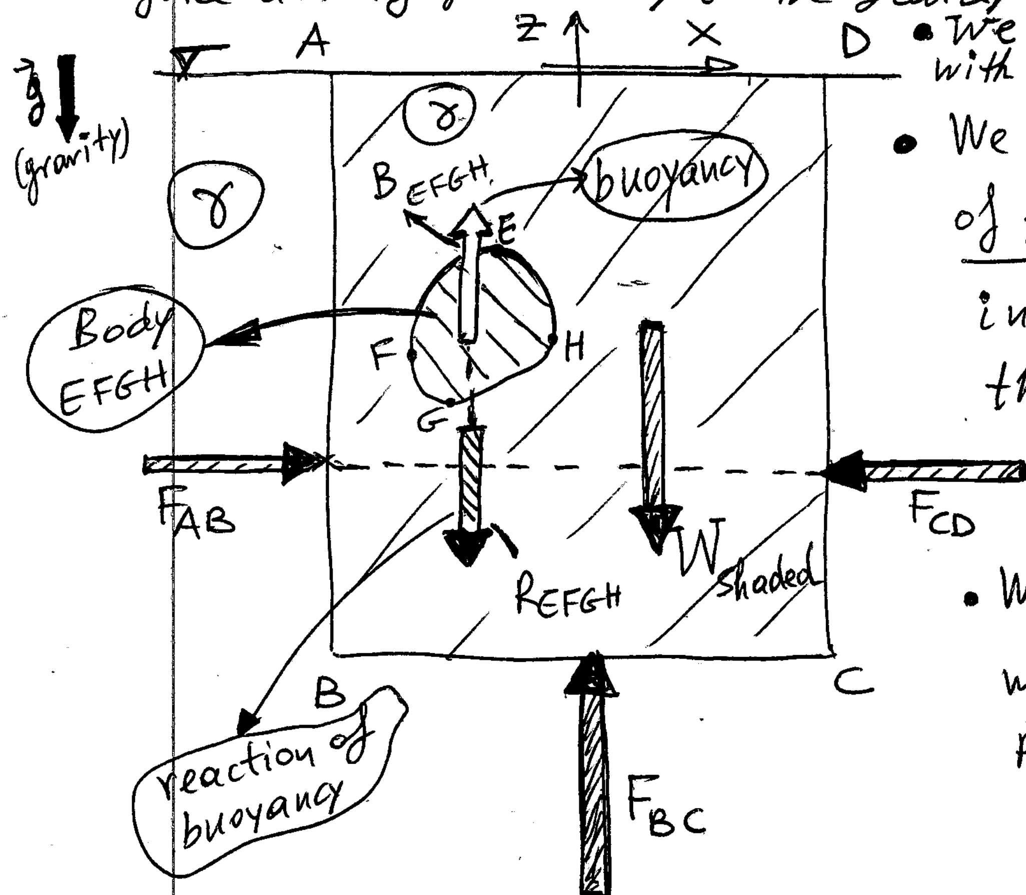
CE319F Archimede's Principle: ©S.A.Kinnas, 2012

We wish to determine the hydrostatic force (buoyancy) acting on submerged body EFGH. We call this force B_{EFGH}. R_{EFGH} is the (equal and opposite) reaction of B_{EFGH} (i.e. the force acting from body to the fluid)



- We surround the body with rectangular domain ABCD
- We call shaded part of fluid the fluid inside ABCD, excluding the part occupied by the body EFGH.
- We assume 2-D case with width b (Note the presented proof can be extended into 3-D)

- We apply equilibrium on shaded part of fluid:

▶ $\Sigma F_x = 0 \Rightarrow F_{AB} - F_{CD} + R^x_{EFGH} = 0$ (1)

Since $F_{AB} = F_{CD}$ (forces acting on sides AB & CD of ABCD, are like forces acting on "gates" AB & CD) and have the same line of action.

(1) $\Rightarrow R^x_{EFGH} = 0 \Rightarrow B^x_{EFGH} = 0$ (i.e. buoyancy has no horizontal component)

▶ $\Sigma F_z = 0 \Rightarrow F_{BC} - W_{shaded} - R_{EFGH} = 0$ (2)

$F_{BC} = p_{BC}(BC)(b) = \gamma(AB)(BC)(b)$ = weight of fluid inside ABCD including that displaced by the body, $= W_{ABCD}$

$W_{shaded} = W_{ABCD} - W_{EFGH}$ → weight of fluid displaced by body

(2) $\Rightarrow R_{EFGH} = W_{EFGH} \Rightarrow$ $\boxed{B_{EFGH} = W_{EFGH}}$ $\Rightarrow$ $\boxed{\text{buoyancy} = \text{weight of displaced fluid}}$

▶ $\Sigma M = 0 \Rightarrow$ Buoyancy applies at centroid of displaced fluid.