CE 311K - McKinney

HW-9 Nonlinear Equations

Problem 1. Use fixed-point iteration to locate the root of: $f(x) = e^{-x} - x$. Use an initial guess of x_0=1.0 and iterate until the approximate error is less than 0.01%.

Problem 2. A mass balance for a pollutant in a well-mixed lake can be written as:

$$V \frac{dc}{dt} = W - Qc - kV\sqrt{c}$$

where c is the concentration of the pollutant in g/m^3 the parameter values are V = 1x10^6 m^3, Q = 1x10^5 m^3/yr, W=1x10^6 g/yr, and k = 0.2 m$^{0.5}$/g$^{0.5}$/yr. The steady-state concentration can be found from the equation:

$$W - Qc - kV\sqrt{c} = 0$$

The root can be found using Fixed-Point Iteration using

Method 1: $c_{i+1} = \left(\frac{W - Qc_i}{kV} \right)^2$ Method 2: $c_{i+1} = \frac{W - kV\sqrt{c_i}}{Q}$

Only one of these equations will work all the time for initial guesses for c > 1.

(a) Use Method 2 and demonstrate that it will work for an initial guess of c_0=2.0 g/m^3.
(b) Demonstrate what happens if you try to use Method 1.

Problem 3. Determine the smallest real root of

$$f(x) = -11 - 22x + 17x^2 - 2.5x^3$$

graphically and (b) using the bisection method using a stopping criterion of 0.05%. (Root is between –1.0 and 0.0).

Problem 4. Starting from an initial guess $x_0 = 2.5$, take 2 iterations of Newton's method to find the **<u>maximum value</u>** of the function:

$$f(x) = 2\sin x - \frac{x^2}{8}$$