4 * 3167 = 1267\,\text{Pa}$$

$$B = \frac{0.622 k^2 \rho_a u_2}{P \rho_w \left[\ln\left(Z_2 / Z_o\right)\right]^2} = \frac{0.622 * 0.4^2 * 1.19 * 3}{101.3 * 997 \left[\ln\left(2 / 3x10^{-4}\right)\right]^2} = 4.54x10^{-11}\ \text{m/Pa}\cdot\text{s}$$

$$E_a = 4.54x10^{-11}\left(3167 - 1267\right) * (1000\,\text{mm} / 1\text{m}) * (86400\,\text{s} / 1\text{day})$$
$$= 7.45\,\text{mm/day}$$

Example (Cont.)

- Use Combo Method to find Evaporation

z = 2 m

P = 101.3 kPa

u = 3 m/s

R_n = 200 W/m^2

T = 25 degC

R_h = 40%

$$\gamma = \frac{C_p K_h p}{0.622 l_v K_w} = \frac{1005 * 101.3x10^3}{0.622 * 2441x10^3} = 67.1\,\text{Pa/degC}$$

$$\Delta = \frac{4098 e_s}{(237.3+T)^2} = \frac{4098 * 3167}{(237.3+25)^2} = 188.7\,\text{Pa/degC}$$

$$\frac{\Delta}{\Delta+\gamma} = 0.738 \qquad \frac{\gamma}{\Delta+\gamma} = 0.262$$

$$E = \frac{\Delta}{\Delta+\gamma} E_r + \frac{\gamma}{\Delta+\gamma} E_a = 0.738 * 7.10 + 0.262 * 7.45 = 7.2\,\text{mm/day}$$

Example

- Use Priestly-Taylor Method to find Evaporation rate for a water body

z = 2 m
P = 101.3 kPa
u = 3 m/s
R_n = 200 W/m2
T = 25 degC
R_h = 40%

$$E = 1.3 \frac{\Delta}{\Delta + \gamma} E_r$$ **Priestly & Taylor**

$$E_r = 7.10 \text{ mm/day}$$

$$\frac{\Delta}{\Delta + \gamma} = 0.738$$

$$E = 1.3 * 0.738 * 7.10 = 6.80 \text{ mm/day}$$

Evapotranspiration

- **Evapotranspiration**
  - Combination of evaporation from soil surface and transpiration from vegetation
  - Governing factors
    - Energy supply and vapor transport
    - Supply of moisture at evaporative surfaces
  - Reference crop
    - 8-15 cm of healthy growing green grass with abundant water
  - Combo Method works well if ***B*** is calibrated to local conditions

Potential Evapotranspiration

- Multiply reference crop ET by a Crop Coefficient and a Soil Coefficient

$ET = \text{Actual ET}$

$ET_r = \text{Reference Crop ET}$

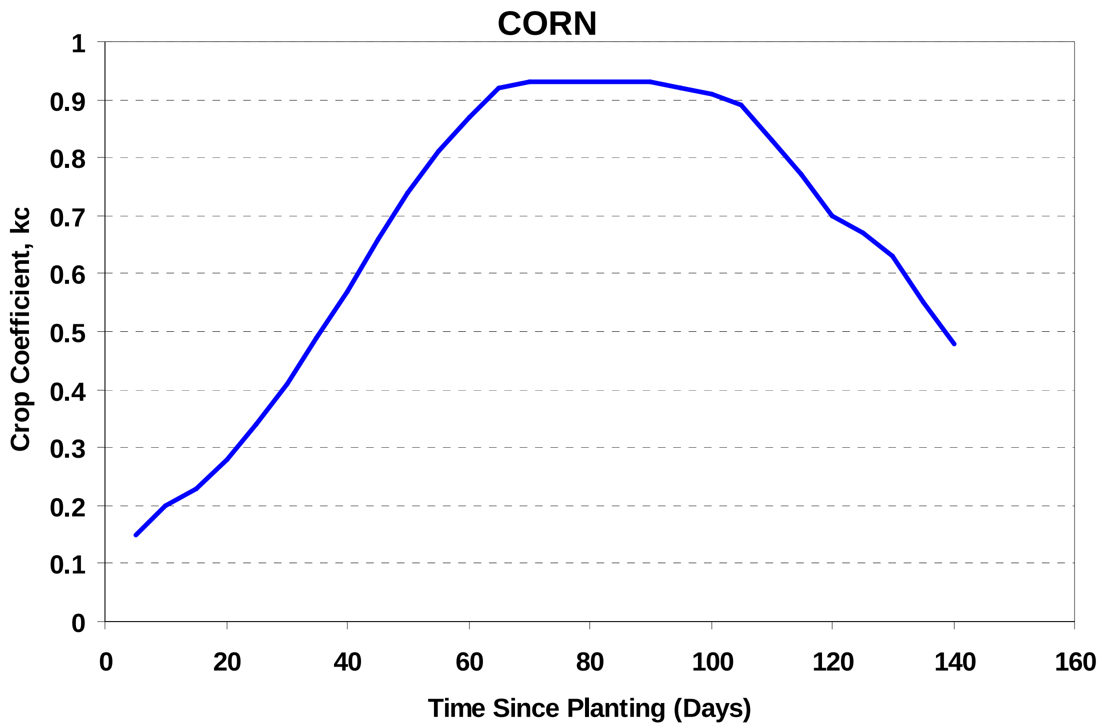
$k_c = \text{Crop Coefficient};$

$0.2 \le k_c \le 1.3$

$k_s = \text{Soil Coefficient};$

$0 \le k_s \le 1$

$$ET = k_s k_c ET_r$$



http://www.ext.colostate.edu/pubs/crops/04707.html

Combined Method

- Evaporation could be calculated by
  - Aerodynamic method: when energy supply is not limiting
  - Energy method: when vapor transport is not limiting
- Normally, both are limiting, so use a combination method
- Sensible heat flux is difficult to estimate
  - Assume it is proportional to the vapor heat flux $H_s = \beta(l_v \dot{m}_v)$
  - Where β = Bowen ratio
  - Energy balance equation (G=0)

$$E = \frac{1}{l_v \rho_w A}(R_n - H_s - G)$$

$$R_n = l_v \dot{m}(1+\beta)$$

Combined Method (2)

- Transport equations for heat and vapor

$$H_s = -\rho_a K_w \frac{dq_v}{dz}$$

$$\dot{m}_v = -\rho_a C_p K_h \frac{dT}{dz}$$

$$\frac{H_s}{\dot{m}_v} = \frac{C_p K_h (T_2 - T_1)}{K_w (q_{v_2} - q_{v_1})}$$

$$H_s = \beta (l_v \dot{m}_v)$$

$$q_v = 0.622 \frac{e}{p}$$

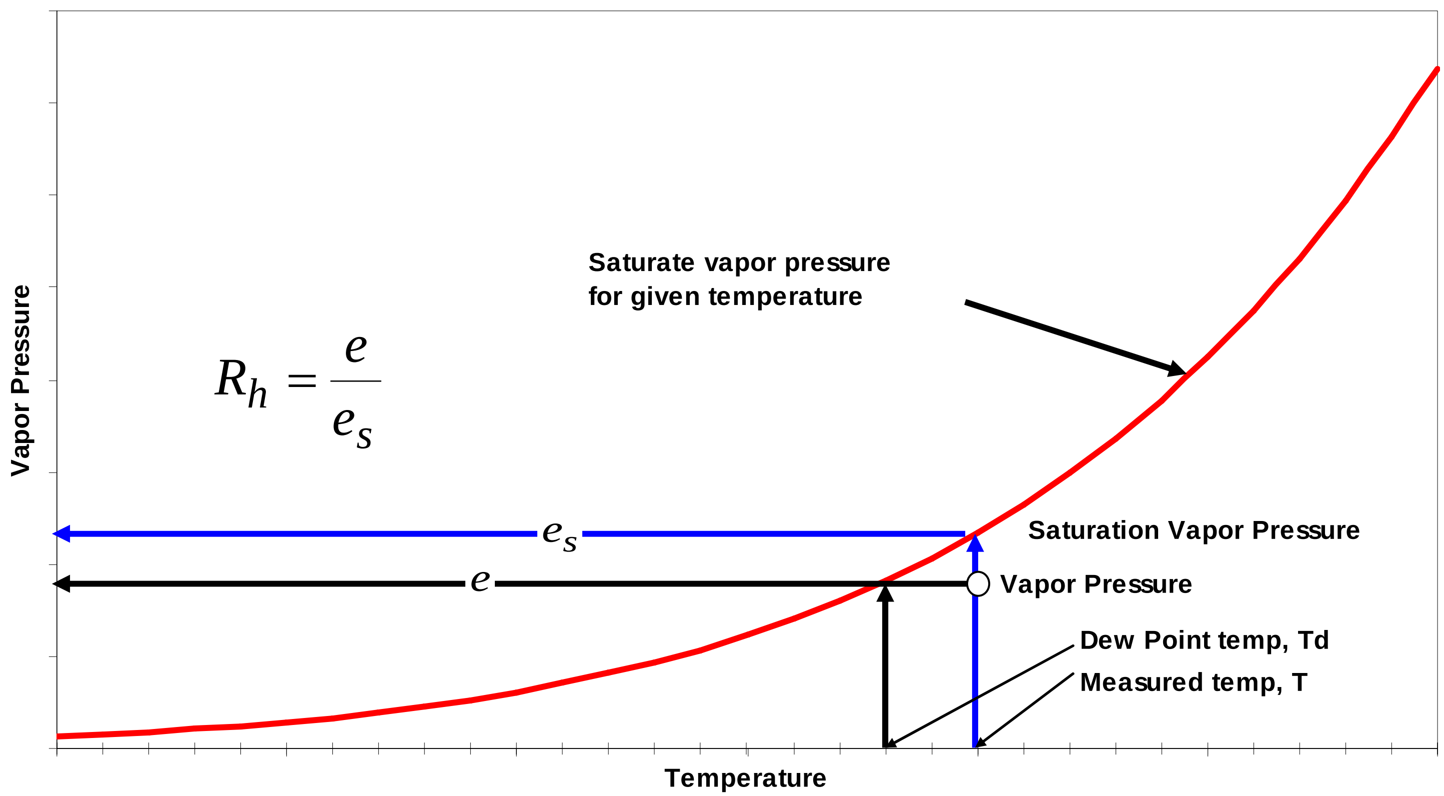
$$\beta = \frac{C_p K_h p (T_2 - T_1)}{0.622 l_v K_w (e_2 - e_1)}$$

$$\gamma = \frac{C_p K_h p}{0.622 l_v K_w}$$

$$\beta = \gamma \frac{(T_2 - T_1)}{(e_2 - e_1)}$$

$$\frac{K_h}{K_w} \approx 1$$

Recall Vapor Pressure



$$e_s = 611\exp\left(\frac{17.27T}{237.3+T}\right)$$

$$\Delta = \frac{de_s}{dT} = \frac{4098e_s}{(237.3+T)^2}$$