# Modeling Traffic's Flow-Density Relation: Accommodation of Multiple Flow Regimes and Traveler Types

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by

**Kara M. Kockelman**
Clare Boothe Luce Assistant Professor of Civil Engineering
The University of Texas at Austin
6.9 ECJ, U.T. Austin
Austin, TX 78712
512-471-0210
FAX: 512-475-8744
kkockelm@mail.utexas.edu

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**Abstract:**

This research investigates freeway-flow impacts of different traveler types by specifying and applying a mixture model of congested and uncongested driving behaviors. Results indicate that mixture models are promising tools for traffic data analysis and that information on travelers, their vehicles, and weather conditions explains significant variation in flow data.

**Introduction and Background:**

It is well accepted that distinct roadways accommodate different levels and patterns of vehicular flow, by virtue of their design. Transportation engineers in the United States rely on the *Highway Capacity Manual* (HCM 1998) to estimate what constitutes capacity for a given roadway and what levels of service are experienced by travelers under different traffic conditions. The methods embodied by this manual depend primarily on a roadway's physical design; however, the fractions of heavy vehicles also explicitly enter the equations. Recently, traveler type has been incorporated as playing a vague role in capacity and flow conditions: engineers are now expected to choose a driver-population adjustment factor level between 0.85 to 1.0, depending on how "efficiently" they expect drivers to use the roadway. Essentially, an engineer's guesswork can produce a fifteen-percent difference in flow estimates. Such differences can have serious repercussions in the design, cost, and operation of roadways. They also substantially affect prediction of variables dependent on traffic conditions, such as emissions, route choices, and network optimization strategies.

The HCM suggests speed-flow curves for freeways with ideal geometries and zero heavy vehicles under "free-flow conditions." Given the trivariate relation for stationary traffic (flow equals space-mean speed times density [Edie 1965]), these imply speed-density and flow-density curves under free-flow conditions. Traffic researchers have long been interested in functionally specifying and estimating these relations (*e.g.*, Greenshield 1935 and Drake *et al.* 1967), but without much behavioral and/or statistical sophistication. Greenshield's (1935) data suggested a linear speed-density relation, leading him to propose a parabolic function as an approximation to the flow-density relation. Other functional forms, based on notions like fluid dynamics and car-following decisions, give rise to a variety of forms. For example, Greenberg (1959) proposed a logarithmic form for speed versus density, Underwood (1961) used an exponential form, and Edie (1961) combined these two to accommodate a clear discontinuity in data near critical densities. Segmentation of congested and uncongested data points is performed exogenously (based on the best guess of the researcher), and estimation relies on ordinary least squares methods.

More recent research has focused on the discontinuities observed across near-capacity data points (*e.g.*, Ceder and May 1976, Payne 1984, Banks 1989, Hall *et al.* 1992, Cassidy 1998). With few exceptions, flow-density-speed models assume a single relation for all travelers, neglecting driver characteristics, vehicle type, and environmental conditions (such as weather). The exceptions include Krauss (1998), who simulates random driver behaviors to investigate traffic variability, and Kockelman (1998), who interacts information on travelers, weather, and vehicle type with density for a least-squares polynomial model of flow. Notably missing from the literature is a model with a strong behavioral basis, convincing stochastic assumptions, and an empirical application.

The work described here improves upon prior models and methods by developing flow-density relations in ways that are fundamentally consistent with behavior and by introducing greater statistical sophistication. The functional forms are derived from free-flow-speed and minimum-spacing choices by distinct driver and vehicle types. The statistical specification is a mixture model of congested and uncongested conditions. And model application is illustrated using a pair of San Francisco Bay Area data sets.

**Model Specification:**

The model relies on two behavioral hypotheses: one for "uncongested", unforced, or "free-flow" traffic conditions; the other for "congested" conditions.[1] These are linked by a segmentation model which recognizes the unobserved character of the traffic regime (uncongested versus congested).

The HCM illustrates speed-flow relations only for ideal, "uncongested" conditions, and these relations are not parametrically specified. By using a parametric structure and appropriate data sets, one can regress flow on various explanatory factors (including, for example, mix of traveler types, weather conditions, and vehicle sizes) to examine which qualities are responsible for variation in observed traffic flows – and to what degree.

To devise a realistic parametric structure, however, one must have a strong behavioral model. Under uncongested conditions, freeway traffic data suggest that speeds are relatively constant and chosen by the drivers. Figure 1 shows flow-versus-density data under congested and uncongested conditions for the second of five northbound lanes on Interstate 80 in Hayward, California. Under stationary conditions, the ratio of flow to density is space-mean speed, and this appears to be nearly constant for the uncongested regime. Thus, these data – as with uncongested data presented elsewhere (*e.g.*, Drake *et al.* 1967, Ceder and May 1976, Koshi *et al.* 1983, and Banks 1989) – are consistent with a constant-speed hypothesis. Figure 1 also suggests possible flow-density relations for aggressive and non-aggressive driver types; a mix of such drivers on the road would lead to observed behaviors lying between these functions.

If each class of driver drives at its desired, "free-flow" speed, the observed uncongested flow-density relationship is a weighted average of the desired speeds. Assuming such behavior, a regression of flow on total density interacted with proportions of distinct driver/vehicle classes "*i*" yields estimates of free-flow speeds for these classes. Equation 1 illustrates this relation.

$$\begin{aligned}
&\text{Under Uncongested Conditions:}\\
&q_U = \sum_i v_{free,i}\, p_i k = \text{Total, uncongested flow,}\\
&\text{where } v_{free,i} = \text{Free}\cdot\text{flow speed for driver/vehicle class "}i\text{",} \qquad (1)\\
&p_i k = \text{Density of driver/vehicle class "}i\text{" per unit length of roadway,}\\
&\text{and } p_i = \text{Proportion of roads' vehicles of driver/vehicle class "}i\text{".}
\end{aligned}$$

Under *congested* conditions, the driving situation is very different: speeds are no longer constant for rising densities, and drivers can no longer choose their free-flow speeds. Instead, each class of driver is able to control the *spacing* at which it follows the preceding traveler. One behavioral assumption is that a class's selected spacing is a linear function of speed. Scatterplots of the inverse of density (the average spacing between vehicles) versus speed using the data illustrated in Figure 1 and elsewhere (*e.g.*, Daganzo 1997) suggest that such an assumption is very reasonable for the congested regime. This implies the following: $s_i = a_i + b_i v$ , where $s_i$ stands for inter-vehicle spacing (front-to-front) of the *i*th class, $v$ is space-mean speed, and $a_i$ and $b_i$ are constants defining the *i*th class's behavior[2]. Since total vehicle density is the inverse of average spacing of vehicles on the roadway and average spacing is a proportion-weighted sum of class densities, one can solve for total density as a function of speed. Notationally:

$$k = \frac{1}{\bar{s}} = \frac{1}{\sum_i p_i s_i} = \frac{1}{\sum_i p_i (a_i + b_i v)}$$

Since flow is simply the interaction of density and space-mean speed (under stationary traffic conditions), these assumptions result in a non-linear functional form for congested flow versus speed and a linear form for congested flow versus density. The linear (and negatively sloped) form is consistent with much data and theory (*e.g.*, the "inverted lambda" hypothesis of Ceder and May [1976], Koshi *et al.* [1983], Payne [1984], and others). Both forms, however, are non-linear in class proportions ($p_i$) and the unknown behavioral parameters ($a_i$ and $b_i$). These results are illustrated by Equation 2.

$$\begin{aligned}&\text{Under Congested Conditions:}\\&q_C = \text{Total, congested flow} = vk = \frac{v}{\bar{a}+\bar{b}v} = \frac{1-\bar{a}k}{\bar{b}},\\&\text{where } \bar{a} = \sum_i p_i a_i \;\&\; \bar{b} = \sum_i p_i b_i.\end{aligned} \quad (2)$$

In comparing Equations 1 and 2, the clear contrast in hypothesized behavior and, therefore, functional form under uncongested and congested states effectively implies two distinct regression models. Unfortunately, many data points are not clearly in one or the other; they straddle both regimes. To resolve the resulting estimation issues, an endogenous segmentation model across the two regimes is very helpful (see, *e.g.*, Maddala 1983 and Bhat 1997). A logit-type segmentation model (Equation 3) was used to probabilistically estimate observation membership in the two regimes, and an iterative search was conducted for the likelihood-maximizing estimators (Equation 4).

$$\Pr(n\text{th observation} \in \text{congested regime}) = \frac{e^{x'\beta}}{1+e^{x'\beta}} \quad (3)$$

Since there is unobserved heterogeneity among travelers within classes and traffic conditions, the uncongested and congested flow predictions involve error terms[3]. In recognition of this, observed flows are assumed to be distributed around behavioral means with *iid* normal error components, producing the following likelihood for any observation *n*:

$$
\begin{aligned}
Lik_n &= \Pr(Flow_n = q_n) \\
&= \Pr(Flow_n = q_n \,|\, \text{Uncong.}) \Pr(\text{Uncong.}) + \Pr(Flow_n = q_n \,|\, \text{Cong.}) \Pr(\text{Cong.}) \\
&= \phi\left(\frac{q_n - \sum_i v_{free,i}\, p_i k}{\sigma_U}\right)\frac{1}{1+e^{x'\beta}} + \phi\left(\frac{q_n - \dfrac{v}{\sum_i p_i a_i + \sum_i p_i b_i v}}{\sigma_C}\right)\frac{e^{x'\beta}}{1+e^{x'\beta}},
\end{aligned}
\tag{4}
$$

where $\phi =$ standard normal density function.

Note that Equation 4's likelihood conditions on density ($k$) for uncongested-traffic estimates and speed ($v$) for congested-traffic estimates. This non-standard set-up is a result of the behavioral assumptions. In reality, the causal structure is not necessarily one-way. For example, downstream bottlenecking typically governs congested flows upstream; spacings (or densities) and speeds then develop simultaneously on the roadway.

**Data Sets and Estimation:**

Estimation of unknown parameters in the two traffic regimes ($a$'s, $b$'s, and $v_{free}$'s) and in the segmentation model ($\beta$'s) was performed after merging three data sets. The data come from the San Francisco Bay Area in the early 1990s. The first consists of 30-second traffic data (including counts, densities, speeds, and vehicle lengths) gathered on a Thursday and a Friday in February 1993. These were collected by paired loop detectors embedded in the second inside lane of a five-lane northbound section of Interstate 880, in Hayward, California[4]. These observations are linked – by time of day – to driver information in the 1990 Bay Area Travel Survey's trip data. The third data set is rainfall data from the National Oceanic and Atmospheric

Administration (NOAA 1993), which indicated the Thursday to be dry and the Friday to be wet. All variables are described in Table 1.

**Results:**

The results of the estimation were produced using GAUSS's maximum likelihood procedure (Aptech 1999)and are shown in Table 2. Models of increasing complexity are illustrated, and the variation falls significantly when introducing classification of driver types in the uncongested regime (Model 2) and when adding explanatory information to the segmentation model (Model 4). On average, it appears that truck drivers and females choose higher free-flow speeds than males (roughly 64 mph vs. 59 mph) and shorter minimum spacings (averaging about 20 feet vs. $34^+$ feet), suggesting more aggressive behaviors. However, the results also indicate that males maintain tighter spacing under congested conditions when speeds increase, exhibiting more aggressive behaviors in the 10 mph to 60 mph range. While the levels of almost all shown parameter estimates are consistent with expectations, the models do not control for several important variables (such as driver age and experience), and they rely on a highly indirect pairing of data sets. The 1990 travel surveys are taken from tens of thousands of households across the region, while the 1993 detector data involve only vehicles in a single lane on a particular section of roadway on specific days. Thus, the travel survey proportions of male and female drivers are gross proxies for the loop-detected population. Upon further disaggregation of the data (by age, as well as gender and truck proportions) many of the resulting estimates were not as intuitive; this is probably due to the error in measurement from coupling diverse data sets, and it may also be due to correlations in traveler characteristics and the congested times of day on this particular roadway.

Since much error arguably arises from measurement error in class proportions, parameter results are expected to be biased toward zero (as is the case in the standard, non-segmented OLS model [Greene 1993]).  Thus, estimates of differences across genders, age groups, and other driver characteristics are likely to be more pronounced in a data set that is able to control for these without measurement errors.

**Extensions to this Work:**

This work can be furthered through data improvements, more flexible stochastic assumptions, and different behavioral assumptions.  This work's coupling of distinct data sets permits illustration of the methodology; however, without a data set that correctly couples actual driver characteristics with loop-detected information, the estimated coefficients may be biased toward zero and behavioral implications may not be valid.  Unfortunately, it will not be inexpensive to collect the data needed for this enhanced resolution.

In terms of stochastic flexibility, modelers may desire non-negative, integer flow estimates and heteroskedasticity within the two behavioral models; such an extension may be realized using a negative binomial regression model, where average rates are as illustrated in Equations 1 and 2.  Additionally, the assumption of independent error terms in the two behavioral models is not consistent with serial, loop-detector data in the presence of lagged, random effects.  There may also be correlation in unobserved information across the three model segments (engendering simultaneity and, as referred to in Maddala 1983, "endogenous switching"); for example, a line of trucks in an adjacent lane may be unobserved but would increase the probability of congested behavior and reduce expected flows under either behavioral model.

Due to their complexities, these more flexible stochastic specifications require more demanding estimation techniques, such as set generation, simulated likelihood or integral approximation. For example, inclusion of auto-regressive normally-distributed dependencies of the first degree in each of the behavioral models entails the following:

$$\Pr(\vec{q} = \vec{q}_{obs'd}) = \sum_{s} \begin{bmatrix} \Pr(\text{subset m} = \text{uncongested})\phi_U\left(\vec{q}_m = \vec{q}_{m,obs'd}\right) \\ \Pr(\text{remaining subset } \overline{\text{m}} = \text{congested})\phi_C\left(\vec{q}_{\overline{m}} = \vec{q}_{\overline{m},obs'd}\right) \end{bmatrix}$$

where $s$ = set of combinations of congested/uncongested points,

and $\phi(\ )$ = multivariate normal density function, (5)

$$\text{with variance - covariance matrix:} \quad \Omega_i = \sigma_i^{\,2} \begin{bmatrix} 1 & \rho_i & \rho_i \\ \rho_i & \ddots & \rho_i \\ \rho_i & \rho_i & 1 \end{bmatrix},$$

(where $i$ denotes uncongested or congested).

To apply Equation 5's AR(1) set-up with just 500 data points (rather than the 2621 used here), one would need to generate on the order of 2.6E120 combinations – and thus probabilities – for the set. This level of computation is not currently practical for most modelers, but it is feasible.

Behavioral modifications to the model presented here appear more practical in the near term. Additional traffic regimes – for example, "transition" regimes between congested and uncongested conditions and/or non-stationary-traffic behaviors – may be incorporated via a multinomial segmentation model. And incorporation of less linear behavior (in free-flow-speed and spacing choices) may prove more realistic, particularly on lower-level facilities where flow-density curves may exhibit more curvature.

**Conclusions:**

This work illustrates the methodology and value of latent traffic-regime segmentation and driver-class identification for purposes of modeling traffic flows. Recognition of traveler characteristics permits assessment of the impacts that different traveler types have on traffic conditions. Here the development of flow-density-speed relations from believable behavioral bases under congested and uncongested conditions and the allowance for endogenous segmentation reveal several traffic behaviors and facilitate more reliable traffic prediction and improved roadway design. The empirical results suggest that significant differences in free-flow and car-following behavior exist across traveler classes.

Ultimately, the methods illustrated here permit researchers to examine such questions as the effects of an aging driver population on the capacity of our nation's roadways and to what extent more experienced drivers and/or smaller vehicles offset these. Such work also supports modifications to the *Highway Capacity Manual*, changes in roadway design, and greater realism in traffic simulation. Model improvements like these are expected to produce more reliable estimates of roadway use and traveler behavior, allowing engineers and planners to better accommodate present and future demands on transportation facilities through more optimal design.

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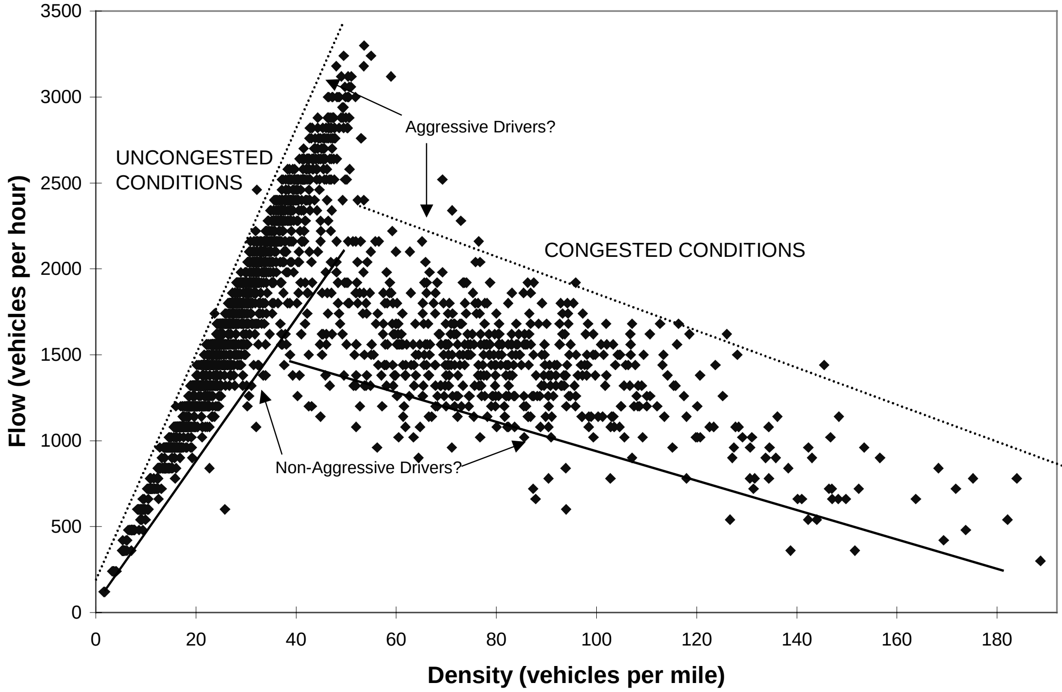


# FIGURE 1. Flow versus Density

| | | |
|---|---|---|
| **Dependent Variable:** | | |
| | Flow ($q$) | Count of vehicles in a 30-second interval |
| **Explanatory Variables:** | | |
| | Density ($k$) | Density computed for 30-second interval [vehicles per lane mile] |
| | Speed ($v$) | Space-mean speed of detected vehicles (via harmonic averaging of spot speeds) [mph] |
| | Male | Fraction of drivers on included trips who are male |
| | Female | Fraction of drivers on included trips who are female |
| | Truck | Fraction of counted vehicles with length > 20 feet (in 30-second interval) |
| | Rain | Indicator variable for rain falling in the vicinity during the hour |

*Note:* "Included trips" involve a personal vehicle (cars, light-duty trucks, vans, and motorcycles), are at least 2.5 Euclidean miles long (as measured between origin and destination Census-tract centroids), and had some portion occuring during the 15-minute interval used for computing the male/female proportions. To estimate the proportions, trips were weighted by their time lengths ocurring in the interval; however, these do not include the first three and last three minutes of each trip (which are assumed to occur on local roads).

## TABLE 1. Description of Variables

| | | Variable: | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|---|---|
| **Segmentation Logit:** | **Beta "$\beta$"** | Constant or Male* | -0.6168 | -0.6269 | -0.6288 | -10.44 |
| | | Female | | | | -12.90 |
| | | Truck | | | | -11.49 |
| | | Density | | | | 0.2308 |
| | | Rain | | | | 1.984 |
| **Congested Behavior:** | **Min. Spacing "*a*"** [feet] | Constant or Male* | 22.46 | 22.27 | 34.34 | 57.93 |
| | | Female | | | 7.117† | -8.437† |
| | | Truck | | | 27.08† | 33.42† |
| | **Add'l Space "*b*"** [feet per 10 mph] | Constant or Male* | 24.09 | 24.21 | 13.89 | 4.572† |
| | | Female | | | 37.11 | 40.84 |
| | | Truck | | | 26.89† | 25.36 |
| **Uncongested Behavior:** | **Free-flow Speed "$v_{free}$"** [mph] | Constant or Male* | 61.56 | 58.88 | 59.37 | 59.37 |
| | | Female | | 64.92 | 64.41 | 64.23 |
| | | Truck | | 64.22 | 64.06 | 63.28 |
| | $\sigma_{cong}$ | | 243.4 | 241.4 | 241.4 | 220.2 |
| | $\sigma_{uncong}$ | | 98.00 | 97.51 | 96.63 | 106.1 |
| | Loglik | | -17762.1 | -17743.5 | -17738.2 | -16743.6 |
| | dof | | 6 | 8 | 12 | 16 |
| | LRI | | 0.1214 | 0.1223 | 0.1226 | 0.1718 |
| | *N* = 2621 | | | | | |

* Where only one parameter is estimated, the variable is a constant. Where more than one has been estimated, the first corresponds to the fraction of males driving long trips during that period.

† *All variables are highly statistically significant – except* those six carrying this symbol.

## TABLE 2. Empirical Results

## ENDNOTES:

[1] These definitions of "uncongested" and"congested" differ from those used by some researchers, who label "congested" those conditions under which addition of traffic produces reductions in speed.

[2] At jam densities, speed is zero, so $a_i$ can be thought of as the inverse of jam density and may be expected to be about 20 feet per vehicle. (Note: The average length of light-duty vehicles sold in 1997 is about 17 feet [Wards 1998].) As speed increases – but traffic remains forced/congested, one may expect a spacing increase of one vehicle's length for every 10 mph increase in speed (a rule of thumb often given to new drivers); such behavior results in a $b_i$ value of roughly 17 feet/10 mph. These values are very similar to the results estimated and shown in Table 2.

[3] In the present application, most of the error arguably arises from measurement error, since class proportions are estimated from the 1990 Bay Area Travel Surveys. This form of error is not accommodated by the *iid* normal assumption used here, and parameter results are likely to be biased toward zero (as is the case in the standard, non-segmented OLS model [Greene 1993]).

[4] The inside lane was designated as a high-occupancy lane during rush hours, so its data were not chosen. The nearest ramp is 0.23 miles downstream from recording detectors, so merging movements are not expected to be significant in this second of five lanes.