3.0 POSITIONING THE NEXRAD STAGEIII CELLS RELATIVE TO THE DIGITAL ELEVATION MODEL CELLS

In order to define the flow length from each NEXRAD precipitation cell to the watershed outlets, a mesh of NEXRAD cells must be created in the same coordinate system as the digital elevation data. This apparently simple task is complicated by the fact that the original NEXRAD mesh is defined on a spherical earth datum but the locations of all the radars are recorded in latitudes and longitudes on an ellipsoidal earth datum. The distinction between these two datums is sufficiently important to require a careful examination of their definitions and how to make transformations between them.

3.1 GEODETIC (ELLIPSOIDAL) AND GEOCENTRIC (SPHERICAL) COORDINATES

In a spherical coordinate system, the latitude of a point on the earth's surface is the angle between the equatorial plane and a line drawn from a point on the earth's surface to the center of the earth, and the longitude is the angle a plane drawn through a particular meridian makes with a similar plane drawn through the Greenwich meridian. These are called geocentric coordinates. But the earth is an oblate spheroid rather than a sphere, being flattened slightly at the poles compared to the equator such that the polar radius (6357 km) is about 1/300 shorter than the equatorial radius (6378 km). If the earth is represented as a sphere of constant radius, such as 6371 km, the "earth surface" or equivalent of mean sea level is about 7 km below the ocean surface at the equator, and about 14 km above the ocean surface in the polar regions. To avoid this discrepancy, geographers represent the earth mathematically by an ellipsoid of rotation with the major axis of the ellipse in the equatorial plane and the minor axis between the poles. This change does not alter longitudes but it means that latitudes normally used for mapping are not defined by a line passing through the center of the earth.

Snyder (1987, p. 13) gives the following definition: "the geographic or geodetic latitude, which is normally the latitude referred to for a point on the Earth, is the angle which a line perpendicular to the surface of the ellipsoid at the given point makes with the plane of the equator. It is slightly greater than the geocentric latitude, except at the equator and the poles, where it is equal. The geocentric latitude is the angle made by a line to the center of the ellipsoid with the equatorial plane." For a sphere, the geodetic and geocentric latitudes are identical because a line normal to the surface always passes through the center but for an ellipsoid they are different.

Figure 3.1 is a conceptual diagram illustrating geodetic (ϕ) and geocentric latitude (ϕ_g) of a point B on the earth's surface where the center of the earth is at point O.

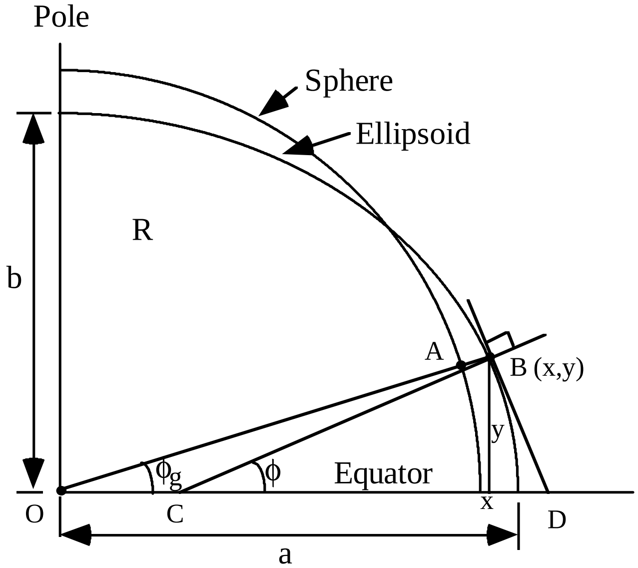


Figure 3.1: Geocentric and Geodetic Latitud

Geometrically, the geocentric latitude of a point B on the earth's surface is given by ϕ_g = angle BOD, and the geodetic latitude by ϕ = angle BCD, where the line BC is normal to the surface at the point B and the line BD is tangent to the ellipsoid at point B. An equivalent point with geocentric latitude ϕ_g is located at point A on a sphere of radius R. It can be seen that the shape of the curve of the ellipse produces a geodetic latitude ϕ greater than the geocentric latitude ϕ_g. The relation between these two latitudes can be derived in the following manner. The equation for an ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (3.1)$$

where the coordinates (x,y) of any point B are measured from the center O. By differentiating Eq. (3.1), the gradient of the surface at B is given by

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

or
$$\frac{dy}{dx} = -\frac{b^2}{a^2}\frac{x}{y} \qquad (3.2)$$

which is equal to the slope of the tangent line BD. The slope of the line CB which is normal to the surface and perpendicular to BD is given by the negative inverse of the slope of BD, or

$$\tan \quad = \frac{a^2}{b^2}\frac{y}{x} \qquad (3.3)$$

The line OAB has slope y/x, so the geocentric latitude satisfies the relation

$$\tan \;{}_g = \frac{y}{x} \qquad (3.4)$$

Thus the geodetic and geocentric latitudes are related by

$$\tan \quad = \frac{a^2}{b^2}\tan \;{}_g \qquad (3.5)$$

Snyder (1987, p. 17) presents an equivalent equation to Eq. (3.5) in the form

$${}_g = \arctan[(1 - e^2)\tan \quad] \qquad (3.6)$$

where the eccentricity e is given by

$$e^2 = 1 - \frac{b^2}{a^2} \qquad (3.7)$$

It can be verified that Eqs (3.5) and (3.6) are equivalent.

3.1.1 Conversions Between Geodetic and Geocentric Latitudes

As discussed in Section 1.5, three ellipsoids used for mapping in the United States are GRS 80, Clarke 1866, and WGS 72. The values of the major and minor axis lengths and the corresponding eccentricities of these three ellipsoids are as follows (Snyder, 1987, p.12-13)

GRS 80: a = 6378.137 km, b = 6356.7523 km, e^2 = 0.00669438

Clarke 1866: a = 6378.2064 km, b = 6356.5838 km, e^2 = 0.006768658

WGS 72: a = 6378.135 km, b = 6356.7505 km, e^2 = 0.00669432

By substituting values for a and b for GRS 80 into Eq. (3.5), the following relations are found between GRS 80 geodetic latitude, ϕ , and geocentric latitude, ϕ_g:

$$\phi = \tan^{-1}(1.00673950 \tan \phi_g) \quad (3.8)$$

$$\phi_g = \tan^{-1}(0.99330562 \tan \phi) \quad (3.9)$$

The simplest connection between the sphere and an ellipsoid is to use either the GRS 80 ellipsoid or the WGS 72 ellipsoid because the NAD 83, WGS 84, and WGS 72 datums use their respective ellipsoids in a geocentered position. In NAD 27, the Clarke ellipsoid is offset from a geocentric position so that it more nearly fits the earth's surface in North America. Applying Eq. (3.8) to compute ϕ from ϕ_g, Table 3.1 is derived,

Table 3.1: Differences Between Geodetic and Geocentric Latitudes for the GRS 80 Ellipsoid

Geocentric latitude, ϕ_g	**Geodetic latitude, ϕ**	**$\phi - \phi_g$**
0°	0°	0
10°	10° 3' 58"	3' 58"
20°	20° 7' 26"	7' 26"
30°	30° 10' 01"	10' 01"
40°	40° 11' 23"	11' 23"
50°	50° 11' 21"	11' 21"
60°	60° 9' 59"	9' 59"
70°	70° 7' 24"	7' 24"
80°	80° 3' 56"	3' 56"
90°	90°	0

from which it can be seen that the difference between the two values in the mid-latitudes is approximately 10', or (10/60)*(π/180) = 0.00291 radians. For a constant earth radius of R = 6371.2 km, this corresponds to 6371.2 * 0.00291 = 18.5 km or approximately 12 miles shift on the earth's surface. The shift is always in the same direction — a point on a spherical geocentric grid having 30° geocentric latitude corresponds to a geodetic latitude of 30° 10' 01" so the "map" location of this mesh point is actually about 10' further North than its geocentric latitude would indicate if the geocentric latitude is interpreted as a map latitude. Likewise, a measurement station at 30°N geodetic latitude corresponds to a location on a spherical grid of approximately 29°50' N.

Snyder (1987, p. 18) presents a similar latitude comparison table for the Clarke (1866) ellipsoid. Comparing values of $\phi_g - \phi$ in Snyder's table with the above values shows that the two sets of values are within 9" (0.15') of one another in all cases. Thus, it appears, that differences in the geodetic latitudes of a point on the earth as a function of the eccentricity of the ellipse are considerably smaller than is the difference between the geocentric and geodetic latitudes of that point. However, the offset from an earth-centered position of the Clarke (1866) ellipsoid in NAD 27 will also have some effect on NAD 27 latitude and longitude values which is not accounted for by the calculations presented here. Experiments with Arc/Info datum conversions discussed in Section1.5 show that differences in projected coordinates among ellipsoidal datums are on the order of tens of meters in the Tenkiller area where 1” is about 31 m.

3.2 THE HRAP COORDINATE SYSTEM

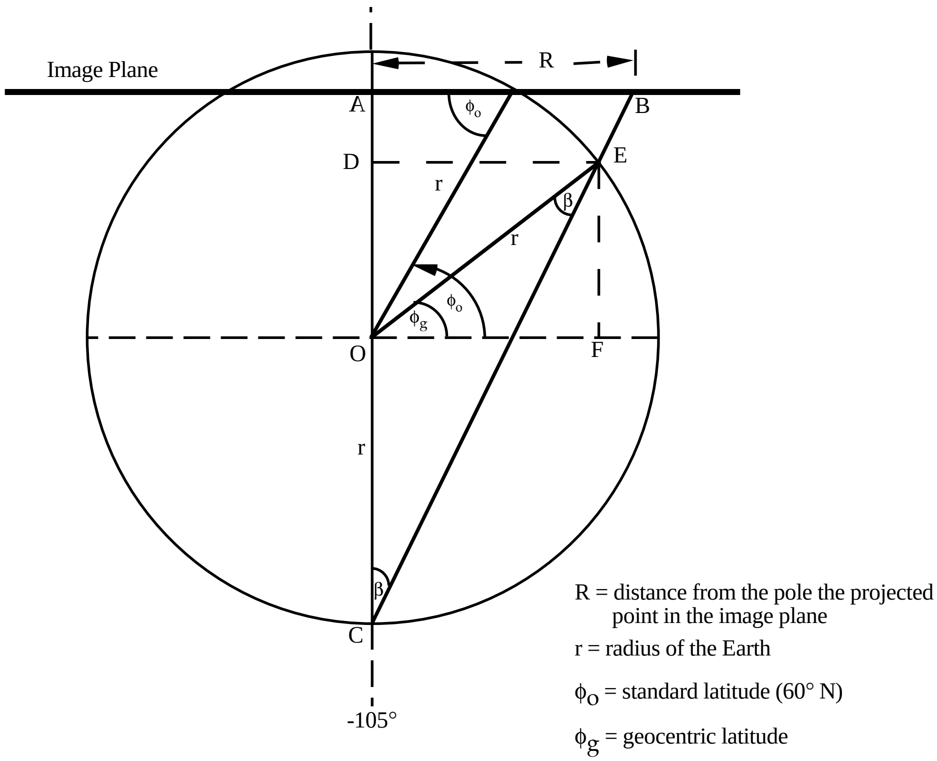
The Hydrologic Rainfall Analysis Project (HRAP) grid as defined by Greene and Hudlow (1982) is used to define the location of each average precipitation value in a NEXRAD StageIII data set. The HRAP cell coordinates are defined in the image plane of a secant polar Stereographic map projection on a spherical, earth-centered datum of radius 6371.2 km. The secant polar Stereographic projection has a standard (true) latitude of 60° North and a standard longitude (longitude of the projection center) of 105° West. In other words, features on the earth’s surface are projected onto a plane perpendicular to the axis of rotation and passing through the earth at $\phi_g = 60°$. This map projection is used by the Air Force Global Weather Central (AFGWC) and also by the National Meteorological Center (NMC) to define several reference grids (Hoke *et al.*, 1981). The Air Force Global Weather Central defines a whole-mesh reference grid in this map projection and several finer-resolution grids relative to the whole mesh grid. By definition, the whole-mesh grid length is 381 km. Finer resolution grids are defined as one-half, one-quarter, one-eighth, and one sixty-fourth of the whole-mesh grid. The half-mesh grid length is 190.5 km (Hoke *et al.*, 1981). The HRAP grid size is equivalent to one-eightieth of the whole-mesh grid, resulting in a grid length of 381/80 = 4.7625 km. However, the standard longitude of the Air Force Global Weather Central reference grids (10° E) is different than the standard longitude for HRAP (105° W) which means that the orientation of HRAP cells relative to map features is different than that of whole-mesh grid cells. In other words, HRAP cells do not fall in alignment with the whole-mesh grid cells. The grid cell lengths cited above are in the projected plane. Lengths in the projected plane are only equivalent to lengths on the surface of the spherical earth datum at 60° N.

3.2.1 Forward Transformation from Geocentric to HRAP Coordinates

The procedure for projecting spherical coordinates in degrees on the earth's surface to Cartesian coordinates in meters on a flat surface can be thought of in two parts. First, convert the geographic coordinates in latitude and longitude (ϕ_g,λ) into equivalent polar coordinates (R,λ) on a flat plane. Second, convert the polar coordinates (R,λ) to Cartesian coordinates (x,y) in the same plane. Finally, a grid mesh is defined in the projected plane by specifying a point of origin (x_o,y_o) and then a mesh size and orientation about that point. The steps are the same when an ellipsoidal datum is used but the conversion from ellipsoidal coordinates to polar coordinates in a plane is more complex because the radius of curvature on an ellipsoidal earth varies continuously with latitude and also varies continuously with direction at any point.

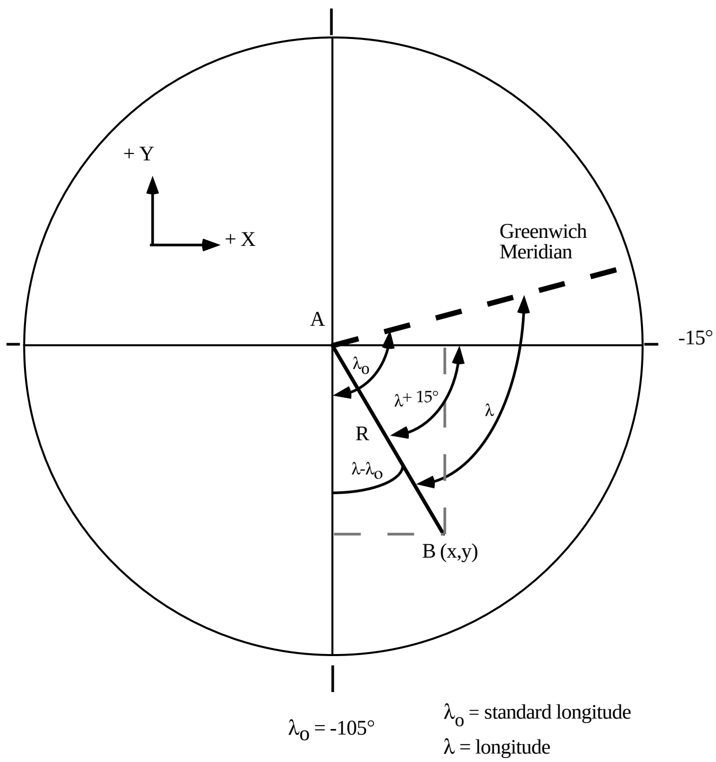
3.2.1.1 From Geographic to Polar Coordinates

The polar Stereographic projection is an azimuthal projection (projection onto a plane). Figure3.2 illustrates the geometry of the polar Stereographic map transformation with a standard latitude of 60° N. A light shining from point C will project point E on the Earth's surface onto the image plane at point B. In the polar Stereographic projection, the point B on the image plane can be defined by two parameters, the distance from the pole R and the longitude (λ) measured from the Greenwich Meridian. An expression for R as a function of latitude (ϕ_g) can be derived from Figure 3.2 .



(Adapted from Greene and Hudlow, Figure 1)

Figure 3.2: Elevation View of a Polar Stereographic Map Projection



(Adapted from Green and Hudlow, Figure 2)

Figure 3.3: Plan View of a Polar Stereographic Map Projection

$$\phi \quad = \frac{}{} = \frac{}{} \tag{3.10}$$

$$\sin\phi_g = \frac{EF}{r} = \frac{DO}{r} \tag{3.11}$$

$$\sin\phi_o = \frac{AO}{r} \tag{3.12}$$

$$CD = DO + r = r\sin\phi_g + r \tag{3.13}$$

$$CA = AO + r = r\sin\phi_o + r \tag{3.14}$$

Noting that triangles CDE and CAB are similar triangles,

$$\frac{CD}{CA} = \frac{DE}{AB} \tag{3.15}$$

Further manipulation and substitutions from Eqns. (3.10) - (3.15) yields:

$$AB = \frac{DE * CA}{CD} = r\cos\phi_g \frac{r(1+\sin\phi_o)}{r(1+\sin\phi_g)} \tag{3.16}$$

$$R = AB = r\cos\phi_g \frac{(1+\sin\phi_o)}{(1+\sin\phi_g)} \tag{3.17}$$

3.2.1.2 From Polar to Cartesian Coordinates

The point B is located in the projected plane as shown in Figure 3.3 which is from the vantage point of an observer at point C of Figure 3.2 (the South Pole). Meridians of constant longitude (λ) on the globe become radial lines in the projected plane. The United States lies between longitudes 65° W and 125° W or ($-65 < \lambda < -125$) and a central meridian of 105° W or λ_o = -105° was chosen as the vertical axis (standard latitude) in the HRAP plane, as shown in Figure 3.3. This meridian lies approximately at the west longitude of Denver, Colorado. Thus, points east of Denver have positive x-values while those west of Denver have negative x-values.

The displacement, $\lambda - \lambda_o$, of a projected point from the central meridian is further modified by subtracting 90° to rotate the coordinate system and the Cartesian coordinates in the projected domain are computed with:

$$x = R^* \cos(\lambda - \lambda\, 90^o)$$
$$y = R^* \sin(\lambda - \lambda\, 90^o)$$
or
$$x = R^* \cos(\lambda + 15^o)$$
$$y = R^* \sin(\lambda + 15^o) \qquad (3.18)$$

For these equations, values of λ west of Greenwich are negative and values of λ east of Greenwich are positive.

3.2.1.3 From Cartesian to HRAP Coordinates

A mesh of HRAP coordinates is then defined in terms of the Cartesian coordinates as follows. Since HRAP is defined in grid units and not kilometers, the radius of the model earth (r) is scaled by the mesh length (4.7625 km).

$$a = \frac{r}{\text{meshlength}} = \text{scaled radius} \sim 1{,}337.78 \text{ units} \qquad (3.19)$$

In order to keep all the HRAP coordinate values within the United States positive, the North Pole is assigned coordinates (401,1601). Taking into account the scaled radius and the origin shift, HRAP coordinates can be written in terms of polar Stereographic coordinates (with x and y in kilometers).

$$\text{hrapx} = \frac{x}{4.7625} + 401 \qquad (3.20)$$

$$\text{hrapy} = \frac{y}{4.7625} + 1601 \qquad (3.21)$$

3.2.2 Reverse Transformation from HRAP to Geocentric Coordinates

A slightly different geometric derivation is required to go from HRAP to geocentric coordinates. The inverses of Equations (3.20) and (3.21) can be used to determine the polar Stereographic coordinates (x,y) given HRAP coordinates (hrapx,hrapy). To go from polar Stereographic (x,y) to polar coordinates (R,λ), determining R is simple.

$$R = \sqrt{x^2 + y^2} \quad (3.22)$$

Computation of the longitude (λ) requires the use of an inverse tangent function. In FORTRAN 77, the function ATAN2D(y,x) returns the value of the angle in degrees that the line segment passing through the origin and the point (x,y) makes with the x-axis. The range of angles returned by atan2(y,x) is 0° to 180° in the counter-clockwise direction and 0° to -180° in the clockwise direction. Measured in the clockwise direction with a range 0° to 360°, the angle(λ') between the Greenwich Meridian and a line segment passing through the origin and a point (x,y) can be computed as follows:

If y>0, then

$$\lambda' = 270° - \lambda_o - \mathrm{atan2d}(y,x)$$

If y<0, then

$$\lambda' = -90° - \lambda_o - \mathrm{atan2d}(y,x) \quad (3.23)$$

(use λ_o = -105°)

To be consistent with the convention that west longitude values range from 0° to -180° and east longitude values range from 0° to 180°,

If $\lambda' < 180°$, then

$$\lambda = (-1) * \lambda'$$

If $\lambda' > 180°$, then

$$\lambda = 360° - \lambda' \quad (3.24)$$

Referring again to Figure 3.2 , an expression for ϕ_g in terms of R can be derived. Since the triangle EOC is isosceles,

$$90° + \phi_g + 2\beta = 180° \quad (3.25)$$

With manipulation,

$$\beta = 45° - \frac{\phi_g}{2} \tag{3.26}$$

From trigonometry and substitution of Equation (3.14),

$$\tan\beta = \frac{AB}{AC} = \frac{R}{r(1+\sin\phi_o)} \tag{3.27}$$

Substituting (3.26) into (3.27) and manipulating yields:

$$\phi_g = 90° - 2\mathrm{atand}\left[\frac{R}{r(1+\sin\phi_o)}\right] \tag{3.28}$$

3.3 USING NEXRAD DATA WITH AN ELLIPSOIDAL DATUM

All discussion of the HRAP coordinate system in previous sections assumes that the earth is a sphere, but when locating radars or other ground control points, the latitudes and longitudes read from a map are in geodetic coordinates referenced to an ellipsoidal datum. It follows that if the HRAP grid is formally defined on a spherical earth datum, a shift to an ellipsoidal datum is needed to correctly register a radar to the rainfall in the HRAP cell that contains other map features. This involves an approximately 10' shift in latitudes as specified in Table 3.1. With further information described subsequently, we found that this datum shift was apparently not made when the National Weather Service prepared software using the HRAP grid. Hence, a mathematical transformation appropriate for data in spherical coordinates was applied to map locations in ellipsoidal coordinates when radars and other map reference points were entered into NEXRAD map displays. This action introduces a further distortion, beyond that created by the HRAP projection itself, to the size and shape of HRAP cells relative to ellipsoid-based locations. To treat these subjects in sequence, the correct method for carrying out the transformations is now presented. Figure 3.4 is a conceptual diagram of the correct steps to transform coordinates from a sphere-based map projection into an ellipsoid-based map projection. The correct steps are (1) transform HRAP coordinates into latitude/longitude geocentric coordinates, (2) convert geocentric latitudes to geodetic latitudes using a datum shift from sphere to ellipsoid, and (3) perform datum transformation between ellipsoids if necessary and project geodetic coordinates into Albers.

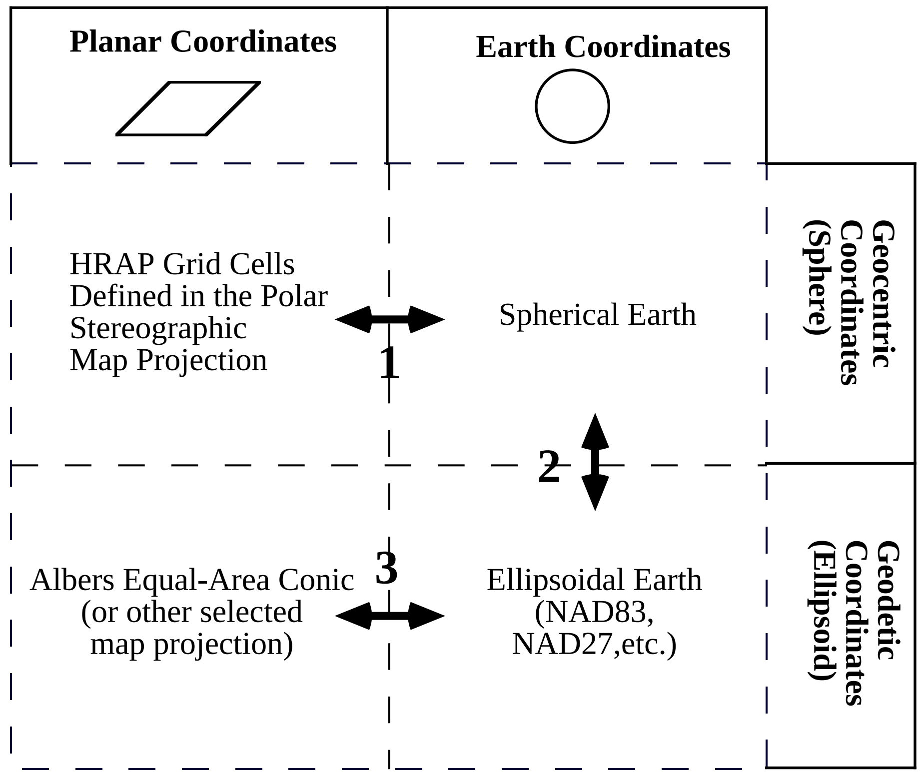


Figure 3.4: Conceptual Diagram of the Correct Steps in the HRAP to Albers Transformation

Evidence provided by Arkansas-Red Basin River Forecast Center and correspondence with the National Weather Service at Silver Spring, Maryland, (Seo and Miller, personal communication, 1995) revealed that the National Weather Service has used geodetically defined coordinates to position radars and other map features (i.e. hydrologic features or political features) in the HRAP plane. The ellipsoidal datum used to define these map features is not known at this time. In other words, geodetic coordinates (ϕ,λ) were treated as geocentric coordinates (ϕ_g,λ) or equations for a spherical earth were used to project geodetically defined features. Therefore, in order to reproduce (as closely as possible) the positions of the HRAP cells relative to the radar sites used by the Arkansas-Red Basin River Forecast Center in radar computations, step 2 in Figure3.4 should be omitted. That is, the treatment of geodetic latitudes as geocentric latitudes should be emulated in reverse by interpreting geocentric latitudes for HRAP cells as geodetic latitudes.

For convenience in discussion, the transformation involving all steps (1-3) will be referred to as the "true" transformation and the transformation omitting step 2 will be referred to as the "matching" transformation. Using the matching transformation appears to be the most accurate use of the data available at this time. The matching transformation should precisely replicate the relative position of a radar site and the HRAP grid used by Arkansas-Red Basin River Forecast Center at individual radar sites. The situation becomes complicated when information from several radars is merged because the sphere-ellipsoid distortion varies with latitude. If one begins with geodetic coordinates (ϕ,λ) and forward transforms these points to planar coordinates, the values (x',y') so produced are different than the values (x,y) that would have been produced if geocentric coordinates (ϕ_g,λ) were used. Although the reverse transformation applied to (x',y') will recover (ϕ,λ) in geodetic coordinates, there is a systematic difference between (x,y) and (x',y') that involves not only a shift in North-South location but also a rearrangement of the relative locations of points in the domain.

3.4 DISTORTIONS INVOLVED WITH USING THE HRAP COORDINATE SYSTEM

It is difficult for us to assess the magnitude of radar mapping errors without more knowledge about how distances traced out by a NEXRAD radar beam are converted to distances in the HRAP plane. It is also recognized that discussion of errors in this section is strictly limited to distortion due to map transformations without considering other errors, such as beam refraction, associated with radar detection of rainfall — other errors may be considerably larger than map transformation distortions (Seo and Miller, personal communication, 1995).

There are two types of distortions inherent in the mapping of NEXRAD products in the HRAP plane and using a spherical transformation on ellipsoidal coordinates: (1) variation of the scale factor (defined below) with latitude; (2) distortion of the scale factor because of the spherical transformation applied to ellipsoidal coordinates.

3.4.1 Scale Factor

When a map is produced, the dimensions on the earth's surface are first reduced to dimensions on a globe in proportion to the map scale, where

$$\text{map scale} = \frac{\text{globe distance}}{\text{earth distance}} \tag{3.29}$$

For example, a map scale of 1:100,000 means that 1 cm on the globe corresponds to 100,000 or 1 km on the earth. For HRAP, the map scale is 1 unit = 4.7625 km. The area on the earth's surface that an HRAP cell represents varies with latitude. In the HRAP image plane, all HRAP cells are square and have a side length equal to one unit. When HRAP cell coordinates are converted to polar Stereographic coordinates, the cells remain square in the map plane and the map distance of each cell side is 4.7625 km but this is not the length of a cell side measured on the globe. When a portion of the globe is projected onto a flat plane, the distance between two points on the globe is distorted by a scale factor:

$$\text{scale factor} = \frac{\text{map distance}}{\text{globe distance}} \qquad (3.30)$$

The scale factor is defined as the map distance divided by the distance on the globe. Snyder (1987, p. 21) denotes the scale factor along meridians of longitude with "h" and the scale factor along parallels of latitude with "k." The polar Stereographic projection is "conformal," which means that the scale factor does not vary with direction, so h = k. The Albers projection is "equal area" which means that h = 1/k so that area is preserved even though distances are distorted. For HRAP, the scale factor is equal to 1.0 only along the standard latitude (60°). At other latitudes, the scale factor can be computed with:

$$h{=}k = \frac{1 + \sin\phi_o}{1 + \sin\phi_g} \qquad (3.31)$$

which is equal to the ratio AB/DE in Figure 3.2 .

At latitudes less than 60° the scale factor is greater than 1.0 and at latitudes greater than 60° the scale factor is less than 1.0. Table 3.2 lists the scale factors at several latitudes. A scale factor greater than 1.0 means that a distance measured on a map is larger than the actual distance on the earth's surface. Thus, one HRAP unit corresponds to 4.7625 km at 60° N and to smaller distances at lower latitudes. The North-South extent of the United States is from approximately 25° N to 49° N, so HRAP cells range in size from 3.6 km in Miami, Florida, to 4.4 km in Minneapolis, Minnesota, and the corresponding range in HRAP cell areas is from approximately 13 km^2 in Miami to 19 km^2 in Minneapolis.

The HRAP cell size is often stated to be 4 km. This statement is accurate for a scale factor of 4.7625/4.00 = 1.1906 which applies at ϕ_g= 34.56° N, approximately the latitude of Little Rock, Arkansas. If a nominal cell area of 16 km^2 is assumed everywhere in the United States, the assumed cell area is 18% larger than the true area in Miami and 19% smaller than the true area in Minneapolis. Significant variation of the scale factor also occurs within the scanning range (230 km) of a WSR-88D radar. For a radar located at 37.05° N latitude, Joplin, Missouri, the scale factor varies from 1.1475 at the northern limit of the radar range to 1.1820 at the southern limit of the radar range. This amounts to a 3.45% variation. The corresponding earth areas of an HRAP cell are 17.22 km^2 at the northern limit and 16.23 km^2 at the southern limit. It seems likely that some accounting for variations in scale factor is included in radar algorithms, but we don't know if that is the case. In general, the polar Stereographic projection is not well suited for local radar mapping due to large variations in the scale factor over short distances.

Table 3.2: Approximate Scale Factor at Different Latitudes

Latitude (ϕ_g)	Representative Location	Scale Factor	HRAP Cell on the Earth's Surface: Side Length (km)	HRAP Cell on the Earth's Surface: Area (km^2)
25	Miami, FL	1.3117	3.63	13.18
30	Houston, TX	1.2440	3.83	14.66
35	Memphis, TN	1.1858	4.02	16.13
40	Indianapolis, IN	1.1359	4.19	17.58
45	Minneapolis, MN	1.0931	4.36	18.98
50	Winnipeg, Manitoba	1.0566	4.51	20.32

3.4.1.1 The Shape of HRAP Cells

It is clear from this discussion that cell sizes vary with latitude. In addition, each cell represents an area of unique size and shape on the earth's surface. This occurs because the HRAP grid is defined in a projected plane rather than the earth's surface itself. An HRAP cell at 60° N latitude has an approximate globe length and width of 4.7625 km. The cell side is not precisely 4.7625 km in globe length because no side of an HRAP cell coincides exactly with the 60° latitude line. Strictly speaking, all sides of an HRAP cell have slightly different lengths in globe distance. Figure 3.5 illustrates the size and shape an HRAP cell in planar and earth coordinates. This particular HRAP cell is located at approximately (92°20'W, 32°37'N) in the southeast corner of the Arkansas-Red Basin River Forecast Center study area. Note that the apparent area of an HRAP cell in the polar Stereographic plane (22,681,406 m^2) differs significantly from that in the Albers Equal-Area plane (15,369,703 m^2) which is the true earth area because, by definition, for an equal-area projection the product h*k = 1.0 at all points. Since h=k for a polar Stereographic projection, the ratio of the polar Stereographic area to the Albers area is very nearly equal to the square of the polar Stereographic scale factor at the average geocentric latitude of the cell. This means that it is critical that when NEXRAD StageIII data are used, the map cell area should not be literally interpreted as a true area

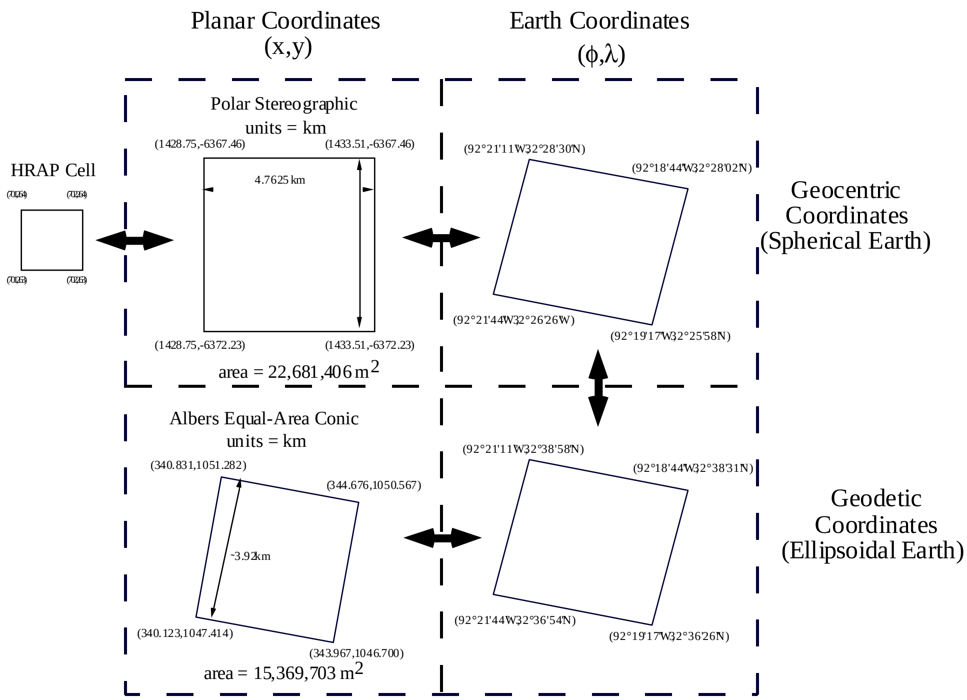


Figure 3.5: Size and Shape of an HRAP Cell (701, 263) in Several Coordinate Systems

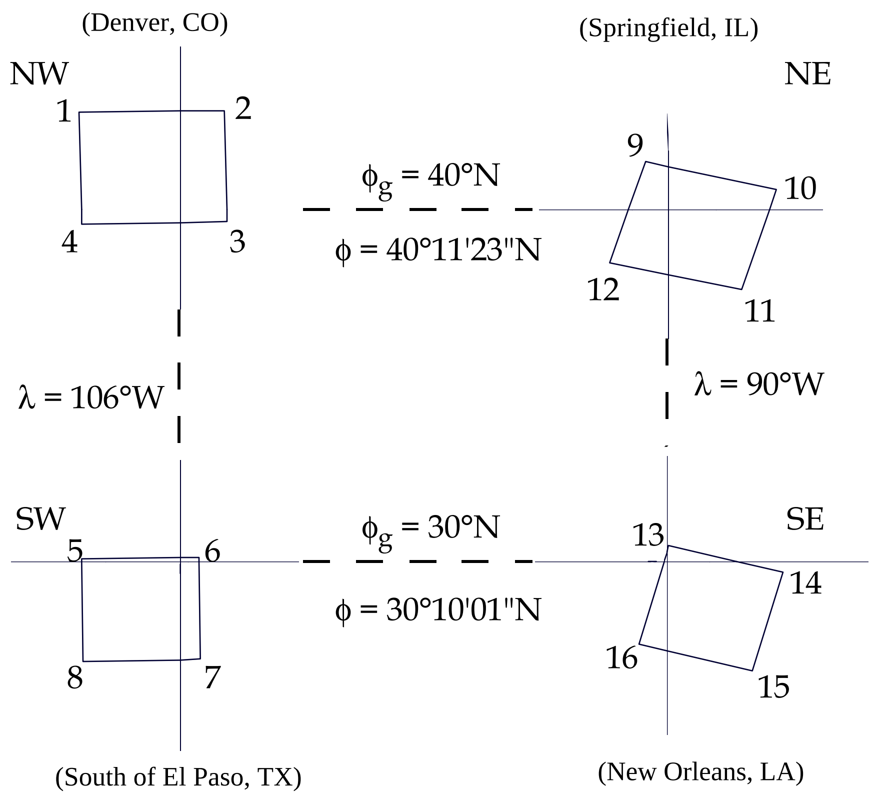


Figure 3.6a: Four HRAP Cells Plotted in Geographic Coordinates

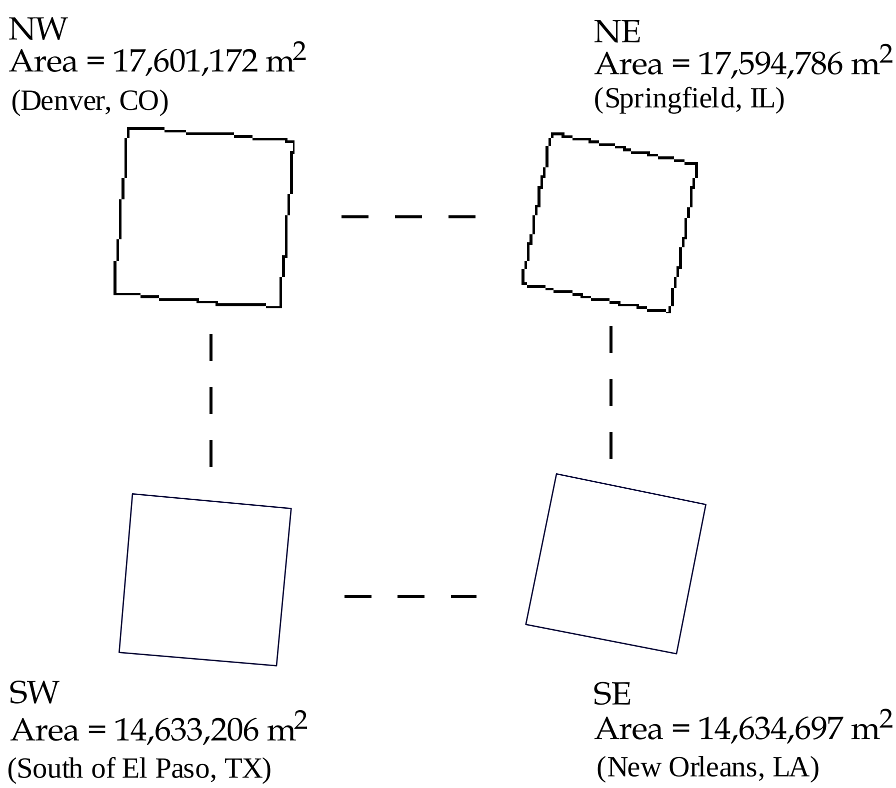


Figure 3.6b: Four HRAP Cells Transformed into Albers Equal-Area Projection

on the ground, particularly if a polar Stereographic projection is used. A more reliable approach is to project the cells into an equal-area projection so that the true earth area is closely approximated even if each cell still has a unique size and shape.

To demonstrate the uniqueness in size and shape of each HRAP cell, Figure 3.6a shows four HRAP cells that contain the intersection of a meridian of longitude and a parallel of latitude. For example, the upper left cell contains the intersection of the 106° W meridian of longitude and the 40° N parallel of latitude (geocentric). Table 3.3 lists the geocentric and geodetic (NAD83) coordinates computed for the cells of Figure 3.6a. Figure 3.6b illustrates the same four cells in the Albers projection used for this study. The differences in areas between cells can be explained by the scale factor of the polar Stereographic projection. Clearly the NW and NE cells have a smaller scale factor than the SW and SE cells and therefore represent a larger area on the earth's surface. The reason that the NE cell contains less area than the NW cell can be understood by looking back at Figure3.6a. More of the NE cell lies below the 40° N parallel; therefore, the scale factor is larger.

Table 3.3: Coordinate Values for Figure 3.6a

						NAD83 Coordinates					
	HRAP Coordinates		Geocentric Latitude			Longitude			Geodetic Latitude		
NW	hrapx	hrapy	deg	min	sec	deg	min	sec	deg	min	sec
1	380	438	40	1	58	106	2	4	40	13	21
2	381	438	40	2	0	105	59	6	40	13	23
3	381	437	39	59	45	105	59	3	40	11	7
4	380	437	39	59	42	106	2	0	40	11	5
SW											
5	375	160	30	0	3	106	2	1	30	10	4
6	376	160	30	0	5	105	59	38	30	10	6
7	376	159	29	58	1	105	59	35	30	8	2
8	375	159	29	57	59	106	1	58	30	8	0
NE											
9	702	477	40	1	2	90	0	29	40	12	24
10	703	477	40	0	26	89	57	38	40	11	49
11	703	476	39	58	15	89	58	24	40	9	38
12	702	476	39	58	50	90	1	15	40	10	13
SE											
13	774	209	30	0	19	89	59	57	30	10	20
14	775	209	29	59	47	89	57	39	30	9	47
15	775	208	29	57	47	89	58	16	30	7	47
16	774	208	29	58	19	90	0	34	30	8	20

3.4.2 Shape Factor, C_s

A shape distortion is introduced due to the use of a spherical transformation on ellipsoidal coordinates as described by Snyder, (1987), pp. 24-27. For a Stereographic projection, this distortion means that (h/k) is not equal to one as it should be for a conformal projection. The actual value of the ratio (h/k) can be determined using the local shape factor (C_s). C_s obeys the relationship,

$$\left(\frac{h}{k}\right)_s = C_s \left(\frac{h}{k}\right)_e \qquad (3.32)$$

where s = sphere and e = ellipsoid. This means that if the ratio $(h/k)_e$ is calculated using spherical map projection equations applied to ellipsoidal coordinates then the actual scale factor $(h/k)_s$ may be computed using Equation 3.32. Since $(h/k)_e$ is forced to unity by the polar Stereographic projection equations, the relationship between h and k on the spherical version is simply $h_s = k_s * C_s$. C_s can be computed for different ellipsoids using Equation 3.33 which corresponds to Equation (4-31) of Snyder, 1987, p. 26.

$$C_s = \frac{\left(1 - e^2 \sin^2 \phi\right)}{\left(1 - e^2\right)} \qquad (3.33)$$

where ϕ is the geodetic latitude and e is the eccentricity of the ellipse. For Houston, Texas, at approximately 30° N latitude, $C_s = 1.00511$ based on the Clarke 1866 ellipsoid with a = 6378.206, $e^2 = 0.006768658$. Due to this distortion, a circle on the earth maps to an ellipse in a projected plane whose major (East-West) axis and minor (North-South) axis have the ratio 1.00511:1. This distortion would cause a 230 km radius circle associated with a radar beam to map as an ellipse with a difference between the lengths of its major and minor axis as much as 230 km* 0.00511 or 1.175 km. This difference arises because the radius of curvature of an ellipsoidal earth varies continuously with latitude and also with direction at any point on the earth's surface. Using the Clarke 1866 ellipsoid, the radius of curvature at 30° N in the North-South direction is 6351.148 km and the corresponding radius of curvature in the East-West direction is 6383.609 km. These values were computed using Equations 3.34 and 3.35 which correspond to Equations (4-18) and (4-20) given by Snyder (a = 6,378,206 m).

$$R' = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \phi)^{3/2}} \qquad (3.34)$$

R' = radius of curvature in the plane of the meridian

$$N = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}} \qquad (3.35)$$

N = radius of curvature in the plane perpendicular to the meridian and also perpendicular to the tangent surface

The ratio of the two radii of curvature at 30° N is 6383.609/6351.148 = 1.00511 which is how the shape factor C_s is determined.

Although the mapping error associated with using a spherical transformation on ellipsoidal coordinates might seem large (about 0.5% in the example above), it is important to keep in mind that maps extending over large areas typically introduce larger variations in the scale factor (Snyder, p.27). For example, with the national Albers projection used in this study both h and k vary by 0.8% between 30° N and 40° N.

3.5 RECONSIDERING THE MAPPING PROBLEM

Figure 3.7 shows circles representing the 230 km beam coverage of radars under the jurisdiction of the the Arkansas-Red Basin River Forecast Center in an Albers Equal-Area projection. Each circular coverage is a plane, traced out by the radar beam as it rotates. The angle of elevation of the beam changes during rotation to better sense the rainfall at different distances from the radar, but essentially each radar coverage is like a flat map, drawn in the plane of the beam, in which distance and bearing are measured in polar coordinates relative to the center of rotation, which is the radar location.

There is a class of map projections called azimuthal projections formed by a flat map tangent to the earth at a given location. The Stereographic projection is one of these (polar Stereographic means that the projection plane is perpendicular to the polar axis). If each radar is treated as an individual map and the mosaicing of radar data to form a composite map is desired, the operation in GIS terms is very straightforward. Each input "map" is in an azimuthal projection with the projection origin at the radar location and is forward transformed to a common map reference frame. If the Lambert Azimuthal Equal-Area projection is used on the input side and Albers Equal-Area on the output side, the area of radar cells will be preserved in this process. The overlay and compositing of the precipitation from various radars on the output map is a standard GIS

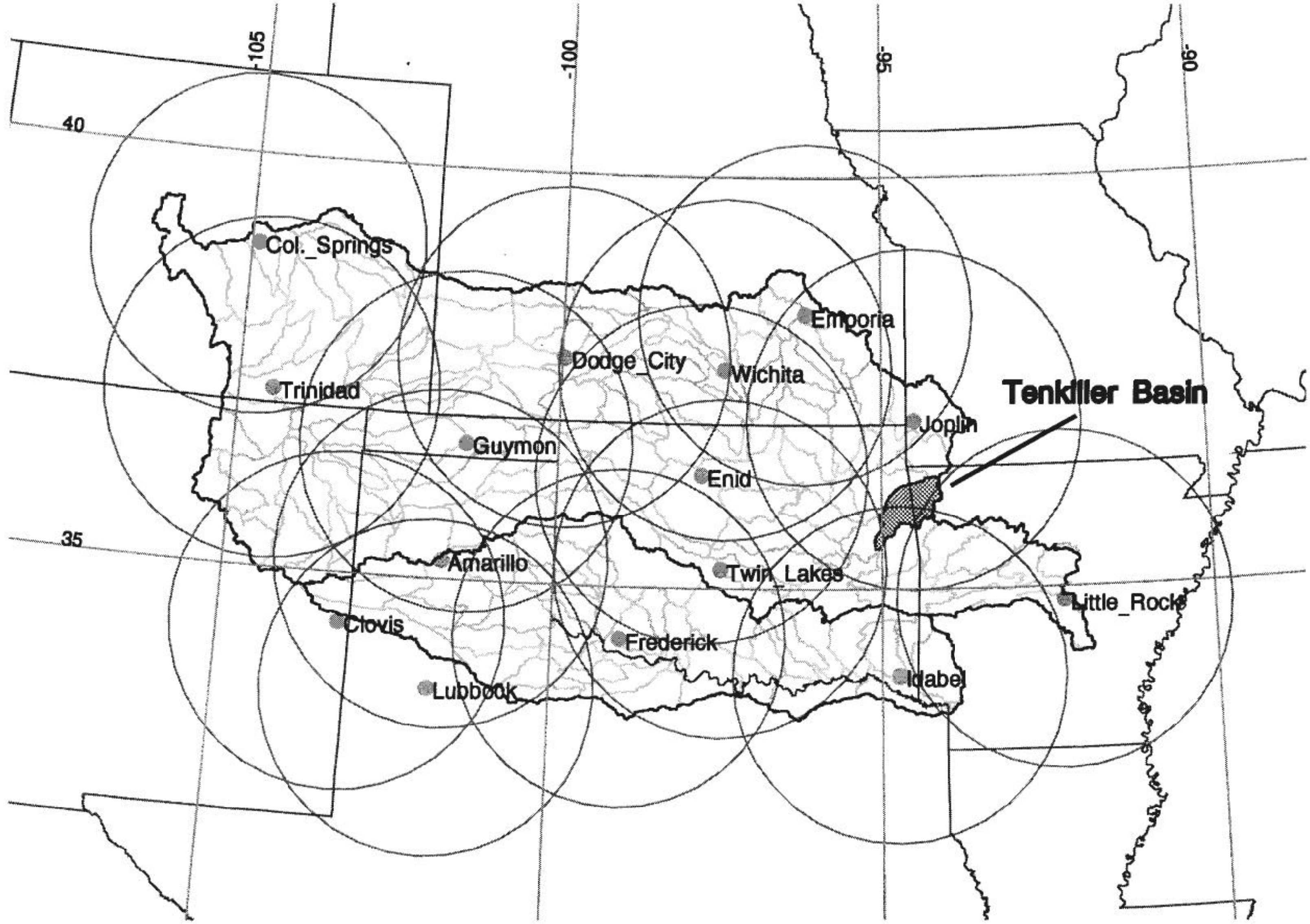


Figure 3.7: Arkansas-Red River Basin

operation. This GIS-based radar data transformation includes all of the required steps in Figure 3.4 and eliminates the shape factor distortion discussed above.

3.6 VERIFYING CONSISTENCY WITH ARKANSAS-RED BASIN RIVER FORECAST CENTER HRAP CELLS

Because of the complications involved with reproducing the HRAP cells in an ellipsoid-based coordinate system, some checks were made at control points provided by the Arkansas-Red Basin River Forecast Center to show that the sphere-ellipsoid transformation (Eqn. 3.8) should not be used for a matching transformation. Norm Bingham at the Arkansas-Red Basin River Forecast Center provided five "GIF" snapshots from their software depicting HRAP cells, state boundaries, and a few USGS gaging stations in the HRAP plane. The five snapshots are in the vicinity of state corners including the northwest corner of the Texas panhandle, the southeast corner of Colorado, the northwest corner of Louisiana, the northeast corner of Oklahoma, and the southwest corner of Missouri. Figure 3.8 (a-e) shows HRAP cells generated in GIS along with state boundaries from Environmental Systems Research Institute's ArcUSA CD-ROM in comparison with Arkansas-Red Basin River Forecast Center snapshots. The state boundaries from ArcUSA were transformed from geodetic coordinates into the HRAP coordinate system using Equations 3.17, 3.18, 3.20, and 3.21 for a spherical earth. After aligning the HRAP cells in our reproduction (first column in Figure 3.8) and those depicted in the ABRFC snapshots (second column in Figure 3.8), the distances between the corners of the state boundaries given by the two sources were approximated. The largest discrepancy occurred at the northeast corner of Oklahoma and was about 3.3 km measured in the polar Stereographic plane which corresponds to about 2.8 km on the earth's surface. A likely explanation for the discrepancy is that the two sets of state boundaries came from different sources and are inconsistent which is obvious in Figures 3.8b and e where the two sets of state boundaries clearly have a different shape. The ArcUSA data is at a relatively large scale, 1:2 million.

In addition to state boundaries, three USGS gaging stations were identified on the Arkansas-Red Basin River Forecast Center snapshots, two in Missouri and one in Oklahoma. The geodetic latitude and longitude for these stations were obtained via Internet at the address http://h2o.usgs.gov:81/swdata.html. Again, these points were transformed into HRAP coordinates using Equations 3.17, 3.18, 3.20, and 3.21. These gaging stations, along with their geographic and HRAP coordinates are listed in

Table3.4. Figure 3.9 shows that our GIS reproduction of the gaging station HRAP coordinates listed in Table 3.4 along with our generated HRAP cell boundaries compares well with the position of these stations shown by the Arkansas-Red Basin River Forecast Center snapshots. The positions of the gaging stations in our reproduction are close enough to those in the ABRFC snapshots such that if the two maps were overlaid, with HRAP cells aligned, the map symbols used to identify gaging stations would overlap. The length of a side on the triangles used to represent gaging stations in the ABRFC snapshots is approximately 0.9 km in the polar Stereographic plane. The noted discrepancies between our reproduction and the ABRFC snapshots are far less than the differences between a geodetic and geocentric latitudes at the locations in Figures3.8 and 3.9.

Table 3.4: USGS Gaging Stations Identified from Arkansas-Red Basin River Forecast Center Snapshots

Gaging Station	λ	ϕ	hrapx	hrapy
TIFM7: Elk River Near Tiff City, MO	-94.5867	36.6314	627.779	366.993
JOPM7: Shoal Creek Above Joplin, MO	-94.5161	37.0231	627.358	377.766
QUA02: Spring River Near Quapaw, OK	-94.7469	36.9344	622.858	374.490

3.8 DESCRIPTION OF FORTRAN AND AML CODES TO GENERATE CELLS AND TRANSFORM TO THE COMMON COORDINATE SYSTEM

Executing a FORTRAN code, genhrap.f, and an AML, genhrap.aml, (both listed in the Appendix) generates an Arc/Info coverage of HRAP cells in the chosen Albers projection. The geographic extent to be covered by these HRAP cells may be specified by the user. The user may specify the extent by geodetic latitudes and longitudes of the corners of the study region or by specifying the HRAP coordinate of the lower left hand corner of the study region and the number of columns and rows of cells to be created. Because the rainfall cells only need to be defined once, in whatever map projection a study is being made, it might be convenient at some point in the future to generate the HRAP grid for the entire United States and store this in a location from which users can cut out desired pieces. However, the Arkansas-Red Basin River Forecast Center radar coverage alone includes 53,365 cells and running simple codes has been easier than working with such a large file up to this point.

The code for genhrap.f consists of a main program and four subroutines. If the user chooses to specify the study extent with latitudes and longitudes, the subroutine llinput computes the corresponding extent of the area in HRAP coordinates. Llinput uses equations 3.17, 3.18, 3.20, and 3.21 to compute the HRAP coordinate corresponding to each geographic coordinate specified by the user. From the computed HRAP coordinates, llinput assigns the minimum hrapx and hrapy coordinates to be the lower left corner of the study area, computes the approximate number of rows and columns to span the geographic extent, and returns these values to the main program.

Given the geographic extent of the study area, genhrap.f first writes a file listing the HRAP coordinates of all corner points to be created beginning with the lower-left corner of the study area. Coordinates are written for the bottom row moving left to right, followed by the next row up and so on. This task is performed in the main program and the file generated is hrap.*cod* where *cod* is a user-defined code that is unique for a given run. For Tenkiller, the file hrap.tk3 was generated. HRAP coordinates from hrap.*cod* are converted to geocentric coordinates by the subroutine wll and written to geoc.*cod*. The subroutine wll uses the inverses of Equations 3.20 and 3.21, and Equations 3.22, 3.23, 3.24, and 3.28. The subroutine topoly reads the list of corner points from geoc.*cod* and creates a file (inputgc.*cod*) in the appropriate format for generating a polygon coverage. The last subroutine called by genhrap.f, crdat, writes a file, hrap.*cod*.dat, used to attach the correct HRAP-IDs as attributes to the final coverage in Albers. The hrapx and hrapy coordinates of its lower-left hand corner serve as the ID for an HRAP cell. Precipitation estimates available on Internet are listed according to the HRAP coordinates of a cell's lower-left hand corner; therefore, the HRAP-IDs are the only link between the geographic position of a cell and a precipitation depth obtained from Arkansas-Red Basin River Forecast Center.

Given the files inputgc.*cod* and hrap.*cod*.dat, genhrap.aml generates a polygon coverage called *cod*geocc, projects this coverage into the chosen projection producing *cod*geoccalb, creates an INFO data file (hrapxy.dat) and adds data from hrap.*cod*.dat to this file, and joins the newly created INFO file to the PAT of *cod*geoccalb. The file inputgc.*cod* is set up so that the polygons in geocentric coordinates that are created from this file are numbered from left to right starting with the bottom row followed by the next row up and so on. The numbering system is important because this is the key to joining the HRAP-IDs to the correct polygons. These polygon numbers get stored in the field *COD*GEOCCALB-ID of *cod*geoccalb.pat. The values in *COD*GEOCCALB-ID correspond to the values in the first column of the file hrap.*cod*.dat. Figure 3.10 illustrates the sequence of data files created by genhrap.f and genhrap.aml and how they are linked. A sample session showing execution of genhrap.f and genhrap.aml for Tenkiller is provided in Table 3.5.

Table 3.5: Sample Session for Generating a NEXRAD Mesh

unix% **genhrap**
Enter 1 if you wish to specify the region by latitudes and longitudes of the corners of the study region. Enter 2 if you would like to specify region by hrap coordinates and number of columns and rows.
2
Enter the hrap(x,y) for the lower left hand corner of the region of interest:
614 331
Enter the number of grid columns and rows to be created:
32
32
Enter a 3 character suffix to uniquely identify your grid:
tk3
nile.crwr.utexas.edu% **arc**
Arc: **&r genhrap**
Enter the 3 character suffix used to ID hrap files:: **tk3**

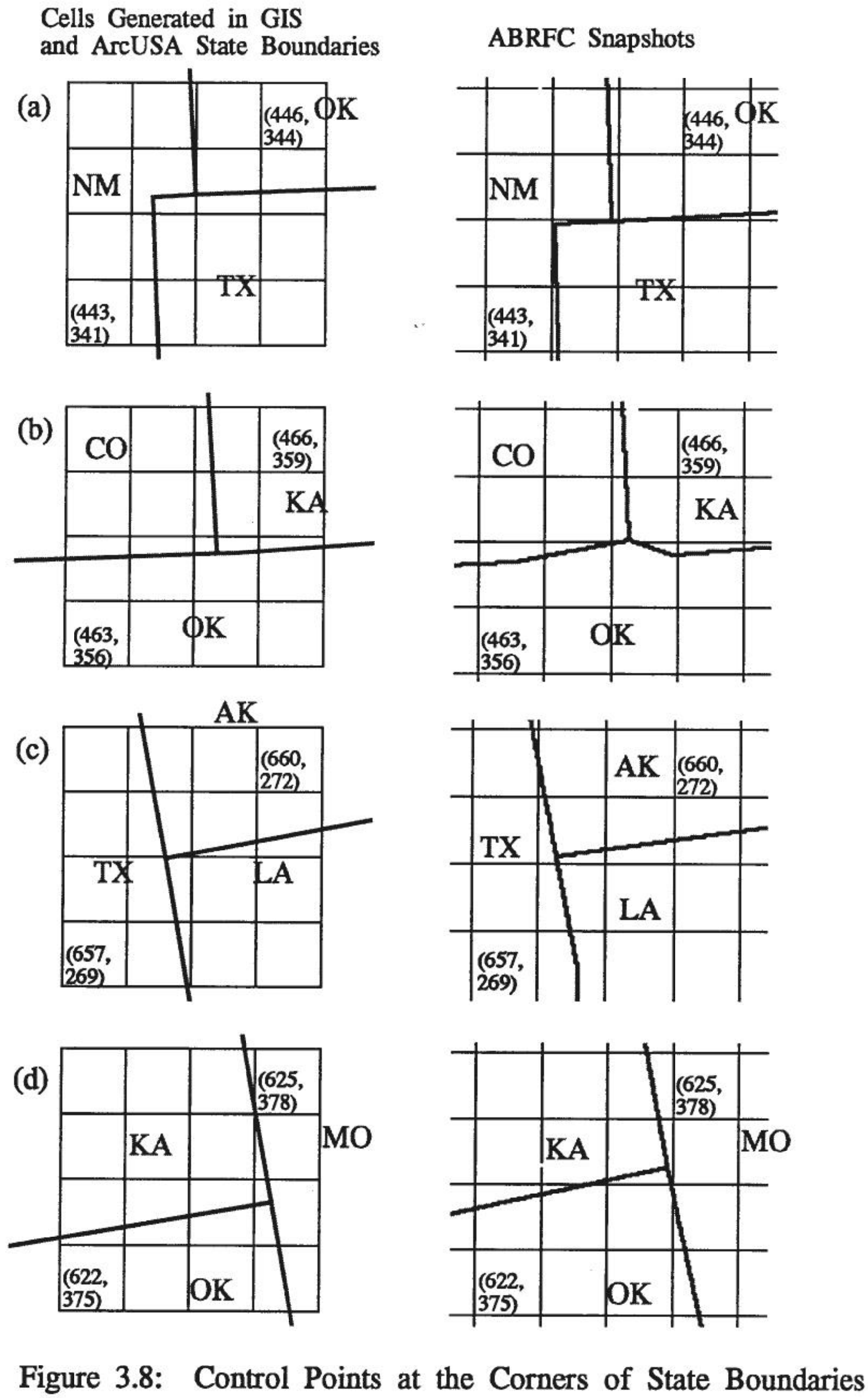
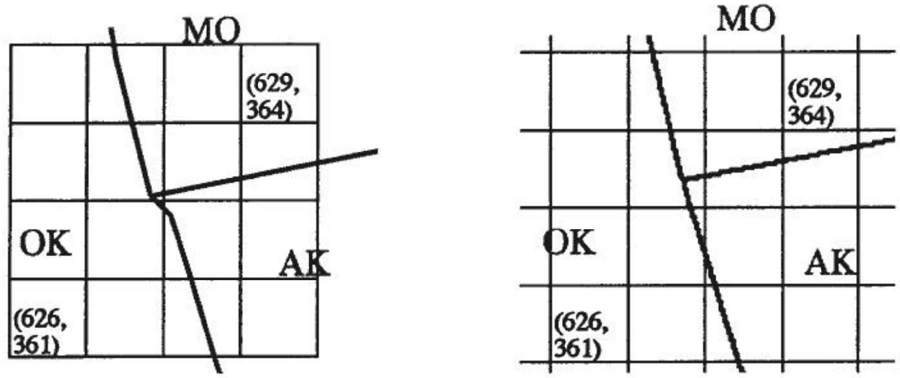


Figure 3.8: Control Points at the Corners of State Boundaries

Figure 3.8 (cont.)

(e)



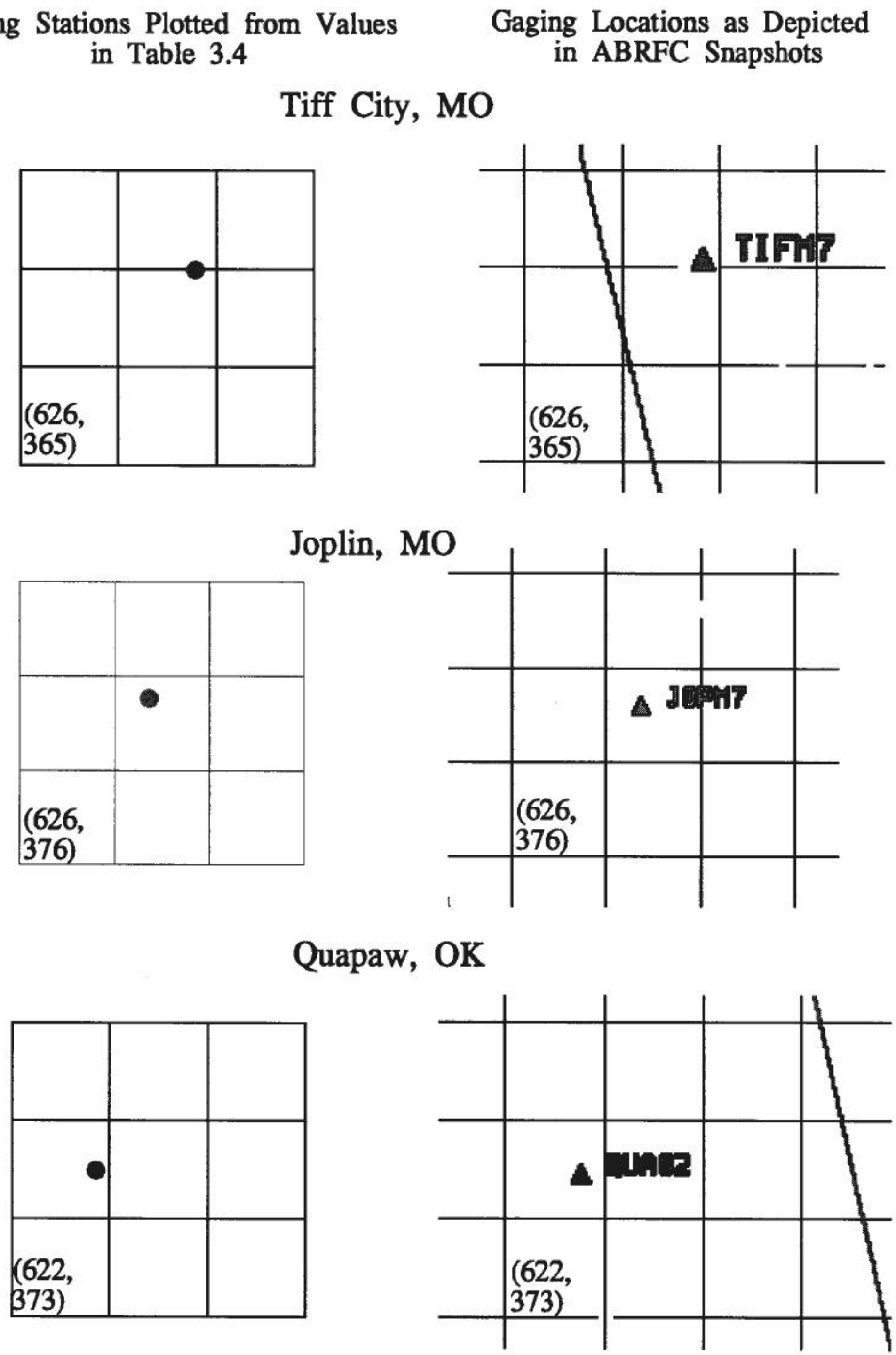
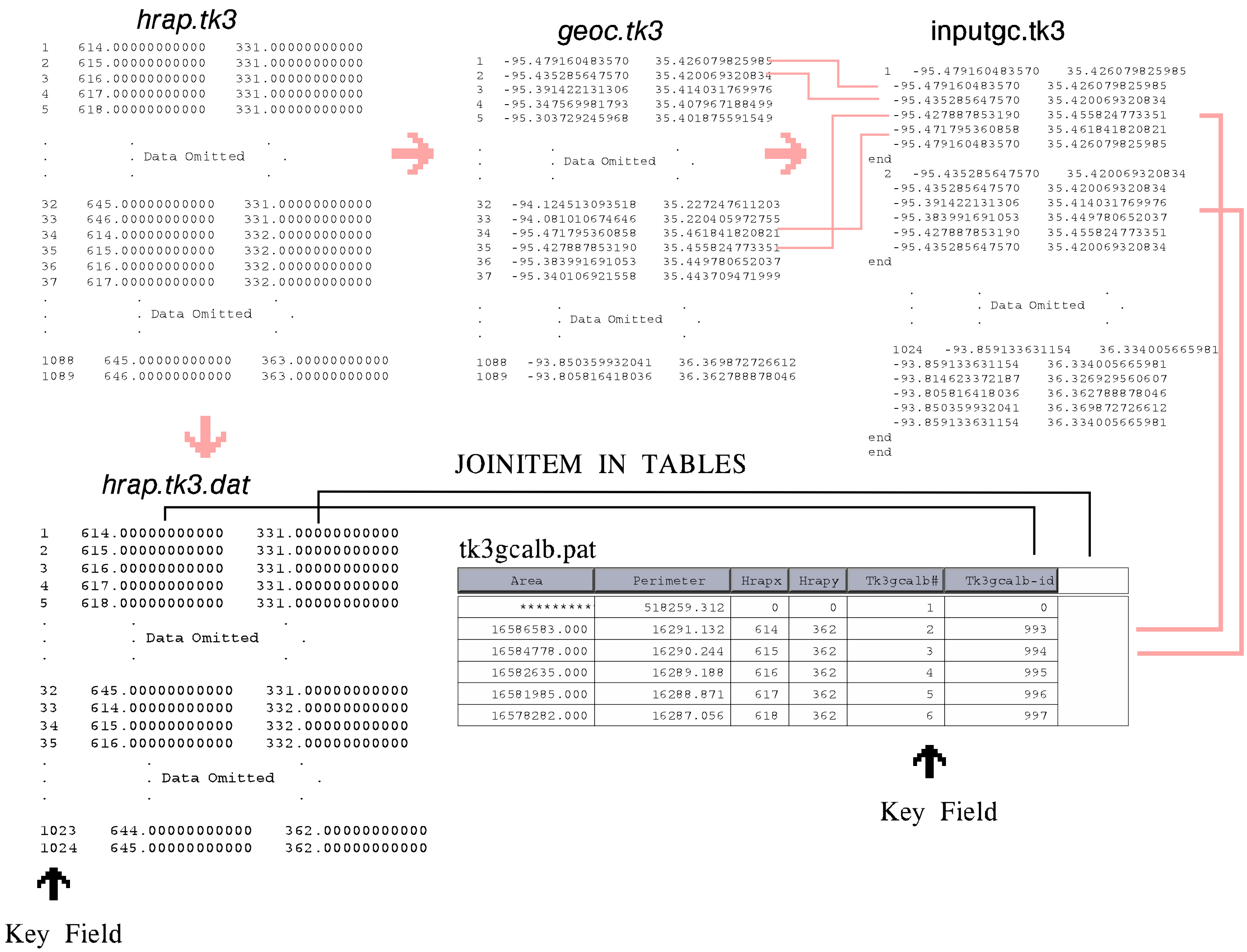


Figure 3.9: USGS Gaging Stations in the HRAP Plane



Area	Perimeter	Hrapx	Hrapy	Tk3gcalb#	Tk3gcalb-id
*********	518259.312	0	0	1	0
16586583.000	16291.132	614	362	2	993
16584778.000	16290.244	615	362	3	994
16582635.000	16289.188	616	362	4	995
16581985.000	16288.871	617	362	5	996
16578282.000	16287.056	618	362	6	997

Figure 3.10: Files Used to Create a Coverage of HRAP Cells