

CE 311K - McKinney
HW-10 Linear Equations

Problem 1. Given the system of equations

$$\begin{aligned} 0.77x_1 + 6x_2 &= 14.25 \\ 1.2x_1 + 1.7x_2 &= 20 \end{aligned}$$

- Solve graphically
- On the basis of the graphical solution, what do you expect regarding the condition of the system?
- Solve by elimination of unknowns.

$$\begin{aligned} 0.77x_1 + 6x_2 &= 14.25 \\ 1.2x_1 + 1.7x_2 &= 20 \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{20 - 1.7x_2}{1.3} \\ 0.77\left(\frac{20 - 1.7x_2}{1.3}\right) + 6x_2 &= 14.25 \\ x_2 &= 0.289 \\ x_1 &= \frac{20 - 1.7(0.289)}{1.3} = 16.25 \end{aligned}$$

Problem 3. Use Gauss Elimination to solve the following systems of linear equations. Show all steps in the computation.

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ \text{a. } \quad 3x_1 + 2x_2 + x_3 &= 10 \\ -2x_1 + 3x_2 - 2x_3 &= -2 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ -2 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 10 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -30 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 2 \\ x_3 &= 3 \end{aligned}$$

Problem 4. The series expansion for sine x is $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$. Starting with the simplest version, $\sin x \approx x$, add terms, one at a time in order to estimate $\sin(\frac{\pi}{2})$. After each new term is added, compute the true error and the approximate relative error. Add terms until the absolute value of the approximate relative error falls below a stopping criterion of 0.001%. Use a spreadsheet.

Problem 5. Start from an initial guess of $x_1^0 = x_2^0 = x_3^0 = 0.0$, show the first 2 iterations of the Gauss-Seidel method for the solution of the system of equations in problem 3. Compute the error after the second iteration.

$$-12x_1 + x_2 - 8x_3 = -80$$

$$x_1 - 6x_2 + 4x_3 = 13$$

$$-2x_1 - x_2 + 10x_3 = 90$$

$$x_1^{i+1} = \frac{-80 - (x_2^i - 8x_3^i)}{-12}$$

$$x_2^{i+1} = \frac{13 - (x_1^{i+1} + 4x_3^i)}{-6}$$

$$x_3^{i+1} = \frac{90 - (-2x_1^{i+1} - x_2^{i+1})}{10}$$

$$x_1^1 = \frac{-80}{-12} = 6.6667$$

$$x_2^1 = \frac{13 - (6.6667)}{-6} = -1.05556$$

$$x_3^1 = \frac{90 - (-2 * 6.6667 + 1.05556)}{10} = 10.2278$$

$$x_1^2 = \frac{-80 - (-1.05556 - 8 * 10.2278)}{-12} = -0.2398$$

$$x_2^2 = \frac{13 - (-0.2398 + 4 * 10.2278)}{-6} = 4.61191$$

$$x_3^2 = \frac{90 - (-2 * -0.2398 - 4.61191)}{10} = 9.41323$$

$$e_1 = \left| \frac{-0.2398 - 6.6667}{-0.2398} \right| * 100 = 2880\%$$

$$e_2 = \left| \frac{4.61191 + 1.05556}{4.61191} \right| * 100 = 122\%$$

$$e_3 = \left| \frac{9.41323 - 10.2278}{9.41323} \right| * 100 = 8.6\%$$