

The Use of Computer Applications for Teaching Structural Reliability

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Abstract

It is becoming increasingly common that practicing engineers are being called upon to design buildings and other civil infrastructure systems with an underlying “performance-based” objective. Then, merely *using* design codes (even if they are reliability-based) will not be adequate in verifying performance against different limit states. The teaching of courses in structural reliability often follows a traditional lecture format that leaves the student with an unclear view of the distinctions between the various load- and resistance-related parameters involved in the design process. The ability to include computer applications in such structural reliability courses can greatly help in understanding the fundamental concepts, in the presentation of real-world numerical examples, and in clarifying the role of different variables in safety and performance.

Structured teaching modules are demonstrated here for a computer-based structural reliability course. From a pedagogical viewpoint, these examples might help the student to learn: (i) the role of uncertainty in engineering failures; (ii) the use of analytical tools – such as simulation and optimization techniques – for assessing the reliability of structural components.

Curriculum elements of a traditional structural reliability course such as the first-order reliability methods (FORM), Monte Carlo simulation, and the development of the LRFD (Load and Resistance Factor Design) methodology are the focus of the teaching modules presented. Illustrative examples developed using LabVIEW are presented.

It is expected that the development of a semester-long course using teaching modules similar to those presented here could provide the

necessary introduction to civil engineering students interested in studying structural reliability.

Introduction

Structural reliability is concerned with the ability of a structure to function according to its intended purpose. Traditionally, structures are designed to meet or be checked against one or more limit states of performance. Design philosophy in current times has been increasingly based on reliability. This generally requires “checking” designs against relevant limit states. An established format for such checking is the so-called LRFD methodology employed in structural design codes (e.g., ACI (1995), AISC(1995)). In this design format, load and resistance factors are applied to nominal (conservative) load and resistance values that implicitly guarantee a prescribed safety margin against a limit state of performance. In Teaching Module 1 presented here, we discuss the LRFD methodology and explain how load and resistance factors may be derived for a specified/target reliability.

In contrast to the code-based approach to reliability analysis, engineers are sometimes called upon to assess the reliability of a structural component or system by explicit computations that incorporate uncertainties in all variables that affect performance. This is especially true for structures other than conventional buildings (e.g., for offshore platforms). Then, it is necessary to be able to employ analytical techniques that utilize probabilistic/statistical information on key random variables affecting load and resistance quantities. Two such techniques are the Monte Carlo simulation method and the first-order reliability method (FORM). In Teaching

Modules 2 and 3 presented here, these two methods are applied to two different structural reliability problems. In one case, the ultimate limit state of a steel beam under flexure is considered; in the other, a fatigue limit state based on a fracture mechanics approach is considered for a specific component of a steel bridge.

The three examples presented here represent teaching modules that may usefully be incorporated into a course on structural reliability and may be taught using a computer-based approach. Such an approach offers the student a hands-on understanding of the reliability problem being studied. The LabVIEW software, employed here for the teaching modules, with its built-in numerical libraries and user interfaces represents a convenient means for teaching this material.

Teaching Module 1

LRFD Methodology and Derivation of Load and Resistance Factors

A quantitative description of the uncertainty in load- and resistance-related parameters can lead to the derivation of load and resistance factors for use in design checking equations. Target reliability levels may be specified and such factors computed. Design codes employ such load and resistance factors to meet safety (ultimate), serviceability or durability limit states. In an ultimate limit state, for example, a structure may be required to be designed such that its strength or resistance is greater than the effects of all applied loads.

Let S and R be the applied load and the resistance, respectively, of a structure to be

designed. An example of the description of the uncertainty in S and R is shown in Figure 1. For this illustration, we will assume that S and R are independent normal random variables with mean values, μ_S and μ_R , respectively, and standard deviations, σ_S and σ_R , respectively. Nominal load and resistance levels, S_N and R_N , are conservative levels chosen for design and are typically 2-3 standard deviations from their mean values. A nominal safety factor, SF_{nominal} , may then be given as

$$SF_{\text{nominal}} = \frac{R_N}{S_N} > 1 \quad (1)$$

The overlap area between the curves representing probability density functions, $f_S(s)$ and $f_R(r)$, is directly related to the probability of failure of the structure. Clearly, the probability of failure depends on the distribution of S and R and on the parameters, μ_S , μ_R , σ_S and σ_R . We can define a safety margin, Z , as follows:

$$Z(S, R) = R - S \quad (2)$$

If we assume that S and R are statistically independent (normal) random variables, Z will also be described by a normal distribution with mean value equal to $\mu_R - \mu_S$ and standard deviation, $(\sigma_R^2 + \sigma_S^2)^{1/2}$.

The probability of failure, which is also equal to the probability that Z is less than zero, may be given in terms of the reliability index, β , as follows:

$$P_f = 1 - \Phi(\beta) \quad (3)$$

If we define a parameter, ε , as follows:

$$\varepsilon = \frac{\sqrt{(\sigma_R^2 + \sigma_S^2)}}{\sigma_R + \sigma_S}, \quad (4)$$

the reliability index, β , can be written in terms of ε as:

$$\beta = \frac{\mu_R - \mu_S}{\varepsilon(\sigma_R + \sigma_S)} \quad (5)$$

It is usually the case that design checking is done by defining nominal loads and resistances at conservative levels (relative to their mean values). We define two quantities, k_S and k_R , to define the degree of conservatism in these nominal values. A value of 2 for k_S implies that the nominal load level is 2 standard deviations above its mean value. The nominal load and

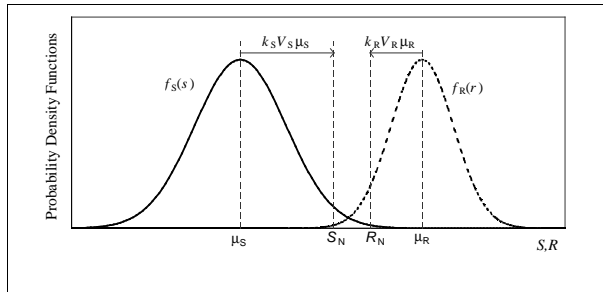


Figure 1 Probability Density Functions for the random variables, S and R , in Teaching Module 1.

resistance values may be given as follows (where $V_S = \sigma_S/\mu_S$ is the coefficient of variation on the load, S , and $V_R = \sigma_R/\mu_R$ is the coefficient of variation on the resistance, R):

$$S_N = \mu_S (1 + k_S V_S) \quad (6a)$$

$$R_N = \mu_R (1 - k_R V_R) \quad (6b)$$

In the traditional LRFD format for design checking, a resistance factor, ϕ , and a load factor, γ , are applied to the nominal resistance, R_N , and the nominal load, S_N , respectively. These factors may be derived in terms of various quantities defined above. Thus, we have:

$$\phi = \frac{1 - \epsilon \beta V_R}{1 - k_R V_R} \quad (7)$$

$$\gamma = \frac{1 + \epsilon \beta V_S}{1 + k_S V_S} \quad (8)$$

LabVIEW Illustration of LRFD Methodology

Consider a structure where the load, S , and the resistance, R , are normal random variables. The load is such that $\mu_S = 50$ kips and $V_S = 0.25$, and it is known that $V_R = 0.1$. Assuming that $k_S = 2$ and $k_R = 2$ for defining nominal levels, we are interested in determining load and resistance factors for a target probability of failure. All the relevant parameters are input in LabVIEW. Load and resistance factors may then be calculated as shown in Figure 2.

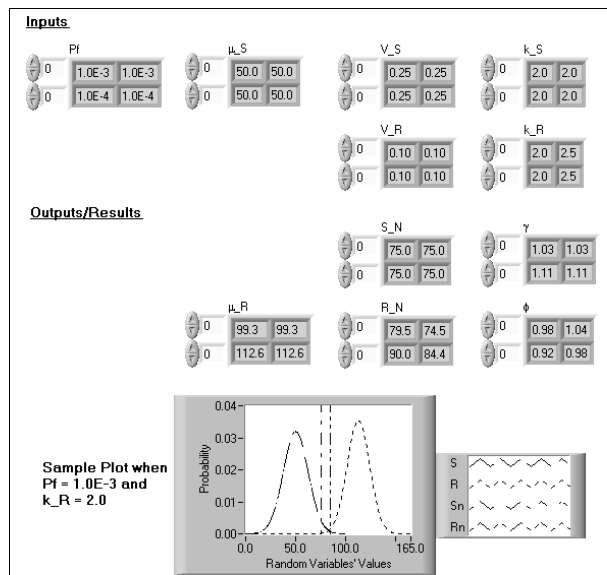


Figure 2 Inputs and outputs from LabVIEW for Teaching Module 1.

From this illustration, the student can gain the following insights:

- The derived resistance and load factors are closely related to the values of k_R and k_S selected to define the nominal values. For the same reliability, a small value of k_R , which might indicate a nominal resistance that is not conservative enough, will require a low resistance factor.
- A change in the target reliability will lead to corresponding changes in the nominal load and resistance or in the derived factors. For example, if the nominal value for S is the same for designs that must guarantee a 10^{-4} probability of failure compared to those for a 10^{-3} value, the associated resistance factor and nominal resistance have to be more conservatively defined as can be seen in Figure 2.

From this LabVIEW application, students can immediately see how the various input parameters affect the nominal capacity reduction factor and nominal load factor in LRFD codes. Thus, the reliability basis of the entire design process is rendered transparent and understandable to the student.

Teaching Module 2

Ultimate Limit State of a Steel Beam in Flexure

In this module, a steel beam in flexure is considered. The limit state function $g(F_y, Z, M)$ for such a problem in reliability is given as:

$$g(F_y, Z, M) = F_y \cdot Z - M \quad (9)$$

where F_y is the yield strength of the steel used, Z is the plastic section modulus, and M denotes the applied moment. The probability of failure, P_f , may be given as follows:

$$P_f = P(g(F_y, Z, M) < 0)$$

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad (10)$$

where β is the reliability index.

LabVIEW Illustration for an Ultimate Limit State Involving a Beam under Flexure

For this illustration, the various random variables that are of importance have the statistical distributions and associated

parameters (mean and coefficient of variation (COV)) as shown in Table 1.

| Variable | Type | Mean | COV |
|----------|--------|--------------------|-------|
| F_y | Normal | 40 ksi | 0.125 |
| Z | Normal | 50 in ³ | 0.05 |
| M | Normal | 1000 kip-in | 0.2 |

Table 1 Random Variables and Distribution Parameters in Teaching Module 2

We will assume that all three random variables are uncorrelated and will employ two different techniques to evaluate the reliability of the beam: Monte Carlo simulation and the first-order reliability method (FORM).

Monte Carlo Simulation

This is a procedure that essentially involves repeated sampling of all the random variables and checking of the limit state in each sampling.

- F_y , Z , and M are sampled N times (N may be selected by the user) to represent their real distributions using a random number generator.
- The number of samples, N_f , that result in failures (i.e., $g(F_y, Z, M) < 0$) is determined.
- The probability of failure is estimated as:

$$P_f = \frac{N_f}{N} \quad (11)$$

From this illustration, the student can learn that the estimate of P_f depends on the number of simulations. The convergence to a stable value of P_f as a function of N is indicated in Figure 3.

First Order Reliability Method

A Newton-Raphson optimization scheme (see Madsen et al, 1985) is employed in a search for the most-likely-to-fail point on the failure surface. Gradients of the limit state function, $g(F_y, Z, M)$, are computed in each iteration and aid in the search. Iterations are performed until the probability of failure converges to within a user-specified tolerance. Thus, we have:

$$\left| \frac{P_{f,n} - P_{f,n-1}}{P_{f,n}} \right| \leq \text{tolerance} \quad (12)$$

in which $P_{f,n}$ and $P_{f,n-1}$ denote the probability of failure estimates in the n^{th} and the $(n-1)^{\text{th}}$ iteration, respectively.

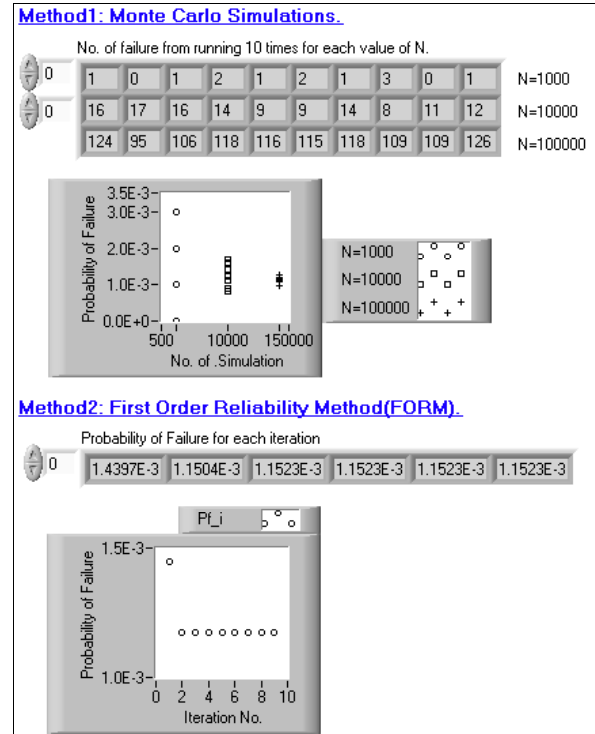


Figure 3 Failure probability calculations from Monte Carlo simulation and FORM in Teaching Module 2.

The same problem that was solved using simulation is also solved using the FORM approach. User inputs can include different starting points for the search as well as different levels of tolerance. As seen in Figure 3, probability of failure values computed based on Monte Carlo simulations and the FORM algorithm are fairly close to each other.

This LabVIEW illustration of reliability analysis using two different techniques is easy to follow and allows the student to alter various inputs in trying to learn the basis for these analytical tools. Various outputs are presented in the form of numerical summaries as well as graphical displays. Useful insights into what parameters affect the reliability of structures can be gained from illustrations such as these.

Teaching Module 3

Fatigue Limit State for a Steel Bridge Component

On the basis of results in numerous metal fatigue experiments, Paris (1964) found a relationship between crack growth rate and

stress intensity range that may be described as follows:

$$\frac{da}{dN} = A(\Delta K)^m \quad (13)$$

where a is the crack length, N is the number of stress cycles, A and m are fatigue constants, and ΔK is the stress intensity range. From the theory of linear elastic fracture mechanics, ΔK can be given as follows:

$$\Delta K = Y(a)(\Delta\sigma)\sqrt{\pi a} \quad (14)$$

where $Y(a)$ is defined as a geometry function related to the crack geometry and shape of the specimen, and $\Delta\sigma$ represents the tensile stress range. Substituting Equation (14) into (13), with separation of variables and integration, leads to the solution:

$$\psi(a_2, a_1) = A(\Delta\sigma)^m (N_2 - N_1) \quad (15)$$

where $\psi(a_2, a_1)$ is defined by Madsen et al. (1985) as follows:

$$\psi(a_2, a_1) = \int_{a_1}^{a_2} \frac{da}{[Y(a)\sqrt{\pi a}]^m} \quad (16)$$

Two types of failure criteria have been proposed to describe limit state functions for fatigue reliability problems.

Type (1): $a_c - a_N \leq 0$

Type (2): $K_{IC} - K_N \leq 0$

In the first type, failure occurs when the crack size, a_N , after N stress cycles, exceeds a critical crack size, a_c . Failure may also occur when the stress intensity factor, K_N , exceeds the fracture toughness, K_{IC} . Madsen et al. (1985) suggest the following limit state function that considers a critical crack size under variable-amplitude loading:

$$g(\mathbf{X}) = \psi(a_c, a_0) - A(\Delta\sigma)^m (N - N_0) \quad (17)$$

The failure probability for such a fatigue limit state can be written as:

$$P_f = P(g(\mathbf{X}) \leq 0) = \Phi(-\beta) \quad (18)$$

LabVIEW Illustration for a Fatigue Limit State Involving a Steel Bridge Component

In order to demonstrate the LabVIEW implementation of this reliability analysis problem, we consider the practical steel bridge fatigue problem studied by Zhao and Haldar (1994).

Consider a full-penetration butt-welded tension flange component with 42 in. width (w) in a 70-ft span steel bridge that is subjected to train loading, 112 stress cycles per day (or a design life of 50 years). The critical crack size, a_c , of this component is 2 in. and a “center-notch crack” mode is considered in this example. The other necessary statistical characteristics of this component are as follows:

| Variable | Type | Mean Value | COV |
|--|-----------|---|------|
| $\Delta\sigma_0$ | Constant | 6.334 | 0.0 |
| a_0 | Lognormal | 0.020 | 0.50 |
| a_c | Constant | 2.000 | 0.00 |
| A | Lognormal | 2.05×10^{-10} | 0.63 |
| m | Constant | 3.000 | 0.0 |
| N | Constant | 1×10^6 to 5.5×10^6 | 0.0 |
| w | Constant | 42.0 in. | 0.0 |
| Note that, for simplicity, m is assumed to be constant, equal to 3 here. | | | |

Table 2 Random Variables and Distribution Parameters in Teaching Module 3.

Paris (1964) proposed the following geometry function to deal with the crack geometry and the shape of center-notched specimens with finite width:

$$Y(\varepsilon) = \frac{1 - 0.5\varepsilon + 0.37\varepsilon^2 - 0.044\varepsilon^3}{\sqrt{1 - \varepsilon}} \quad (19)$$

where $\varepsilon = 2a/w$; a is the crack size and w is the width of the specimen. Zhao and Haldar (1994) expressed the mean-stress effect, $(\Delta\bar{\sigma})^m$, as

$$(\Delta\bar{\sigma})^m = [\sqrt{2}(\Delta\sigma_0)]^m \Gamma\left(\frac{m}{2} + 1\right) \quad (20)$$

Substituting Equations (19) and (20) into the limit state function of Equation (17) with the variable transformation from a to ε , we obtain:

$$g(\mathbf{X}) = \left(\frac{1}{2}\right) \int_{\varepsilon_0}^{\varepsilon_c} \frac{w d\varepsilon}{\left[\frac{\varepsilon^{\frac{1}{2}} - 0.5\varepsilon^{\frac{3}{2}} + 0.37\varepsilon^{\frac{5}{2}} - 0.0447\varepsilon^{\frac{7}{2}}}{\sqrt{1 - \varepsilon}} \sqrt{\frac{\pi w}{2}} \right]^m} - A \left\{ [\sqrt{2}(\Delta\sigma_0)]^m \Gamma\left(\frac{m}{2} + 1\right) \right\} (N - N_0) \leq 0 \quad (21)$$

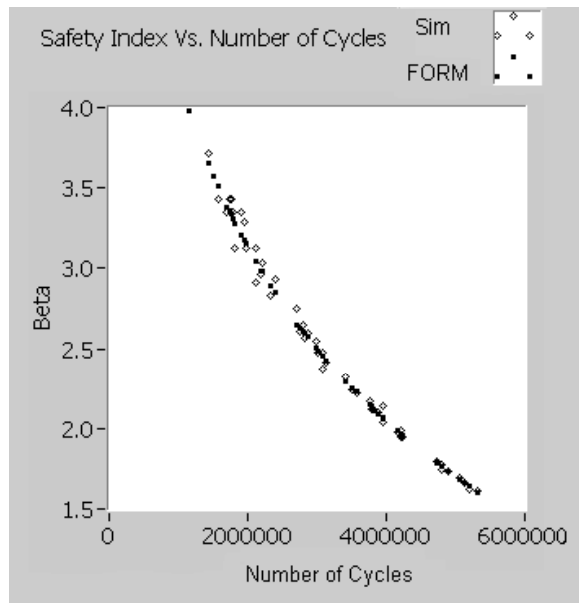


Figure 4 Comparison of β - N curves from Monte Carlo simulations and from FORM in Teaching Module 3.

The variable, ε_0 , is assumed to follow a lognormal distribution with a mean value of 9.524×10^{-4} and a coefficient of variation of 0.5. Equation (21) is defined together with other necessary random variables in the LabVIEW implementation. This fatigue reliability analysis is then solved using two techniques: Monte Carlo simulation and FORM. Results from both methods are presented in the form of a plot showing the β index versus number of cycles, N . From the plot, we see that especially for cases with a large number of simulations, the two methods lead to very similar results.

The advantages of using a LabVIEW implementation to illustrate this problem in fatigue reliability are as follows:

1. The operating interface developed is easy for the student to understand and to modify for any other situation.
2. The values of different statistical parameters may be changed easily and the results can be immediately observed in graphical summaries such as the one in Figure 4. This enables one to evaluate the role of different variables on the reliability of the component.

Discussion and Conclusions

The introduction of a computer environment that may be used as a platform for demonstrating mathematical concepts can greatly improve the learning of material in a course such as structural reliability. Obvious advantages exist over a traditional lecture format since students can take a hands-on approach to understanding the role of load and resistance uncertainties on the safety and performance of structural components and systems. Analytical techniques as well as design choices become more meaningful and, as the three teaching modules presented here have shown, the computer implementations using software such as LabVIEW can offer at once a grasp of the theoretical concepts as well as a platform for demonstrating realistic applications of the theory.

Acknowledgments

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