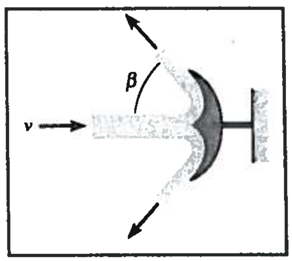
Example 6.5 shows how to calculate the force on an axisymmetric vane, which redirects the flow in the radial direction.

EXAMPLE 6.5 FORCE ON AN AXISYMMETRIC VANE

As shown in the figure, an incident jet of fluid with density ρ, speed v, and area A is deflected through an angle β by a stationary, axisymmetric vane. Find the force required to hold the vane stationary. Express the answer using ρ, v, A, and β. Neglect the influence of gravity.

Sketch: Gravitational effects are negligible.



Problem Definition

Situation: Fluid deflected by axisymmetric vane.

Find: Force required to hold vane stationary.

Assumptions:

1. Flow is steady.
2. Fluid is incompressible.
3. Viscous effects are negligible.

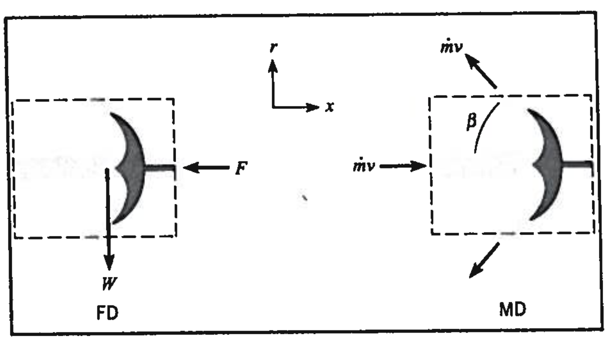
Plan

Because the pressure is constant, the Bernoulli equation shows the inlet and outlet speeds are the same. Application of the continuity equation shows the inlet and outlet mass flows are also the same.

1. Select a control volume with the constraining force on control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the component form of the momentum equation in x-direction, Eq. (6.7a).
5. Evaluate force terms.
6. Evaluate momentum terms.
7. Calculate force.

Solution

1. Control volume selected is shown. Control volume is stationary.



2. The force diagram shows only one force.
3. The momentum diagram shows one momentum flux in and one axisymmetric flux out. The net radial flux of momentum is zero, so only the component in the axial direction contributes to the momentum flux.
4. Momentum equation in x-direction.

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho\, d\mathcal{V} + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

5. Sum of forces

$$\sum F_x = -F$$

6. Evaluation of momentum terms
   - Accumulation term for stationary control volume is

$$\frac{d}{dt}\int_{cv} v_x \rho\, d\mathcal{V} = 0.$$

   - Momentum outflow is $\sum_{cs} \dot{m}_o v_{ox} = -\dot{m} v \cos\beta$.
   - Momentum inflow is $\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v$.

7. Force on vane

$$-F = -\dot{m}v(1 + \cos\beta)$$

$$F = \dot{m}v(1 + \cos\beta)$$

Apply mass flow rate equation, $\dot{m} = \rho A v$,

$$\boxed{F = \rho A v^2(1 + \cos\beta)}$$

and the direction of this force is to the left, as shown in the force diagram.

Review

This type of reverse flow vane is used to reverse thrust on aircraft engines.