Solutions for Sample Problems, CE 374K Spring 2012

Problem 9.3.1:

9.3.1 Calculate the water velocity V, the kinematic wave celerity c_k, the dynamic wave celerity c_d, and the velocities of propagation of dynamic waves $V \pm c_d$ for the channel described in Example 9.3.1 in the text and flow rates of 10, 50, 100, 500, 1000, 5000, and 10,000 cfs. Plot the results to show the variation of the velocities and celerities as a function of the flow rate.

Solution:

9.3.1.

The calculations are carried out as in Example 9.3.1 in the text with results presented in Table 9.3.1 and Fig. 9.3.1. Water flows downstream with velocity v, shown in Col. (3) of Table 9.3.1, while the bulk of the flood wave moves with kinematic wave celerity c_k, shown in Col. (4). Dynamic waves proceed downstream at velocity $v + c_d$ (shown in Col. 6) and upstream with velocity $-(v - c_d)$ (shown in Col. 7). Because $v - c_d$ is always negative over the range of discharges examined, the flow is subcritical and downstream disturbances can propagate upstream.

Table 9.3.1. Computation of the flow velocity and wave celerities.

Col: (1)	(2)	(3)	(4)	(5)	(6)	(7)
Discharge (cfs)	Depth (ft)	Velocity (ft/sec)	c_k (ft/sec)	c_d (ft/sec)	$V+c_d$ (ft/sec)	$V-c_d$ (ft/sec)
10	0.069	0.72	1.20	1.50	2.22	-0.78
50	0.183	1.37	2.28	2.42	3.79	-1.05
100	0.277	1.81	3.01	2.98	4.79	-1.18
500	0.727	3.44	5.73	4.84	8.28	-1.40
1000	1.101	4.54	7.57	5.96	10.50	-1.41
5000	2.893	8.64	14.40	9.65	18.29	-1.01
10000	4.384	11.40	19.01	11.88	23.29	-0.48

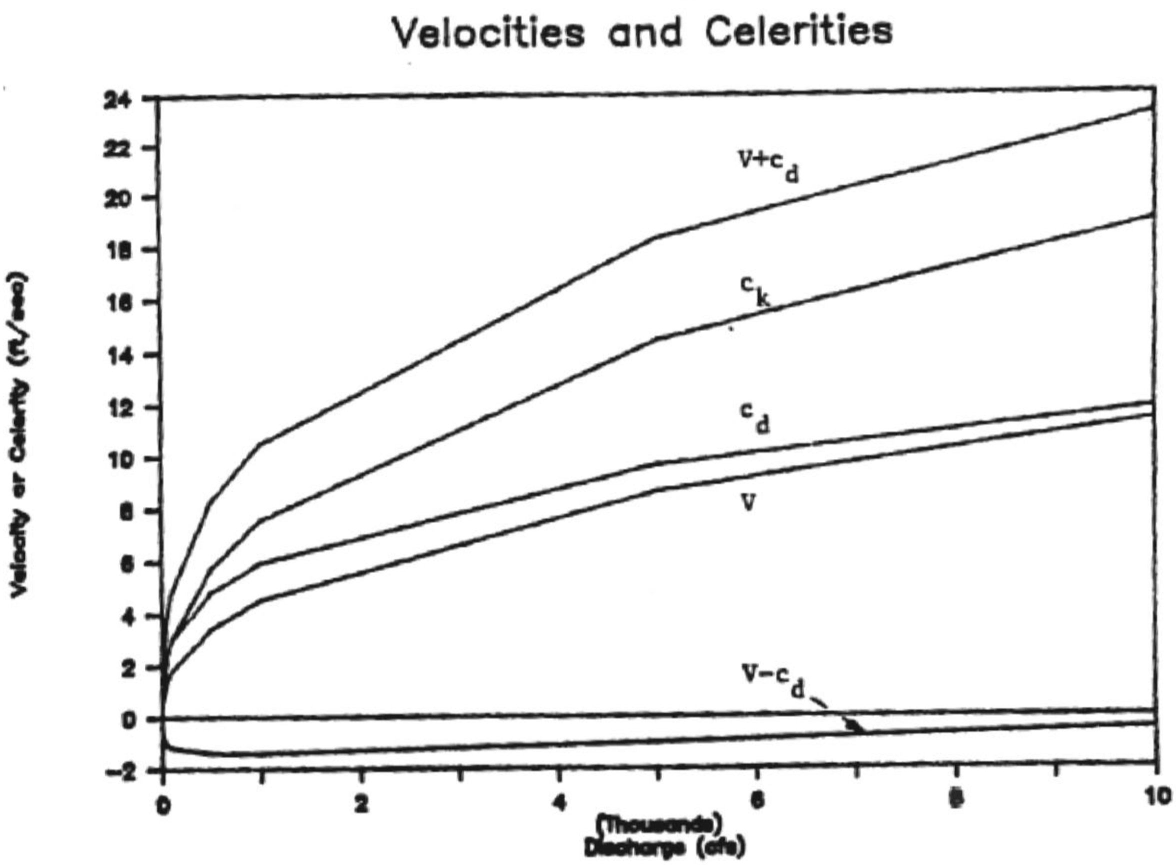

Figure 9.3.1 Flow velocity and wave celerities in a channel

Problem 9.4.2:

9.4.2 Prove that the kinematic wave celerity is $c_k = 5V/3$, where V is the average velocity, when Manning's equation is used to describe the flow resistance in a wide, rectangular channel.

Solution:

9.4.2.

In a wide, rectangular channel, the hydraulic radius is approximately $R = y$, where y is the flow depth. The flow velocity is given by Manning's equation, Eq. (5.6.10) of the textbook

$$V = (\Phi/n)\, y^{2/3} \sqrt{S_f}$$

and the discharge is, with channel width B,

$$Q = Byv = (\Phi/n)\, B\, y^{5/3} \sqrt{S_f}$$

where $\Phi = 1$ for SI units and $\Phi = 1.49$ for English units. Then, the celerity of the kinematic wave, c_k, is

$$c_k = (1/B)(dQ/dy) = (1/B)(\Phi B/n)(5/3)\, y^{2/3} \sqrt{S_f} = (5/3)(\Phi/n)\, y^{2/3} \sqrt{S_f}$$

and, substituting Eq. (9.4.2-1) into the previous equation

$$c_k = (5/3)v$$

Problem 9.4.3

9.4.3 Prove that the travel time T of a kinematic wave in a wide rectangular channel of width B, length L, slope S_o, and Manning roughness n carrying a flow of Q is given approximately by

$$T = \frac{3}{5}\left(\frac{nB^{2/3}}{1.49S_o^{1/2}}\right)^{3/5} Q^{-2/5}L$$

If $B = 200$ ft, $L = 265$ mi, $S_o = 0.00035$, $n = 0.045$, and $Q = 2000$ cfs, calculate the travel time in days.

Solution:

9.4.3.

As shown in Problem 9.4.2, for a wide rectangular channel of width B and slope S_o, the discharge is

$$Q = (\phi/n)\, B\, y^{2/3}\, S_o^{1/2}$$

where $\phi = 1$ for SI units and $\phi = 1.49$ for English units. From the previous equation

$$y = [(Q/n)/(\phi B\, S_o^{1/2})]^{3/5} \qquad (9.4.3\text{-}1)$$

Then, the wave celerity c_k can be written as a function of the discharge, substituting the flow depth into

$$c_k = (5/3)v = (5/3)(\phi/n)\, y^{2/3}\, S_o^{1/2} = (5/3)(\phi/n)[Qn/\phi B\, S_o^{1/2}]^{2/5}\, S_o^{1/2}$$

$$= (5/3)\ [\phi S_o^{1/2} / (nB^{2/3})]^{3/5}\ Q^{2/5}$$

and the travel time is

$$T = L/c_k = (3/5)\ [nB^{2/3}/(\phi\ S_o^{1/2})]^{3/5}\ Q^{-2/5}\ L \qquad (9.4.3\text{-}2)$$

Then with B = 200 ft, L = 265 mi., S_o = 0.00035, n = 0.045 and Q = 2000 cfs

$$T = (3/5)[0.045 \times 200^{2/3}/(1.49 \times 0.00035^{1/2})]^{3/5}(2000^{-2/5})$$

$$\times\ (265)(5280)$$

$$= 445{,}478 \text{ sec} = 123.74 \text{ hrs} = 5.16 \text{ days}$$

Problem 15.6.2

15.6.2 Compute the monthly water balances for the proposed Justiceburg reservoir site near Lubbock, Texas, for the years 1940–1942. Assume the reservoir is initially at a normal conservation storage level, at elevation 2220 ft above MSL (which is also the elevation of the service spillway). The emergency overflow spillway is at elevation 2240 ft above MSL. The elevation–surface area–capacity characteristics are listed below:

Elevation (ft above MSL)	2,130	2,140	2,150	2,160	2,170	2,180
Area (acres)	108	253	506	765	1,046	1,330
Capacity (acre·ft)	608	2,407	6,187	12,515	21,549	33,417
Elevation (ft above MSL)	2,190	2,200	2,205	2,210	2,215	2,220
Area (acres)	1,682	2,045	2,232	2,437	2,651	2,884
Capacity (acre·ft)	48,485	67,065	77,737	89,414	102,108	115,937
Elevation (ft above MSL)	2,225	2,230	2,235	2,240		
Area (acres)	3,197	3,589	4,094	4,784		
Capacity (acre·ft)	131,153	148,069	167,194	189,268		

Consider an annual demand of 26,100 acre·ft with the following demand fractions.

Month	1	2	3	4	5	6
Fraction	0.05	0.05	0.05	0.06	0.07	0.13
Month	7	8	9	10	11	12
Fraction	0.15	0.17	0.09	0.07	0.06	0.05

The net evaporation data are given in Table 15.P.4 and the runoff data (reservoir inflows) are given in Table 15.P.5.

Solution:

15.6.2

The output file for the years 1940-1978 is shown in Table 15.6.3. The first thirty-six values under column 9 of Table 15.6.3 represent the end-of-month storages for the years 1940 through 1942. It can be seen that spills occur in the 1941 and 1942 data when the reservoir reaches maximum capacity of 115,937 ac-ft.

Table 15.6.3 Justiceburg reservoir water balance 1940–1978

DEMAND = Y = 26100.ac-ft per year
THE INITIAL RESERVOIR STORAGE = 115937.ac-ft
RESERVOIR CAPACITY = 115937.ac-ft
MINIMUM STORAGE = 5827. ac-ft IN MONTH 460

WATER BALANCE CALCULATIONS

1	2	3	4	5	6	7	8	9	
MONTH	INFLOW	DEMAND FACTOR	WTHDRWL. RATE	NET EVAP.	SURFACE AREA	EVAP. LOSS	SPILL	STORAGE	
t (Jan @ t=1)	I(t) (ac-ft)	D(t)	Y*D(t) (ac-ft)	E(t) (ft)	A(t) (acres)	A(t)*E(t) (ac-ft)	O(t) (ac-ft)	S(t) (ac-ft)	
INITIAL								115937.	
1	20.	0.050	1305.	0.090	2871.	258.	0.	114394.	
2	190.	0.050	1305.	0.040	2848.	114.	0.	113165.	
3	0.	0.050	1305.	0.460	2815.	1295.	0.	110565.	
4	570.	0.060	1566.	0.370	2776.	1027.	0.	108541.	
5	2650.	0.070	1827.	0.560	2753.	1542.	0.	107823.	
6	5570.	0.130	3393.	0.480	2754.	1322.	0.	108677.	
7	100.	0.150	3915.	1.030	2706.	2787.	0.	102075.	
8	15780.	0.170	4437.	0.580	2733.	1585.	0.	111833.	1940
9	6280.	0.090	2349.	0.790	2829.	2235.	0.	113529.	
10	0.	0.070	1827.	0.390	2819.	1099.	0.	110603.	
11	2910.	0.060	1566.	0.040	2804.	112.	0.	111835.	
12	130.	0.050	1305.	0.160	2801.	448.	0.	110211.	
13	0.	0.050	1305.	0.120	2774.	333.	0.	108574.	
14	870.	0.050	1305.	0.090	2754.	248.	0.	107891.	
15	5530.	0.050	1305.	0.020	2784.	56.	0.	112060.	
16	30830.	0.060	1566.	0.180	2884.	519.	24829.	115937.	
17	68820.	0.070	1827.	-.480	2884.	-1384.	68744.	115937.	
18	21700.	0.130	3393.	0.350	2884.	1009.	17236.	115937.	
19	11210.	0.150	3915.	0.500	2884.	1442.	5823.	115937.	1941
20	5670.	0.170	4437.	0.630	2879.	1814.	0.	115356.	
21	10730.	0.090	2349.	0.180	2884.	519.	7269.	115937.	
22	54660.	0.070	1827.	-.280	2884.	-808.	53804.	115937.	
23	2500.	0.060	1566.	0.260	2884.	750.	184.	115937.	
24	890.	0.050	1305.	0.140	2877.	403.	0.	115119.	
25	250.	0.050	1305.	0.180	2857.	514.	0.	113550.	
26	40.	0.050	1305.	0.250	2827.	707.	0.	111578.	
27	10.	0.050	1305.	0.380	2791.	1060.	0.	109223.	
28	3250.	0.060	1566.	0.080	2783.	223.	0.	110684.	
29	760.	0.070	1827.	0.650	2771.	1801.	0.	107816.	
30	3530.	0.130	3393.	0.500	2737.	1368.	0.	106584.	
31	650.	0.150	3915.	0.620	2685.	1665.	0.	101655.	
32	7360.	0.170	4437.	0.530	2656.	1408.	0.	103176.	1942
33	9400.	0.090	2349.	-.030	2729.	-82.	0.	110303.	
34	13950.	0.070	1827.	0.120	2884.	346.	6142.	115937.	
35	730.	0.060	1566.	0.340	2869.	975.	0.	114126.	
36	1430.	0.050	1305.	-.130	2858.	-371.	0.	114622.	
37	700.	0.050	1305.	0.210	2852.	599.	0.	113418.	
38	40.	0.050	1305.	0.350	2823.	988.	0.	111165.	