Momentum Equation

Reynolds’s Transport Theorem

\[
\frac{d\vec{B}}{dt} = \frac{d}{dt} \int_V \vec{\beta} \rho d\mathcal{V} + \int_{CS} \vec{\beta} \rho \vec{V} \cdot d\vec{A}
\]

\[\vec{B} = M\vec{V}\] momentum of the system; \[\vec{\beta} = \frac{d\vec{B}}{dm} = \vec{V}\]

Newton’s Second Law

\[
\frac{d\vec{B}}{dt} = \frac{dM\vec{V}}{dt} = \sum \vec{F}
\]

\[
\sum \vec{F} = \frac{d}{dt} \int_V \vec{V} \rho d\mathcal{V} + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A}
\] Unsteady, nonuniform flow

Nonuniform flow – velocity varies in space
Uniform flow - velocity constant in space
Unsteady flow – velocity varies in time
Steady flow – velocity constant in time
Momentum Equation

\[ \sum \mathbf{F} = \frac{d}{dt} \iiint \mathbf{V} \rho d\mathbf{A} + \iiint \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \]

\[ \sum \mathbf{F} = \iiint \mathbf{V} \rho \mathbf{V} \cdot d\mathbf{A} \quad \text{Steady, nonuniform flow} \]

\[ \sum \mathbf{F} = 0 \quad \text{Steady, uniform flow} \]
Energy Equation

Reynolds Transport Equation
\[ \frac{dB}{dt} = \frac{d}{dt} \left( \int \int \beta \rho d\mathcal{A} + \int \int \rho \tilde{V} \cdot d\mathcal{A} \right) \]

Energy of the System
\[ B = E = E_u + \frac{1}{2} MV^2 + Mgz \]

First Law
\[ \frac{dE}{dt} = \frac{dH}{dt} - \frac{dW}{dt} \]

Combining
\[ \frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt} \left( \int \int \left( \frac{V^2}{2} + e_u + gz \right) \rho d\mathcal{A} + \int \int \left( \frac{p}{\rho} + \frac{V^2}{2} + e_u + gz \right) \rho \tilde{V} \cdot d\mathcal{A} \right) \]

\[ \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_f = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 \]
Internal Energy

• Sensible Heat – related to temperature

\[ \Delta e_u = C_p \Delta T \]

Specific heat \( C_p \)

• Latent Heat – related to phase changes
  – Fusion/Melting
    • ice – water, 0.33x10^6 J/kg
  – Vaporization/Condensation
    • water – water vapor, 4.2x10^3 J/kg
  – Sublimation
    • ice – water vapor, 2.5x10^6 J/kg

– Main internal energy change in hydrology
Steady Uniform Flow in an Open Channel

- **Continuity**
  \[ \oint_{CS} \vec{V} \cdot d\vec{A} = 0 \]

Steady flow \( Q_1 = Q_2; \quad V_1 = V_2 \)
Uniform flow \( A_1 = A_2; \quad y_1 = y_2 \)

Uniform channel
Steady Uniform Flow in an Open Channel

- **Energy**

\[
\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - h_f = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2
\]

\[
v_1 = v_2 \\
y_1 = y_2
\]

\[
z_1 - z_2 = h_f \\
\frac{h_f}{L} = \frac{z_1 - z_2}{L} = S_f = S_0
\]
Flow in an Open Channel

Steady, uniform flow

- **Momentum** \( \sum \vec{F} = 0 \)

- **3 forces on CV:**
  - Pressure: cancels
  - Friction: \( \vec{F}_f = -\tau_0(PL) \)
  - Gravity: \( \vec{F}_g = \gamma AL \sin \theta = \gamma ALS_f \)

- **Sum:** \( \sum \vec{F} = 0 = -\tau_0(PL) + \gamma ALS_f \)

\[ \tau_0 = \gamma RS_f \]
Open Channel Flow

- **Darcy – Weisbach Equation**: head loss due to wall friction
  \[ h_f = f \frac{L V^2}{D 2g} \]
  \[ S_f = \frac{h_f}{L} \]
  \[ D = 4R = 4 \frac{A}{P} \]

- **Chezy’s Equation for open channel flow**
  \[ V = C \sqrt{RS_f} \]
  \[ C = \sqrt{\frac{8g}{f}} \]

- **Manning’s Equation for open channel flow**
  \[ V = \frac{1}{n} \frac{2^{2/3}}{S_f^{1/2}} \]
  \[ C = \frac{1}{n} \frac{1}{R^{1/6}} \]
Manning’s Equation

- Manning’s Equation for open channel flow

\[ V = \frac{1}{n} R^{2/3} S_f^{1/2} \quad V = \frac{1.49}{n} R^{2/3} S_f^{1/2} \quad R = \frac{A}{P} \]

- Valid for fully turbulent flow

\[ n^6 \sqrt{RS_f} \geq 1.1 \times 10^{-13} \quad \text{As } n \uparrow, V \downarrow \]

- Laminar flow: use Chezy with \( f \) from Moody diagram

Manning, Robert, "On the Flow of Water in Open Channels and Pipes,“
Transactions of the Institution of Civil Engineers of Ireland, 1891
## Manning’s n

<table>
<thead>
<tr>
<th>Material</th>
<th>Manning n</th>
<th>Material</th>
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</table>
Ethics Question

- (http://www.lmnoeng.com/manningn.htm)

- *Is it ethical to use an engineering software program to solve a problem if you cannot complete the calculations manually?*
Manning’s $n$

- Hydraulic computations related to discharge require an evaluation of the roughness of the channel.
- This is an art developed through experience.
- The appearance of some typical channels whose roughness coefficients are known can be studied on the web page:
  
  [wwwrcamn1.wr.usgs.gov/sws/fieldmethods/Indirects/nvalues/](wwwrcamn1.wr.usgs.gov/sws/fieldmethods/Indirects/nvalues/)
Example

- Manning’s equation: Steady, uniform flow in an open channel. Find velocity and flow rate

Given:
- $H = 5 \text{ ft}$
- $S = 0.03 \%$
- $B = 200 \text{ ft}$; and
- $n = 0.015$

$P = B + 2xH = 200 + 2(5) = 210 \text{ ft}$

$R = \frac{A}{P} = \frac{200 \times 5}{210} = 4.76 \text{ ft}$

$Q = VA = 4.87 \times 200 \times 5 = 4870 \text{ ft}^3 / s$

You can check that flow is, indeed, turbulent