CE 374 K – Hydrology

Systems and Continuity

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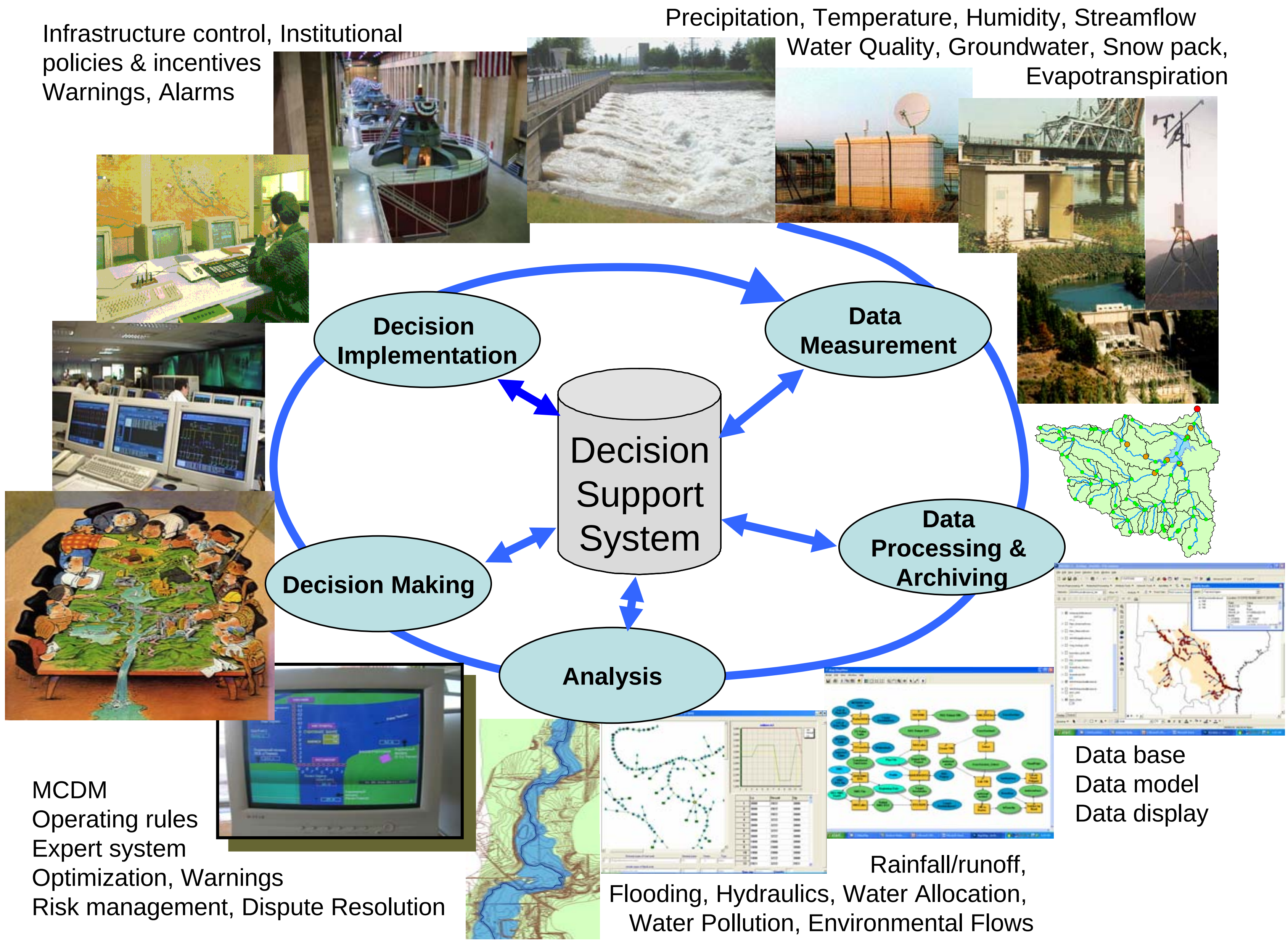
River Basin Management



Water Resources Planning and Management

- Identification, formulation and analysis of projects and designs
- Based on scientific, legal, ethical, economic, …, concepts
- Problems Considered
  - Municipal and industrial supply
  - Irrigation
  - Flood control
  - Hydroelectric power
  - Navigation
  - Water quality
  - Recreation
  - Fisheries
  - Drainage & sediment control
  - Preservation and enhancement of natural water areas, ecological diversity, archeology, etc

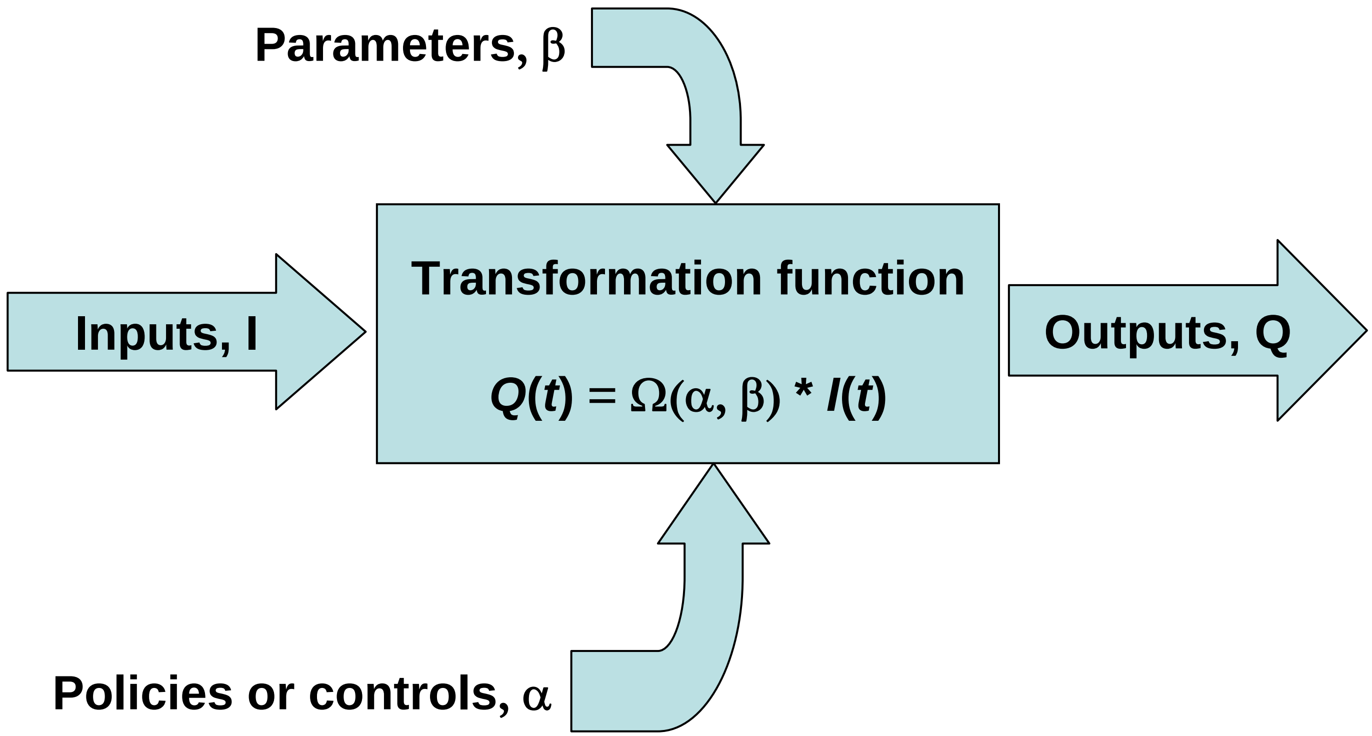
Water Resource Systems Analysis

- Water resources problems are
  - Complex, interconnected, and overlapping
  - Involving water allocations, economic development, and environmental preservation
- Systems analysis
  - Break complex system down into components and analyze the interactions between the components
  - Central method used in water resources planning

System

<u>Some Systems:</u>

- Watershed
- Aquifer
- Development Area
- Detention Basin



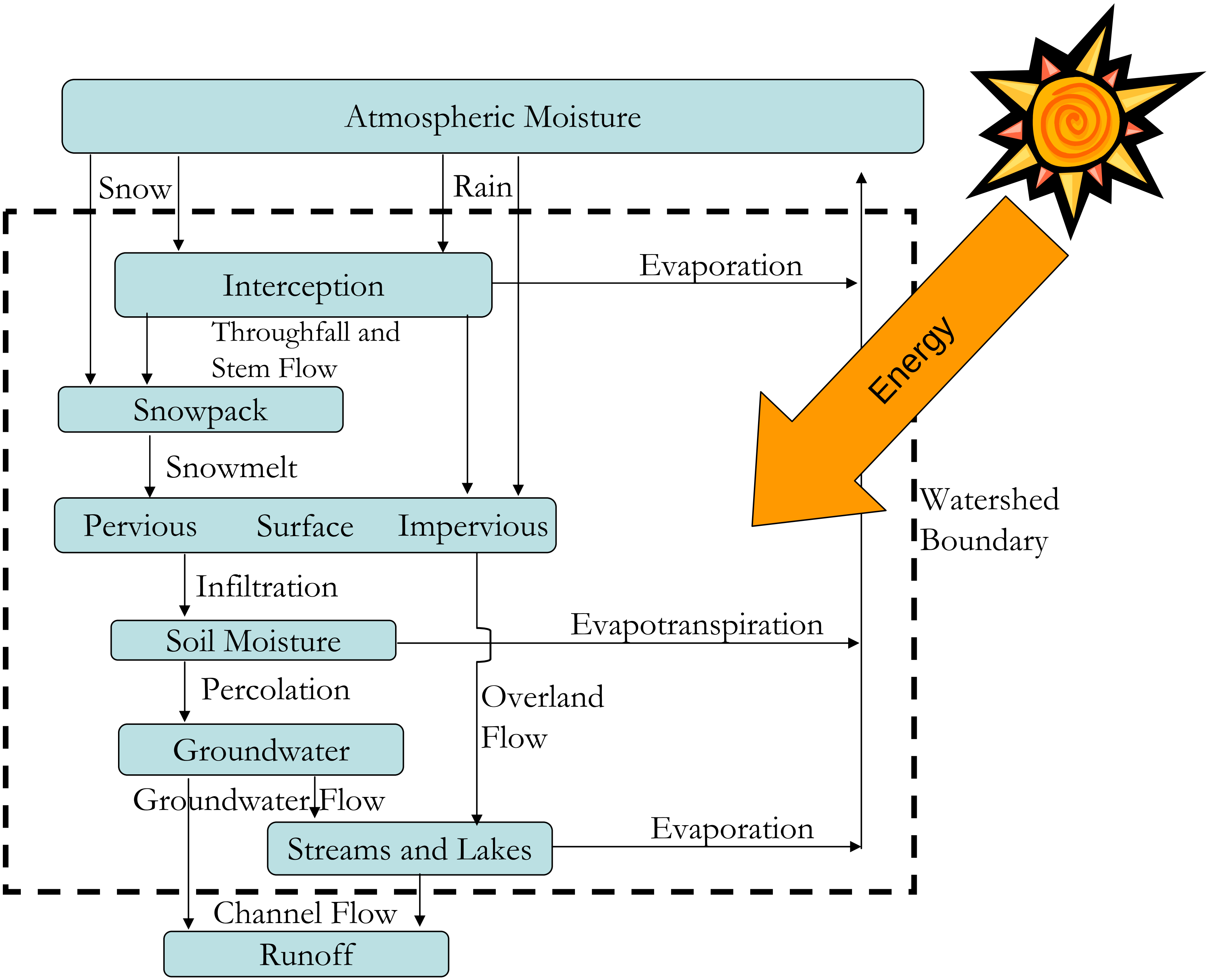
System Transformation Function

$$\boldsymbol{Q(t) = \Omega(\alpha, \beta) * I(t)}$$

- Mathematical model
- Typically a set of algebraic equations
- Derived from differential equations of
  - Conservation of Mass (e.g., continuity)
  - Conservation of Momentum (e.g., Manning)
  - Conservation of Energy (e.g., friction loss)

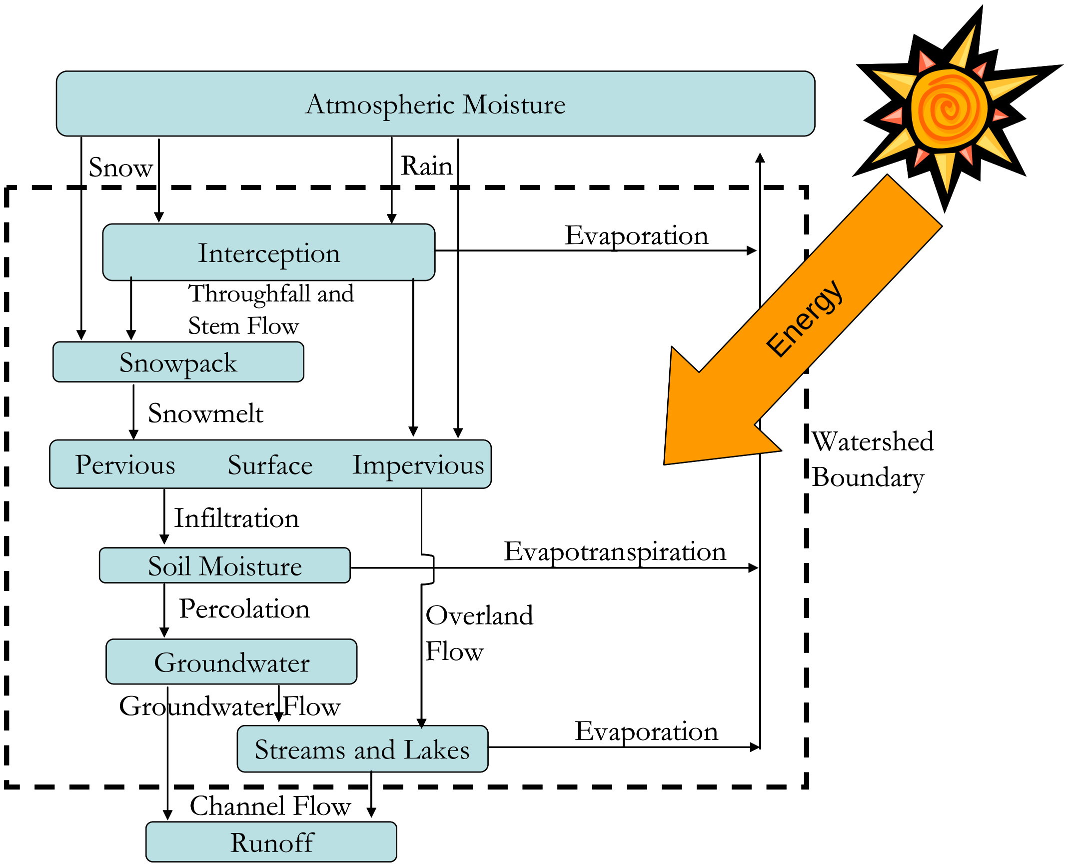
System Characteristics

- Linear vs nonlinear
  - Linear - superposition is valid
    - *If* $I_1 \rightarrow Q_1$ and $I_2 \rightarrow Q_2$
    - *Then* $I_1 + I_2 \rightarrow Q_1 + Q_2$
- Lumped vs distributed parameter (spatially varying)
- Steady-state vs transient (time dependent)
- Deterministic vs stochastic (random)


Atmospheric Moisture
Snow
Rain
Interception
Evaporation
Throughfall and
Stem Flow
Snowpack
Snowmelt
Pervious
Surface
Impervious
Energy
Watershed
Boundary
Infiltration
Soil Moisture
Evapotranspiration
Percolation
Overland
Flow
Groundwater
Groundwater Flow
Streams and Lakes
Evaporation
Channel Flow
Runoff

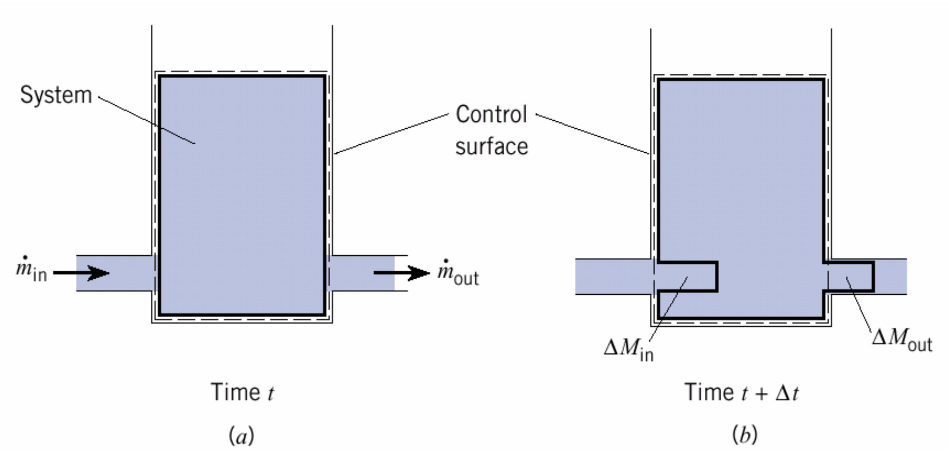
Hydrologic Processes
(Precipitation, Evaporation, Infiltration, Runoff)

- Transform the distribution of water in the hydrologic cycle
- Governed by fundamental conservation principles
- **Reynold's Transport Theorem** allows us to derive these fundamental principles



Systems

- **Laws of Mechanics**
  - Written for systems
  - System = arbitrary quantity of mass of fixed identity
  - Fixed quantity of mass, *m*



- **Conservation of Mass**
  - Mass is conserved and does not change

$$\frac{dm}{dt} = 0$$

- **Momentum**
  - If surroundings exert force on system, mass will accelerate

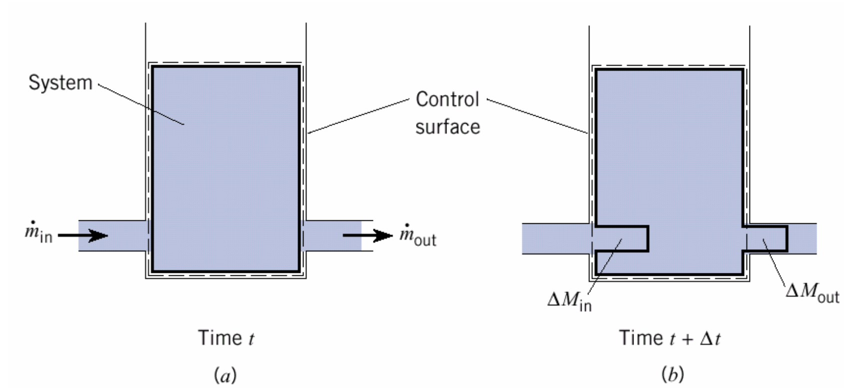
$$\vec{F} = \frac{d(m\vec{V})}{dt}$$

- **Energy**
  - If heat is added to system or work is done by system, energy will change

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Control Volumes

- Solid Mechanics
  - Follow the system, determine what happens to it
- Fluid Mechanics
  - Consider the behavior in a specific region or Control Volume
- Convert System approach to CV approach
  - Look at specific regions, rather than specific masses
- Reynolds Transport Theorem
  - Relates time derivative of system properties to rate of change of property in CV



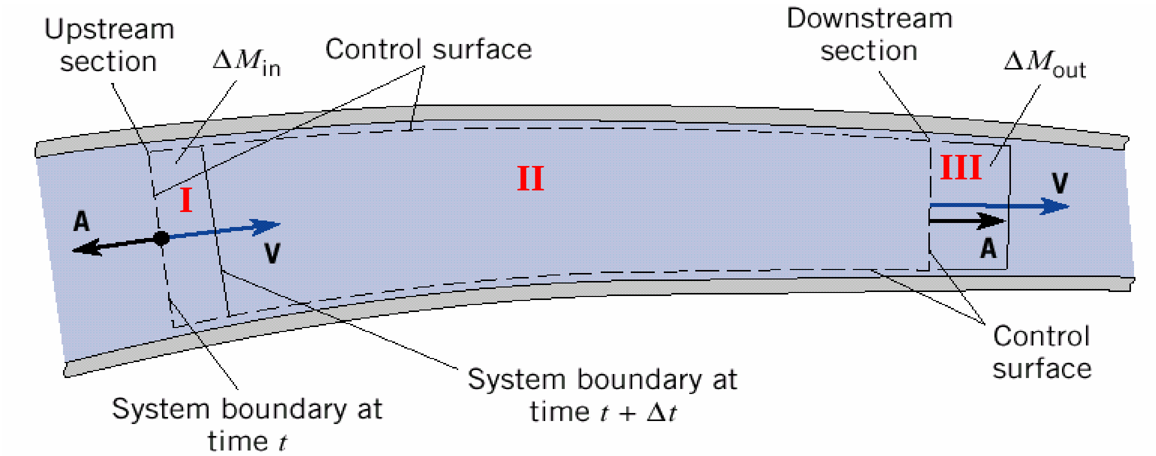
$$B = \int_{CV} \beta dm = \int_{CV} \beta \rho d\forall$$

$$= \text{mass, momentum, energy (extensive)}$$

$$\beta = \frac{dB}{dm}$$

$$= \text{amount of } B \text{ per unit mass (intensive)}$$

Reynolds Transport Theorem



$$\frac{dB}{dt} = \lim_{\Delta t \to 0} \frac{(B_{II} + B_{III})_{t+\Delta t} - (B_I + B_{II})_t}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{(B_{II})_{t+\Delta t} - (B_{II})_t}{\Delta t} + \frac{(B_{III})_{t+\Delta t} - (B_I)_t}{\Delta t}$$

$$\frac{dB}{dt} = \frac{d}{dt} \iiint_{CV} \beta \rho d\forall + \iint_{CS} \beta \rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

Continuity Equation

(Conservation of Mass)

$$B = M \quad \text{mass of the system; } \beta = {}^{dM}\!/_{dm} = 1$$

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{CV} \beta\rho d\forall + \iint_{CS} \beta\rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}} \longrightarrow 0 = \frac{d}{dt}\iiint_{CV} \rho d\forall + \iint_{CS} \rho \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}}$$

if $\rho = \text{constant}$

$$\frac{d}{dt}\iiint_{CV} d\forall + \iint_{CS} \vec{\mathbf{V}} \cdot d\vec{\mathbf{A}} = 0$$

$$\frac{dS}{dt} + Q(t) - I(t) = 0$$

$$\boxed{\frac{dS}{dt} = I(t) - Q(t)}$$

Inflow – Outflow = Change in Storage

Discrete Time Continuity

$$\frac{dS}{dt} = I(t) - Q(t)$$

Units of each term $= \frac{L^3}{T} \; (\frac{m^3}{s}, \frac{ft^3}{s})$

$$dS = I(t)dt - Q(t)dt$$

$$\int_{S_{j-1}}^{S_j} dS = \int_{(j-1)\Delta t}^{j\Delta t} I(t)dt - \int_{(j-1)\Delta t}^{j\Delta t} Q(t)dt$$

$$S_j - S_{j-1} = I_j - Q_j$$

Units of each term $= L^3 \; (m^3, ft^3)$

$$\boxed{S_j = S_{j-1} + I_j - Q_j}$$

Volume of water in storage at the end of the next time period Δt, S_j, equals the volume in storage at the beginning of that period, S_{j-1}, plus the volume of inflow, I_{j-1}, minus the volume of outflow, Q_{j-1}

Shoal Creek Flood
Memorial Day 1981

- Normal flow = 90 gpm
- Storm peak = 6.5 million gpm
- 13 lives lost

Shoal Creek Flood Memorial Day 1981

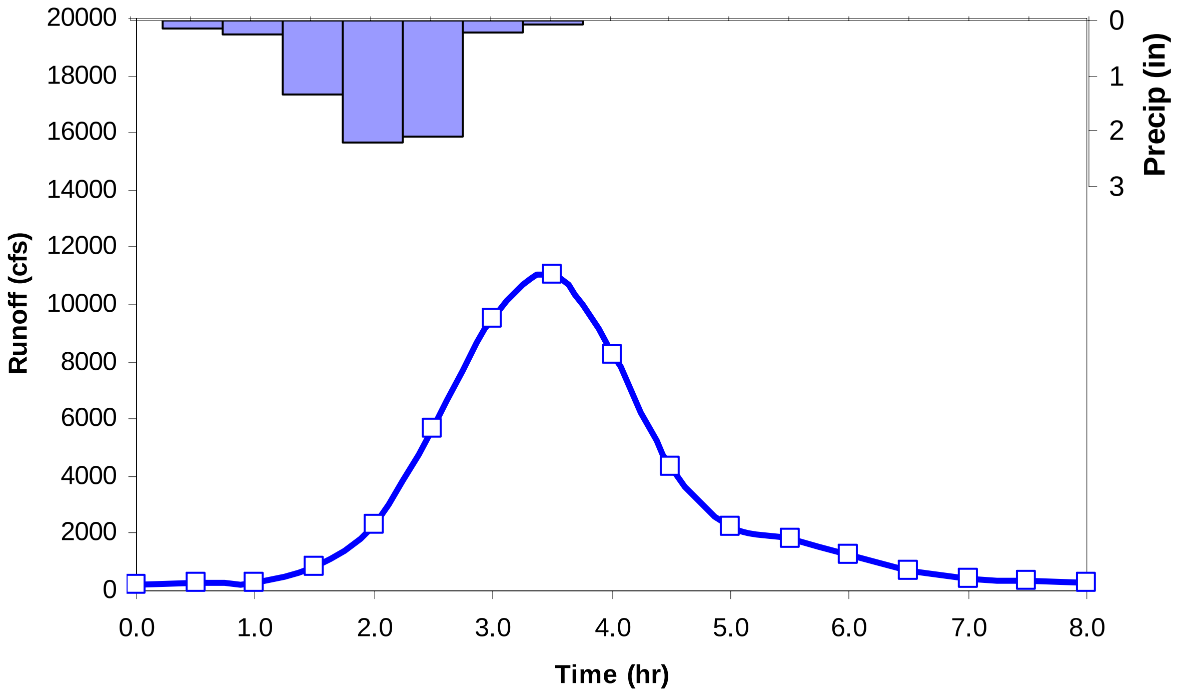
- 6.31 in. of rain fell uniformly over 7.03 sq. mi.
- What was the equivalent volume of water?

$$6.31\,\text{in} * \frac{1\,\text{ft}}{12\,\text{in}} * 7.03\,\text{mi}^2 * (5280\,\text{ft/mi})^2$$

$$= 103{,}055{,}525\,\text{ft}^3 * 7.48052\,\text{gal/ft}^3$$

$$= 770{,}908{,}921\,\text{gal}$$

770 million gallons in 8 hours



Example – Shoal Creek

- Given:
  - Incremental precipitation over the watershed, pulse
  - Streamflow measured at the outlet, continuous
- Find: Storage as function of time

- Convert streamflow to pulse data $\Delta t = \frac{1}{2}\,\text{hour}$

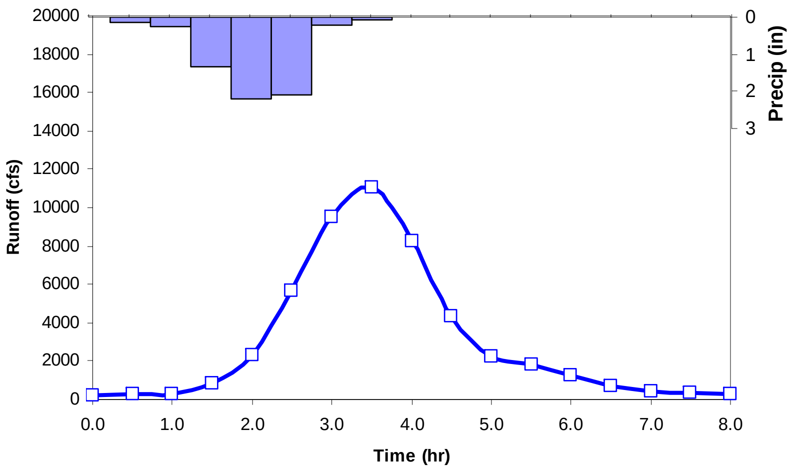
- Average streamflow over time interval $\frac{1}{2}\left(Q_i + Q_{i+1}\right)\Delta t$

- Equivalent depth over the watershed $\frac{1}{A}\frac{1}{2}\left(Q_i + Q_{i+1}\right)\Delta t$

- Continuity Eq. $S_j = S_0 + \sum_{i=1}^{j}\left(I_i - Q_i\right)$

$$S_0 = 0; \quad S_1 = S_0 + I_0 - Q_0; \quad S_2 = S_1 + I_1 - Q_1$$

Shoal Creek Flood



Time Interval	Time	Incremental Precip	Instantaneous Streamflow	Incremental Streamflow	Incremental Storage	Cumulative Storage
j	t	I_j	Q(t)	Qj	ΔS_j	S_j
	hr	in	cfs	in	in	in
	0.0	0	203			0.00
1	0.5	0.15	246	0.02	0.13	0.13
2	1.0	0.26	283	0.03	0.23	0.36
3	1.5	1.33	828	0.06	1.27	1.62
4	2.0	2.20	2323	0.17	2.03	3.65
5	2.5	2.08	5697	0.44	1.64	5.29
6	3.0	0.20	9531	0.84	-0.64	4.65
7	3.5	0.09	11025	1.13	-1.04	3.61
8	4.0	0.00	8234	1.06	-1.06	2.55
9	4.5	0.00	4321	0.69	-0.69	1.85
10	5.0	0.00	2246	0.36	-0.36	1.49
11	5.5	0.00	1802	0.22	-0.22	1.27
12	6.0	0.00	1230	0.17	-0.17	1.10
13	6.5	0.00	713	0.11	-0.11	1.00
14	7.0	0.00	394	0.06	-0.06	0.93
15	7.5	0.00	354	0.04	-0.04	0.89
16	8.0	0.00	303	0.04	-0.04	0.86
		6.31				

I_1 Q_1

$$\Delta S = S_1 - S_0$$

$$S_2 = S_1 + I_1 - Q_1$$

$$S_j = S_0 + \sum_{i=1}^{j} (I_i - Q_i)$$

Shoal Creek Flood

Shoal Creek at Northwest Park, Austin, Texas, May 24-25, 1981
Area= 7.03 mi2 195985152

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