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On the Use of Empirical Orthogonal Decomposition of Field Data to Study Deepwater Current Profiles for Risers

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ABSTRACT

Since vortex-induced vibration of deepwater risers and failures resulting from it are generally due to characteristics of the current velocity field, a study of the random spatial characteristics in current velocity and derivation of important velocity profiles based on field data can be of use in the design of risers. The traditional approach used to derive design current profiles, where current velocities at each depth are treated independently, is overly conservative for deepwater riser design. At the same time, while evaluating a riser using field data, it is impractical to deal with large amounts of data to establish full-field joint distributions of current velocity at different depths. Proper Orthogonal Decomposition (POD) is an effective numerical technique that can be employed to characterize the spatial coherence in a random field often with great efficiency. In contrast to other studies that have utilized this technique, the present study involves its use with full two-dimensional current velocity data. POD is employed to identify significant current profiles, especially those associated with observed vortex-induced vibration (VIV), at a site where the water depth is 1,000 meters. The efficiency and accuracy resulting from the use of a limited number of POD modes is studied by comparing measured current velocity profiles with those reconstructed based on the reduced-order truncation. Also, POD is applied for extracting patterns in the current profiles accompanying VIV and non-VIV response

KEY WORDS: Vortex-induced vibration; current profiles; proper orthogonal decomposition; deepwater riser.

INTRODUCTION

Vortex-induced vibration of deepwater risers results from cross-flow response due to current velocity variation over the riser length. Hence, a study of spatial statistics of the current velocity random field and empirical derivation of energetic velocity profiles using field data can be of use in the design of risers. As discussed by Jeans *et al*. (2003) and Meling and Eik (2002), the traditional approach used to derive design current profiles involves treating current velocities at each depth independently. This can be overly conservative for deepwater riser design. At the same time, establishing full-field joint distributions of current velocity at different depths from field data is cumbersome due to the large amounts of data involved. Proper Orthogonal Decomposition (POD) is an efficient numerical technique that can be employed to describe the spatial coherence in a random field. POD relies on an empirical orthogonal transformation of the spatial random field and proves insightful information because it helps in identifying energetic spatial patterns in the field; for risers, current profiles may be directly derived from current velocity measurements obtained at different depths. The application of POD techniques to establish current profiles for risers has been discussed by others (see, for example, Forristall and Cooper, 1997; Jeans and Feld, 2001; Jeans *et al.*, 2003; and Meling and Eik, 2002). In contrast to those other studies, however, the present study addresses the use of POD techniques for full two-dimensional current velocity profiles. The efficiency and accuracy resulting from the use of a limited number of POD modes is studied by comparing measured current velocity profiles with those reconstructed based on a POD-based reduced-order truncation. The procedure is also employed to identify significant patterns in profiles, especially those associated with the occurrence of vortex-induced vibration (VIV).

PROPER ORTHOGONAL DECOMPOSITION

Proper Orthogonal Decomposition (POD) (Lumley, 1970) is a numerical method used for empirically deriving orthogonal basis functions or “POD modes” of a stationary random field. POD techniques have been applied in many engineering fields such as wind engineering to estimate important spatial distribution patterns in pressures on structures, inflow turbulence for wind turbines (see, for example, Saranyasoontorn and Manuel, 2005), etc. A brief review of covariance-based POD, also known as Covariance Proper Transformation (CPT) is presented here.

Given N weakly stationary correlated random processes, $\boldsymbol{V}(t) = \{v_1(t),$

$v_2(t), \ldots, v_N(t)\}^T$, one can establish an $N \times N$ covariance matrix, $\boldsymbol{C}_v$, from measurements of the N-dimensional spatio-temporal vector, $\boldsymbol{V}(t)$. By solving an eigenvalue problem, it is possible to diagonalize $\boldsymbol{C}_v$ so as to obtain the (diagonal) matrix, Λ. Thus, we have:

$$\boldsymbol{\Phi}^T \boldsymbol{C}_v \boldsymbol{\Phi} = \Lambda \,; \quad \boldsymbol{C}_v \boldsymbol{\Phi} = \boldsymbol{\Phi}\Lambda \tag{1}$$

yielding eigenvalues, $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$, where $\lambda_1 > \lambda_2 > \ldots > \lambda_N$, and corresponding eigenvectors, $\boldsymbol{\Phi} = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_N\}$.

It is now possible to rewrite the original N correlated processes, $\boldsymbol{V}(t)$, in terms of N uncorrelated scalar subprocesses, $\boldsymbol{Z}(t) = \{z_1(t), z_2(t), \ldots, z_N(t)\}^T$ such that

$$\boldsymbol{V}(t) = \boldsymbol{\Phi} \cdot \boldsymbol{Z}(t) = \sum_{j=1}^{N} \boldsymbol{\phi}_j z_j(t) \tag{2}$$

where $\boldsymbol{\phi}_j$ represents the j^{th} POD mode shape corresponding to the j^{th} generalized coordinate or scalar subprocess, $z_j(t)$. The energy associated with each subprocess, $z_j(t)$, may be described in terms of its variance, λ_j. A reduced-order representation, $\hat{\boldsymbol{V}}(t)$, may also be reconstructed by including only the first M POD modes and associated generalized coordinates as follows:

$$\hat{\boldsymbol{V}}(t) = \sum_{j=1}^{M} \boldsymbol{\phi}_j z_j(t)\,, \text{ where } M < N. \tag{3}$$

In the present study, $\boldsymbol{V}(t)$ will represent current velocity random processes in two orthogonal directions at 20 elevations over the water depth, so that the number of POD modes, N, is equal to 40.

FIELD DATA

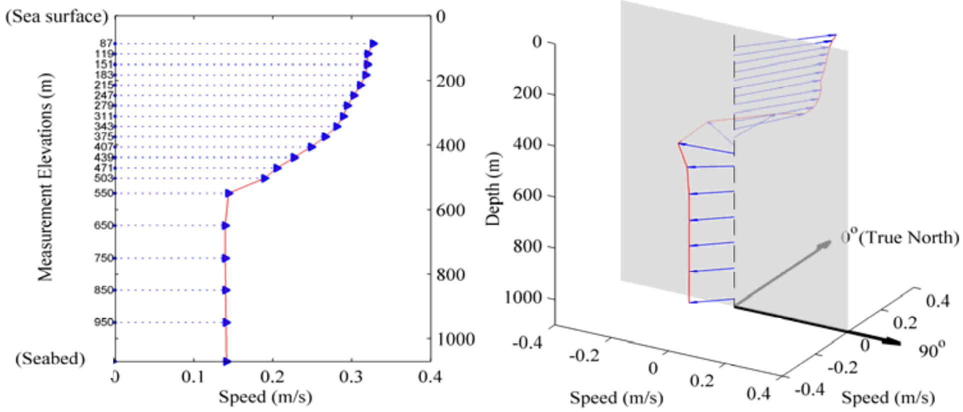
This study involves the analysis of current velocity data from the monitoring of vortex-induced vibrations (VIV) on a deepwater riser at a site where the water depth is 1,000 meters. The data, recorded over a two-month period every four hours, consist of the magnitude of the current speed at each level as well as its direction. Current velocities were measured at various depths along the riser. An “event” here will refer to any single such sample measured every four hours.

DATA ANALYSIS

From the mean current velocity profile shown in Fig. 1, it can be seen that the overall current velocity profile is markedly different over the upper and lower halves of the water column. Currents in the upper layer mostly flow towards the northeast (around 35°) whereas in the lower layer, they are predominantly in a reversed southwesterly direction. High current velocities, at this location, are mostly seen close to the sea surface with maximum values of around 0.8 m/s; while low velocities are seen in the transition between the upper and lower layers. In general, the current velocities are seen to reduce considerably from the upper layer to the transition zone, below which (in the lower layer) their directions gradually change to a direction opposite to that in the upper layer. Also, as distinct from the more variable upper layer currents, it was found that current velocities in the lower layer had a more consistent pattern, fluctuating only slightly from event to event.

POD REPRESENTATION OF THE CURRENT VELOCITY

In applying the POD procedure for the analyses, the measured current velocity data, which represent full two-dimensional speed and direction profiles, were first transformed into velocity components in two orthogonal directions (0° and 90°) at each elevation. Next, the 40×40 covariance matrix based on the current velocity data was estimated. A total of 40 POD modes were derived using the POD procedure described. After arranging modes by decreasing energy (i.e., eigenvalue), the first three dominant POD mode shapes were obtained and are shown in Fig. 2. Also, the eigenvalues, λ_j, associated with energy of each mode are indicated in terms of the percentage of the total energy in the current velocity field. The mode shapes are shown in two formats: (i) components in the 0° and 90° directions; and (ii) resultant and associated direction. The first POD mode in both cases displays dominant contribution in the 90° direction and negligible contribution in the 0° (true north) direction along the water column, especially in the upper layer. In contrast, again primarily in the upper layer, the second POD mode displays almost no contribution in the 90° direction along the water column and relatively much larger contribution in the 0° direction. The mode shape resultants for the first two modes are, therefore, perpendicular to each other. The third POD mode displays a rather sheared current profile: velocities in the uppermost and lowermost depths are in nearly opposite directions. This mode shape for current velocity has similar amplitudes in the 0° and 90° directions; as a result, the resultant for this third mode shape is oriented roughly 45° relative to the first two modes. On studying the eigenvalues, λ_j, with the entire data set, the first POD mode is seen to contribute a large proportion (almost 60%) of the total energy in the current velocity field, while the first two POD modes together account for 90% of the total energy.



(a) (b)

Figure 1: (a) Variation of mean current speed (magnitude) with depth; and (b) Mean current profile (indicating direction of resultant).

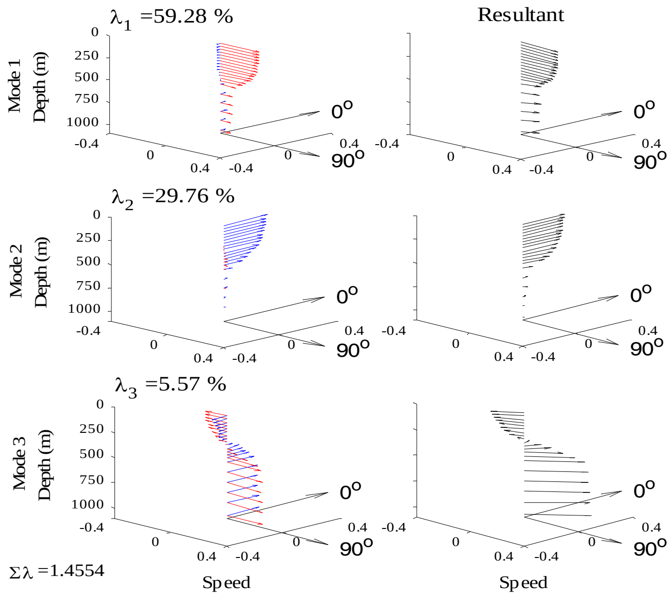


Figure 2: First three POD mode shapes of current velocity for the entire data set.

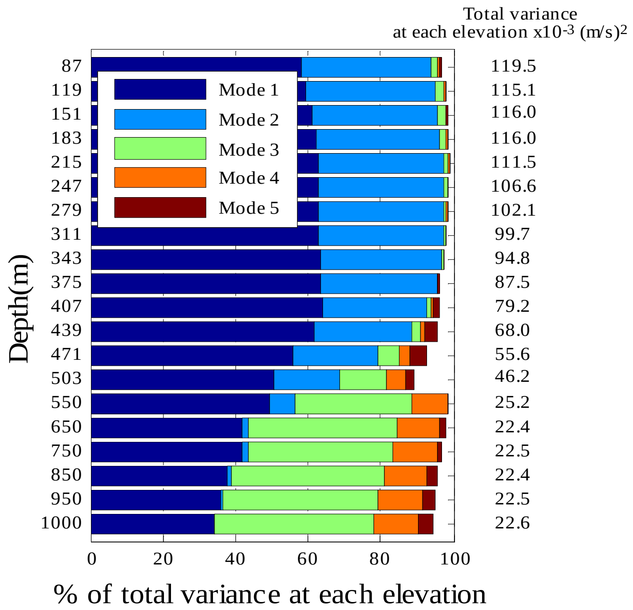


Figure 3: Variance of the current velocity field resulting from reconstruction using a different number of POD modes for the entire data set.

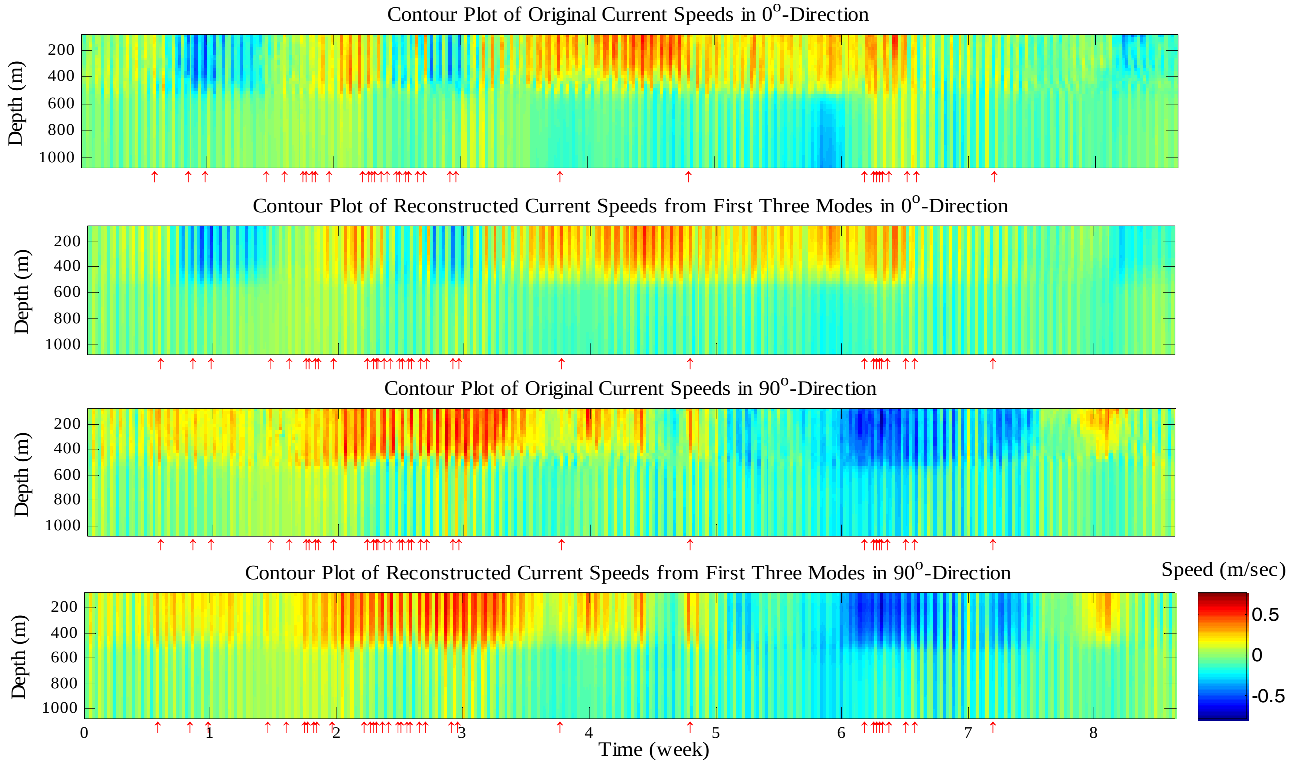


Figure 4: Measured current velocity profiles for all recorded events over the two-month monitoring period and reconstructed profiles based on three POD modes.
(Note: The arrows on the abscissa indicate recorded VIV events.)

When the energy distribution in the current velocity at each elevation is considered separately, as is done in Fig. 3, it is very clear that the first two modes are able to account for more than 95% of the total variance or energy in the upper layer, while in the lower layer, at least four modes are needed to capture around 90% of the total variance at each depth there. Moreover, in the transition zone between the upper and lower layers, more than five modes are needed in order to account for 90% of the total variance at each depth there. Figure 3 also shows the total current velocity variance at each level of measurement along the water column. From these values, it is clear that the current velocities in the upper layer exhibit far greater fluctuations (dispersions about the mean) than is the case for the lower layer.

The current velocity field in the 0° and 90° directions as measured over the entire monitoring period is presented in Fig. 4 along with the reconstructed field based on three POD modes. Events where VIV occurred over the two-month period are indicated by arrows on the abscissa. For both directions, contour plots of reconstructed current velocities are almost exactly the same as those for the measured current velocities. Only in the lower layers, are some differences evident. These plots also clearly reveal a separation of the water column into upper and lower layers as mentioned earlier, and a transition zone in between these two layers where the current velocities are small.

To further investigate how efficiently three POD modes can capture dominant characteristics of the two-dimensional current profiles at all elevations jointly, plots of the correlation coefficients between the measured current speeds at all depths (and among the different directional components) are compared in Fig. 5 with corresponding correlation coefficient estimates from a POD-based reconstruction using different numbers of modes. The plots present correlation coefficients for the current speed data in three different ways: (i) between pairs of 0° current velocity components at different levels; (ii) between pairs of 90° components at different levels; and (iii) between a 0° and a 90° component at different levels. It is seen that reconstructed two-dimensional current velocity profiles based on three POD modes generally preserve the observed correlation for all cases and at all depths fairly well. The only exception is for the case involving a study of the correlations between the 0° and 90° current velocity components where three POD modes do not yield similar estimates to those from the measured data. Four POD modes are needed for that case. A reason for the slower convergence here is the lower correlation levels between the 0° and 90° components of current at different depths. POD convergence is more efficient when correlation levels are large. Figure 5 also indicates that there is very strong correlation among all the current velocity data within the same layer (upper only or lower only) of the water column for each direction, as is evident from the darker regions close to the diagonal in the lower right and upper left sectors.

SPATIAL PATTERNS IN CURRENT PROFILES

POD techniques are employed here to empirically identify predominant energetic spatial velocity profiles of the current velocity field. Using POD, energetic current profiles in the data set that are associated with VIV response of the riser are compared with profiles when VIV was not indicated. From Figs. 6 and 7, it can be seen that the first mode shapes in the two different data sets suggest quite different spatial profiles. In the first mode shape of the non-VIV data set, the current in the upper layer flows nearly perpendicular to the direction of the lower-layer current. On the other hand, for the VIV data set, the dominant current profile demonstrated by the first POD mode shape displays almost the same direction along the water depth. There is more consistency in the cross-flow direction for VIV response along the entire depth. Note, too, that the energy in the current velocity field, denoted by the summation of eigenvalues in Figs. 6 and 7, suggest higher energy levels in the VIV data set than in the non-VIV data set.

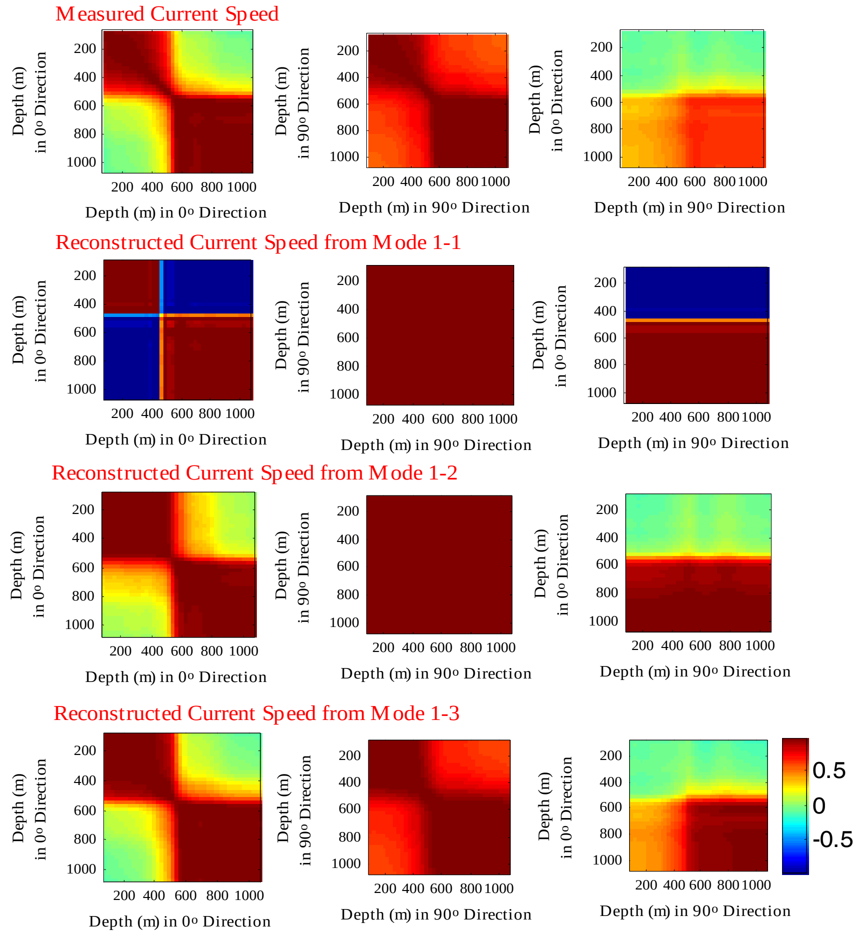


Figure 5: Correlation between current velocity components at various depths – among 0° components; among 90° components; and between 0° and 90° components for the recorded data as well as based on reconstruction using one, two, and three POD modes.

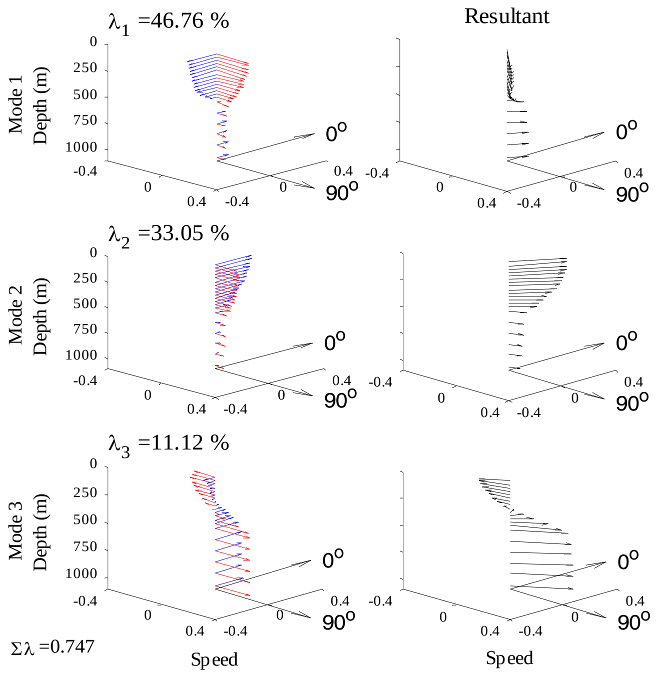


Figure 6: First three POD mode shapes of current velocity for the non-VIV data set

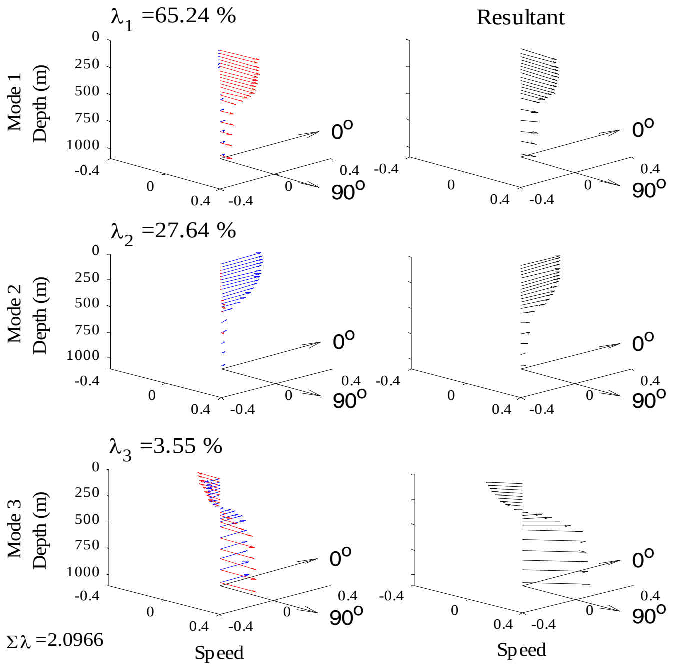


Figure 7: First three POD mode shapes of current velocity for the VIV data set

CONCLUSIONS

In order to account for the coherence in bi-directional current velocities at different vertical levels, the POD approach has been used to derive current profiles instead of using the traditional approach where the currents at different vertical field points are treated independently. At the deepwater location (1,000-meter water depth) studied here, current profiles were shown to be efficiently represented using only three POD modes, which together were able to account for nearly 95% of the variance of the entire current velocity field (i.e., including all measurement locations along the depth). The first two modes alone contributed more than 95% of the total variance in the upper layer of the water column. In the lower layer, their contributions were significantly smaller. The third POD mode, however, provides a significant contribution in the lower layer. As a result, at the lower levels, three POD modes are sufficient to capture more than 80% of the total variance. For the data set studied, then, current velocities in the upper layer were found to converge much faster than in the lower layer. Using POD procedures, dominant patterns in current profiles associated with non-VIV and VIV data sets were compared. Contrasting POD modes associated with the two sets were evident.

At a given site, current velocity measurements such as those used here can form the basis for extracting important spatial distributions of current velocity jointly at different depths. It is then possible to develop design current profiles using simple empirically derived shapes by using a limited number of POD modes. Importantly, these shapes do not independently describe the current velocities at each depth but do so jointly. Hence, POD-based design profiles can be quite efficient in application for deepwater risers.

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