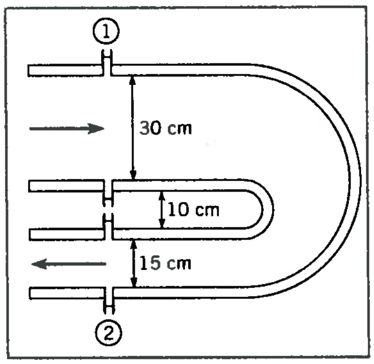
Example 6.7 illustrates how to calculate the force required to restrain a reducing bend, a bend in which the fluid speed and direction both change, by application of the component form of the momentum equation.

EXAMPLE 6.7 WATER FLOW THROUGH REDUCING BEND

Water flows through a 180° reducing bend, as shown. The discharge is $0.25\ \text{m}^3/\text{s}$, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is $0.10\ \text{m}^3$, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500 N. The water density is $1000\ \text{kg}/\text{m}^3$. The bend is in the vertical plane.

Sketch:



Problem Definition

Situation: Water flow through reducing bend.

Find: Force (in newtons) required to hold bend in place.

Assumptions:

1. The Bernoulli equation is valid.
2. Neglect pipe wall thickness.

Properties: $\rho = 1000\ \text{kg/m}^3$.

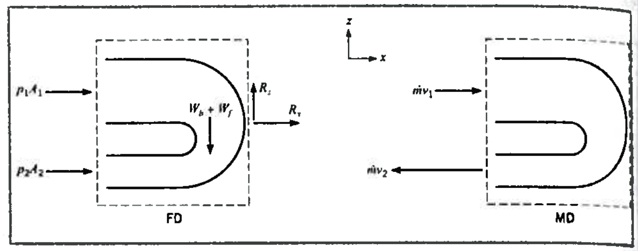
Plan

The flow is steady, so $Q_1 = Q_2 = Q$.

1. Select control volume that encloses bend and the reaction force acts on the control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the component form of the momentum equation in the x- and z-directions, Eqs. (6.7a) and (6.7c).
5. Evaluate the force terms.
6. Evaluate the momentum terms.
7. Solve momentum equations for reaction forces.
8. Calculate the inlet and outlet speed.
9. Apply the Bernoulli equation to find the outlet pressure.
10. Calculate the reaction force.

Solution

1. The control volume selected is shown. The control volume is stationary.



2. There are two forces due to pressure and a reaction force component in the x-direction, and there are weight and reaction forces component in the z-direction.
3. There is inlet and outlet momentum flux in x-direction
4. Momentum equations in x- and z-directions

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho\, dV + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho\, dV + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz}$$

5. Summation of forces in x- and z-directions

$$\sum F_x = p_1 A_1 + p_2 A_2 + R_x$$

$$\sum F_z = R_z - W_b - W_f$$

6. Evaluation of momentum terms
   - Accumulation terms, steady flow

     $\frac{d}{dt}\int_{cv} v_x \rho\, dV = 0$, and $\frac{d}{dt}\int_{cv} v_z \rho\, dV = 0$.
   - Momentum outflow

     $\sum_{cs} \dot{m}_o v_{ox} = \dot{m}(-v_2) = -\rho Q v_2$, and $\sum_{cs} \dot{m}_o v_{oz} = 0$.
   - Momentum inflow

     $\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_1 = \rho Q v_1$, and $\sum_{cs} \dot{m}_i v_{iz} = 0$.

7. Solution for reaction forces

- x-direction

$$p_1A_1 + p_2A_2 + R_x = -\rho Q(v_2 + v_1)$$

$$R_x = -(p_1A_1 + p_2A_2) - \rho Q(v_2 + v_1)$$

- z-direction

$$R_z = W_b + W_f$$

8. Inlet and outlet speeds

$$v_1 = \frac{Q}{A_1} = \frac{0.25\ \text{m}^3/\text{s}}{\pi/4 \times 0.3^2\ \text{m}^2} = 3.54\ \text{m/s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.25\ \text{m}^3/\text{s}}{\pi/4 \times 0.15^2\ \text{m}^2} = 14.15\ \text{m/s}$$

9. Outlet pressure (the Bernoulli equation between sections 1 and 2)

$$p_1 + \frac{\rho v_1^2}{2} + \gamma z_1 = p_2 + \frac{\rho v_2^2}{2} + \gamma z_2$$

From diagram, neglecting pipe wall thickness, $z_1 - z_2 = 0.325$ m.

$$p_2 = p_1 + \frac{\rho(v_1^2 - v_2^2)}{2} + \gamma(z_1 - z_2)$$

$$= 150\ \text{kPa} + \frac{(1000)(3.54^2 - 14.15^2)\text{Pa}}{2}$$

$$+ (9810)(0.325)\text{Pa}$$

$$= 59.3\ \text{kPa}$$

10. Reaction force

- Pressure forces

$$p_1A_1 + p_2A_2 = (150\ \text{kPa})(\pi \times 0.3^2/4\ \text{m}^2)$$

$$+ (59.3\ \text{kPa})(\pi \times 0.15^2/4\ \text{m}^2)$$

$$= 11.6\ \text{kN}$$

- Momentum flux

$$\rho Q(v_2 + v_1) = (1000\ \text{kg/m}^3)(0.25\ \text{m}^3)$$

$$\times (14.15 + 3.54)(\text{m/s})$$

$$= 4420\ \text{N}$$

- Reaction force components

$$R_x = -(11.6\ \text{kN}) - (4.42\ \text{kN})$$

$$= \boxed{-16.0\ \text{kN}}$$

$$R_z = W_b + W_f$$

$$= 500\ \text{N} + (9810\ \text{N/m}^3)(0.1\ \text{m}^3)$$

$$= \boxed{1.48\ \text{kN}}$$