CE 374 K – Hydrology

Precipitation

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Precipitation

- Lifting cools air masses so moisture condenses
- Condensation nuclei
  - Aerosols ($10^{-3} - 10 \, \mu m$)
  - Water molecules attach
- Rising & growing
  - Critical size ($\sim 0.1 \, mm$)
  - Gravity overcomes and drop falls
Terminal Velocity

- **Three forces**
  - Buoyancy, Friction, Gravity
    \[
    \sum F_{\text{vert}} = 0 = F_B + F_D - W
    \]
    \[
    = \rho_a g \frac{\pi}{6} D^3 + C_d \rho_a \frac{\pi}{4} D^2 \frac{V_t^2}{2} - \rho_w g \frac{\pi}{6} D^3
    \]
  - Accelerate until terminal velocity, \( V_t \)
    - Where forces balance
    \[
    F_D = F_B - W
    \]
    \[
    C_d \rho_a \frac{\pi}{4} D^2 \frac{V_t^2}{2} = \rho_a g \frac{\pi}{6} D^3 - \rho_w g \frac{\pi}{6} D^3
    \]

- **Stokes Law**

\[
C_d = \frac{24}{\text{Re}} \quad \text{Re} = \frac{\rho_a V D}{\mu_a}
\]

\[
V_t = \sqrt{\frac{4gD}{3C_d} \left( \frac{\rho_w}{\rho_a} - 1 \right)}
\]
Precipitation Mechanisms

- **Convective**
  - Heating of air at ground level leads to expansion and rise of air
- **Frontal (Cyclonic)**
  - Movement of large air mass systems (warm & cold fronts)
- **Orographic**
  - Mechanical lifting of air masses over windward sides of mountain ranges
Global Precipitation

http://geography.uoregon.edu/envchange/climAnimations/#Global%20Water%20Balance
Greatest Rainfalls

Point rainfall, in.

Minutes

Duration

Hours

Days

Months

Unionville, Md.

Fossen, Bavaria

Plumb Point, Jamaica

Curtea de Arges, Rumania

Holt, Mo.

D'hantis, Tex.

Rockport, W.Va.

Silver Hill Plantation, Jamaica

Funkiko, Formosa

Baguio, Philippine Is.

Thrall, Tex.

Smethport, Pa.

Cherrapunji, India

d(inches) = 15.3 \left[ \frac{tr}{(hours)} \right] 0.486
Texas Precipitation

- Hyetograph – Graph of Precipitation
- Isohyet - Contour of equal precipitation
Precipitation Variation

- Influenced by
  - Atmospheric circulation and local factors
    - Higher near coastlines
    - Seasonal variation – annual oscillations in some places
    - Variables in mountainous areas
    - Increases in plains areas
    - More uniform in Eastern US than in West
    - Precipitation over the US:
      - [http://weather.noaa.gov/radar/mosaic.loop/DS.p19r0/ar.us.conus.shtml](http://weather.noaa.gov/radar/mosaic.loop/DS.p19r0/ar.us.conus.shtml)
Isohyetal Map
Memorial Day Flood, Austin, 1981

Storm of May 24–25, 1981, in Austin, Texas. Maximum rainfall of 11 in. recorded over 3 hours. Isohyets are in inches depth of total rainfall in the storm. (Source: Massey, Reeves, and Lear, 1982.)
# Rainfall Depth and Intensity

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Incremental Rainfall (in)</th>
<th>Cumulative Rainfall (in)</th>
<th>Running Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>10</td>
<td>0.34</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>15</td>
<td>0.1</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>20</td>
<td>0.04</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>25</td>
<td>0.19</td>
<td>0.69</td>
<td>0.69</td>
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<tr>
<td>30</td>
<td>0.48</td>
<td>1.17</td>
<td>1.17</td>
</tr>
<tr>
<td>35</td>
<td>0.5</td>
<td>1.67</td>
<td>1.65</td>
</tr>
<tr>
<td>55</td>
<td>0.31</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>60</td>
<td>0.66</td>
<td>3.81</td>
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<tr>
<td>115</td>
<td>0.15</td>
<td>8.04</td>
<td>8.04</td>
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<tr>
<td>120</td>
<td>0.09</td>
<td>8.13</td>
<td>8.13</td>
</tr>
<tr>
<td>150</td>
<td>0.01</td>
<td>8.41</td>
<td>8.41</td>
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<tr>
<td>Depth</td>
<td>0.76</td>
<td></td>
<td>3.07</td>
</tr>
<tr>
<td>Intensity</td>
<td>9.12364946</td>
<td></td>
<td>6.14</td>
</tr>
</tbody>
</table>

Depth: 3.07
Intensity: 6.14
Incremental Rainfall

![Bar chart showing incremental rainfall over time. The x-axis represents time in minutes, ranging from 5 to 150, and the y-axis represents incremental rainfall in inches per 5 minutes, ranging from 0 to 0.8. The chart shows a peak in rainfall between the 80th and 90th minute.]
Cumulative Rainfall

Cumulative Rainfall (in.)

Time (min.)

0 30 60 90 120 150

0 1 2 3 4 5 6 7 8 9 10

3.07 in

5.56 in

8.2 in

30 min

1 hr

2 hr
Point Precipitation Estimates

- Estimate point rainfall at a given location from recorded values at surrounding sites

\[ D_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 \]  
Distance from ungaged Point \( j \) to gage \( i \)

\[ W_{ij} = \frac{1}{D_{ij}} \]  
Weight

\[ \hat{P}_j = \frac{1}{\sum_{i=1}^{I} W_{ij}} \sum_{i=1}^{I} W_{ij} P_i \]  
Precipitation Estimate at ungaged Point \( j \)
Areal Precipitation Estimates

- Precipitation is highly variable in time and space
- Mean areal precipitation
  - Areal mean
    \[ P_1 = \frac{1}{A} \int_A f(x) dx \]
  - Precipitation at point \( x \)
    \[ f(x) \]
  - Long-term areal average
    \[ P_2 = \frac{1}{T} \frac{1}{A} \sum_{t=1}^{T} \int_A f(x, t_i) dx \]
  - Precipitation at point \( x \) and time \( t_i \)
    \[ f(x, t_i) \]
  - Unfortunately, we don’t know the precipitation value at all points \( x \), so we have to use an approximation
Areal Precipitation Estimates: Arithmetic Mean

\[ \bar{P} = \frac{1}{J} \sum_{j=1}^{J} P_j \]

<table>
<thead>
<tr>
<th>Station, ( j )</th>
<th>Observed Rainfall Mm, ( P_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2</td>
<td>20</td>
</tr>
<tr>
<td>P3</td>
<td>30</td>
</tr>
<tr>
<td>P4</td>
<td>40</td>
</tr>
<tr>
<td>P5</td>
<td>50</td>
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<tr>
<td></td>
<td>140</td>
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</table>

Ave. Rainfall = 140/4 = 35 mm
Areal Precipitation Estimates:
Thiessen Polygon Method

\[ \overline{P} = \frac{1}{A} \sum_{j=1}^{J} A_j P_j \]

<table>
<thead>
<tr>
<th>Station, ( j )</th>
<th>Observed Rainfall, ( \text{Mm, } P_j )</th>
<th>Area, ( A ), ( \text{km}^2 )</th>
<th>Weighted Rainfall, ( A_j P_j ), ( \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>10</td>
<td>0.22</td>
<td>2.2</td>
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<tr>
<td>P2</td>
<td>20</td>
<td>4.02</td>
<td>80.4</td>
</tr>
<tr>
<td>P3</td>
<td>30</td>
<td>1.35</td>
<td>40.5</td>
</tr>
<tr>
<td>P4</td>
<td>40</td>
<td>1.60</td>
<td>64.0</td>
</tr>
<tr>
<td>P5</td>
<td>50</td>
<td>1.95</td>
<td>97.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.14</td>
<td>284.6</td>
</tr>
</tbody>
</table>

Ave. Rainfall = 284.6/9.14 = 31.1 mm
Areal Precipitation Estimates: Isohyetal Method

Ave. Rainfall = 255.2/9.14 = 27.9 mm
Areal Precipitation Estimates
Three Methods

- **Arithmetic Average**
  - Gages must be uniformly distributed
  - Individual variations must not be far from mean rainfall
  - Not accurate for large area where rainfall distribution is variable

- **Thiessen Polygon**
  - Areal weighting of rainfall from each gage
  - Does not capture orographic effects
  - Most widely used method

- **Isoheytal**
  - Most accurate method
  - Extensive gage network required
  - Can include orographic effects and storm morphology
http://geography.uoregon.edu/envchange/climAnimations/#Global%20Water%20Balance