Supplement to notes on Reynolds Transport Theorem:

(Revised 4/12/07)

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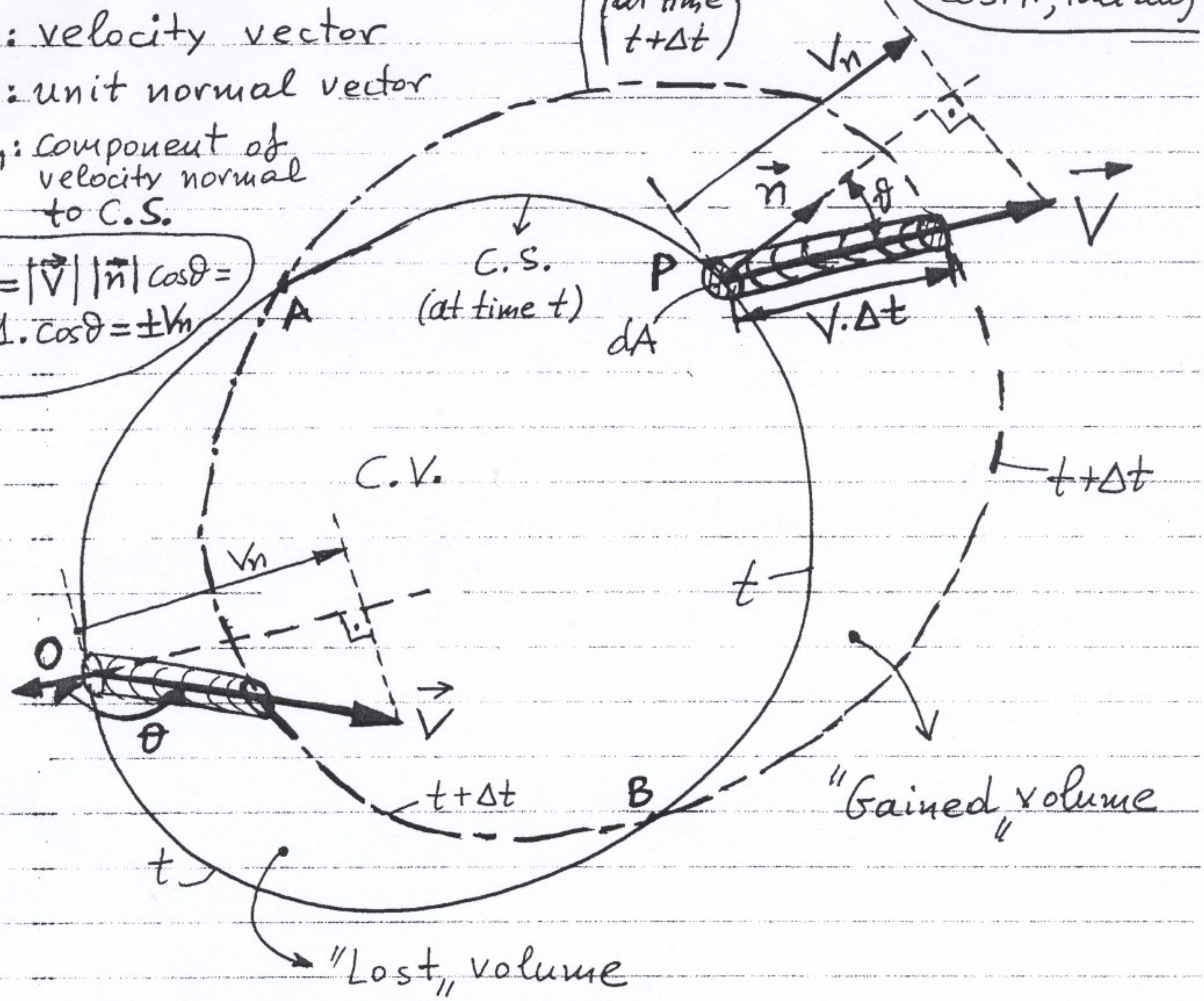
CE319F, Fall 2000

$\vec{V}$: velocity vector

$\vec{n}$: unit normal vector

V_n: component of velocity normal to C.S.

$$\vec{V}\cdot\vec{n} = |\vec{V}|\,|\vec{n}|\cos\theta = V \cdot 1 \cdot \cos\theta = \pm V_n$$



Gained B_{sys} within time $\Delta t = \int_{\text{Gained Volume}} b\rho\,(dVol)$

$dVol$ = volume of shown elementary prism $= dA \times (\text{height of prism}) =$

$= dA \times (V\cdot\Delta t)\cos\theta = dA\cdot\Delta t\cdot\vec{V}\cdot\vec{n} = (\vec{V}\cdot d\vec{A})\,\Delta t$

$d\vec{A}$ is defined as $(dA\,\vec{n})$

Gained $B_{sys} = \int_{APB} b\rho\,\vec{V}\cdot d\vec{A}\,\Delta t = \int_{APB} b\rho V_n\,dA\,\Delta t$

Similarly: Lost $B_{sys} = -\int_{AOB} b\rho\,\vec{V}\cdot d\vec{A}\,\Delta t$ ("−" sign because $\vec{V}\cdot d\vec{A} = \vec{V}\cdot\vec{n}\,dA < 0$)

Finally (assuming steady flow):

$$\frac{\Delta B_{sys}}{\Delta t} = \frac{(\text{Gained } B_{sys}) - (\text{Lost } B_{sys})}{\Delta t} = \int_{C.S.} b\rho\,\vec{V}\cdot d\vec{A}$$