$$\boxed{\frac{d}{dt}\int_{c.v.}\rho\, d\forall + \int_{c.s.}\rho\,\vec{V}\cdot d\vec{A} = 0}$$

equ. 5.24 of Textbook

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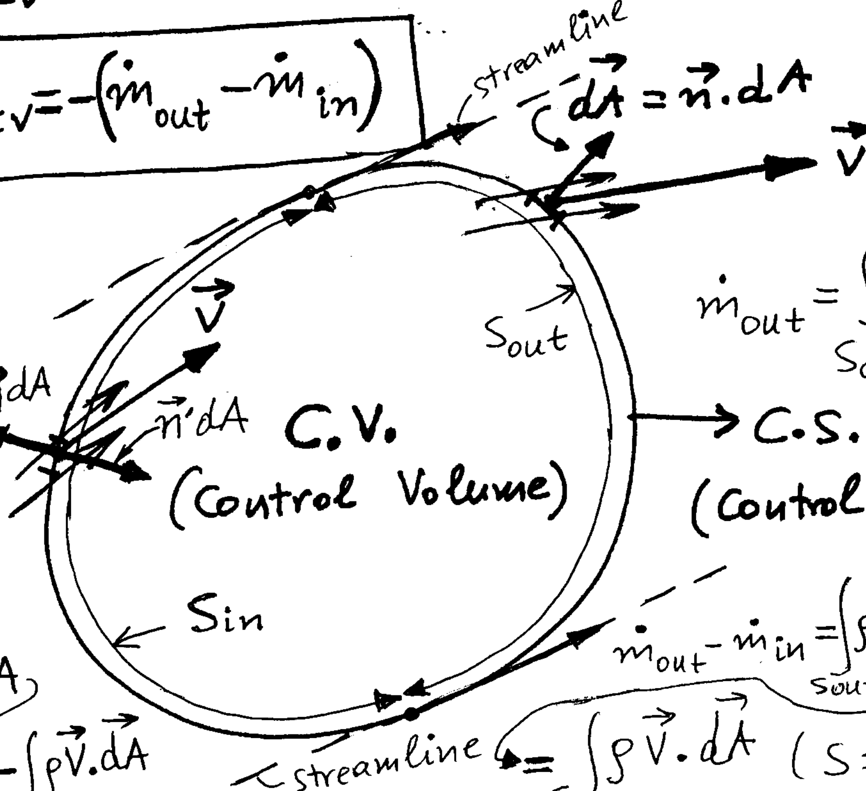
Integral form of continuity equation

- very general
- applies to steady & unsteady flow
- applies to compressible & incompressible flow

or $\dot{m}_{cv} + \dot{m}_{out} - \dot{m}_{in} = 0$

or $\boxed{\dot{m}_{cv} = -(\dot{m}_{out} - \dot{m}_{in})}$

$\vec{n}' = -\vec{n}$



$$\dot{m}_{out} = \int_{S_{out}}\rho\,\vec{V}\cdot d\vec{A} = \int_{S_{out}}\rho\,\vec{V}\cdot\vec{n}\, dA$$

$$\dot{m}_{in} = \int_{S_{in}}\rho\,\vec{V}\cdot\vec{n}'\, dA = -\int_{S_{in}}\rho\,\vec{V}\cdot\vec{n}\, dA = -\int_{S_{in}}\rho\,\vec{V}\cdot d\vec{A}$$

$$\dot{m}_{out} - \dot{m}_{in} = \int_{S_{out}}\rho\,\vec{V}\cdot d\vec{A} - \left(-\int_{S_{in}}\rho\,\vec{V}\cdot d\vec{A}\right) = \int_{S}\rho\,\vec{V}\cdot d\vec{A} \quad (S = S_{out} + S_{in})$$

$\int_{C.S.}\rho\,\vec{V}\cdot d\vec{A} = \dot{m}_{out}$ (rate of outflow out of the C.V.) $- \dot{m}_{in}$ (rate of inflow into C.V.)

$\int_{C.V.}\rho\, d\forall = m_{cv}$ (mass inside C.V.)

$\dot{m}_{cv} = \dfrac{dm_{cv}}{dt}$ = (rate with which mass is increasing in the c.v.)