Proceedings of the Sixteenth (2006) International Offshore and Polar Engineering Conference
San Francisco, California, USA, May 28-June 2, 2006

ISBN 1-880653-66-4 (Set); ISSN 1098-6189 (Set)

Identification and Analysis of Vortex-Induced Vibrations of a Drilling Riser using Empirical and Spectral Procedures

Tanapat Srivilairit and Lance Manuel
Department of Civil, Architectural, and Environmental Engineering,
University of Texas, Austin, Texas, USA

ABSTRACT

The objective of this study is to use full-scale field data on riser motions to better understand the behavior of a deepwater drilling riser. We examine such data from the monitoring of riser accelerations and vortex-induced vibration (VIV) of a drilling riser located at a site where the water depth is 1,000 meters. In order to identify and analyze VIV for this riser, in-line and cross-flow motions in different data segments are studied using spectral methods. We also illustrate the application of Proper Orthogonal Decomposition – a procedure which is used to derive energetic spatial modes defining the motion in a purely empirical approach. Time series data of riser accelerations recorded by accelerometers distributed along the riser over a two-month period are analyzed using both, the spectral and the POD procedures.

KEY WORDS: Vortex-induced vibration; current profiles; proper orthogonal decomposition.

INTRODUCTION

In deepwater marine risers, vortex-induced vibration (VIV) is an important design consideration especially for fatigue. Several numerical analysis tools are in common use for studying VIV of risers. Field measurements of riser motions including VIV response can be extremely valuable in studies seeking to validate the different numerical prediction tools. Such measurements when well resolved in space and time, though, still require robust processing techniques in order to draw meaningful conclusions regarding riser behavior.

The present study is concerned with the analysis of riser acceleration data and accompanying current velocity data obtained from a deepwater drilling riser. The site has a water depth of 1,000 meters. These riser acceleration data consist of measurements in two-orthogonal horizontal directions (denoted as X and Y directions) and one vertical direction (i.e., the Z direction).

It is important to point out that the acceleration data available from the site are likely to be contaminated by gravitational acceleration. Kaasen et al (2000) recommended a procedure for obtaining lateral motions corrected for gravitational acceleration contamination but measurements of angular velocity are needed along the riser. Since no rotational measurements are available here, the riser accelerations and displacements obtained by double integration used later in the analyses may contain some error.

DATA ANALYSIS USING SPECTRAL METHODS

Rotation procedure

The actual orientation of the three accelerometers at each logger was unknown; hence, a vector rotation procedure is employed to separate the measured accelerations into in-line and cross-flow components. This is based on the assumption that the dominant VIV response is transverse to the direction of the current velocity. The vector rotation procedure involves the following steps:

1. A high-pass filter is applied to the acceleration data to remove unwanted frequencies and drift (a cut-off frequency of around 0.03 Hz is used here);
2. The acceleration data are rotated at different angle increments from 0^o to 180^o to obtain orthogonal components of horizontal acceleration at each angle;
3. Power spectra of the rotated acceleration data at each increment are estimated using Welch's method (Welch, 1967) with a 1,024-point Hanning window and a 50% overlap rate on the data segments;
4. Steps 2 and 3 are repeated for each rotation angle until the entire range of rotations is considered;
5. The maximum peak value of the power spectrum in the cross-flow (Y) direction and the associated angle of signal rotation (α°) are found.

An illustration of the application of the rotation procedure using data from a single logger is presented in Fig. 1 where, for different angles of rotation (α) of the X and Y accelerations, Welch spectra for the rotated Y acceleration component are shown.

VIV identification using spectral methods

Following the rotation procedure described above, power spectra in the two orthogonal directions are studied to identify whether or not VIV is indicated in the different data segments. This is achieved by examining whether or not one or more narrow band peaks in power spectra can be easily identified and isolated. A single narrow band peak could be a possible indication of single-mode VIV; multiple peaks could indicate multi-mode VIV. The occurrence of VIV in an event can also be identified or confirmed based on the frequency-doubling phenomenon, whereby the dominant frequency in power spectra of the in-line direction acceleration is found to be twice that of the dominant frequency in the cross-flow direction.

Examples of Welch power spectra for (a) a single-mode VIV event (A), (b) a multi-mode VIV event (B) and (c) a non-VIV event (C) are presented in Fig. 2 for all eight loggers where the narrow band peaks in the cross-flow direction and frequency doubling in the orthogonal in-line direction are evident for the VIV events but absent for the non-VIV event. The relationship between in-line and cross-flow VIV response has been studied by Vandiver and Jong (1987). By comparing the dominant frequencies obtained following the rotation procedure applied to acceleration data from each event with the estimated natural frequencies for VIV response calculated from modal analysis of the riser, we can identify the mode number associated with the VIV response for that event.

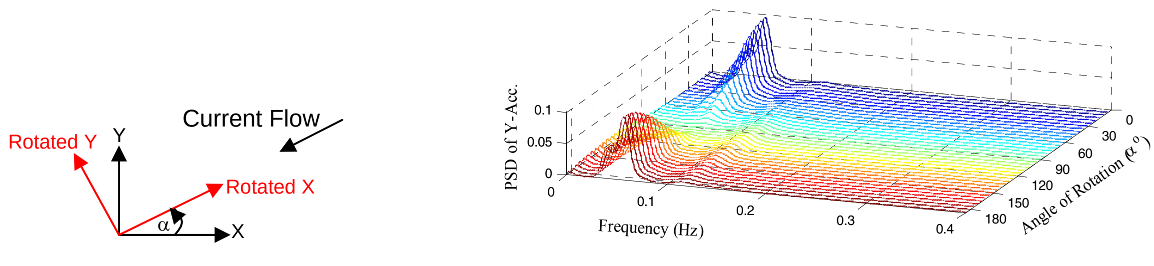
PROPER ORTHOGONAL DECOMPOSITION

Proper Orthogonal Decomposition (POD) (Lumley, 1970) is a numerical method used for empirically deriving orthogonal basis functions or "POD modes" of a stationary random field. POD techniques have been applied in many engineering fields such as wind engineering to estimate important spatial distribution patterns in pressures on structures, inflow turbulence for wind turbines (see, for example, Saranyasoontorn and Manuel, 2005), etc. A brief review of covariance-based POD, also known as Covariance Proper Transformation (CPT) is presented here.

Given N weakly stationary correlated random processes, $\boldsymbol{V}(t) = \{v_1(t), v_2(t), \ldots, v_N(t)\}^T$, one can establish an $N\times N$ covariance matrix, $\boldsymbol{C_v}$ (= $\mathrm{E}[\boldsymbol{V}(t)\cdot\boldsymbol{V}(t)^T]$), from measurements of the N-dimensional spatio-temporal vector, $\boldsymbol{V}(t)$. By solving an eigenvalue problem, it is possible to diagonalize $\boldsymbol{C_v}$ so as to obtain the (diagonal) matrix, Λ. Thus, we have:

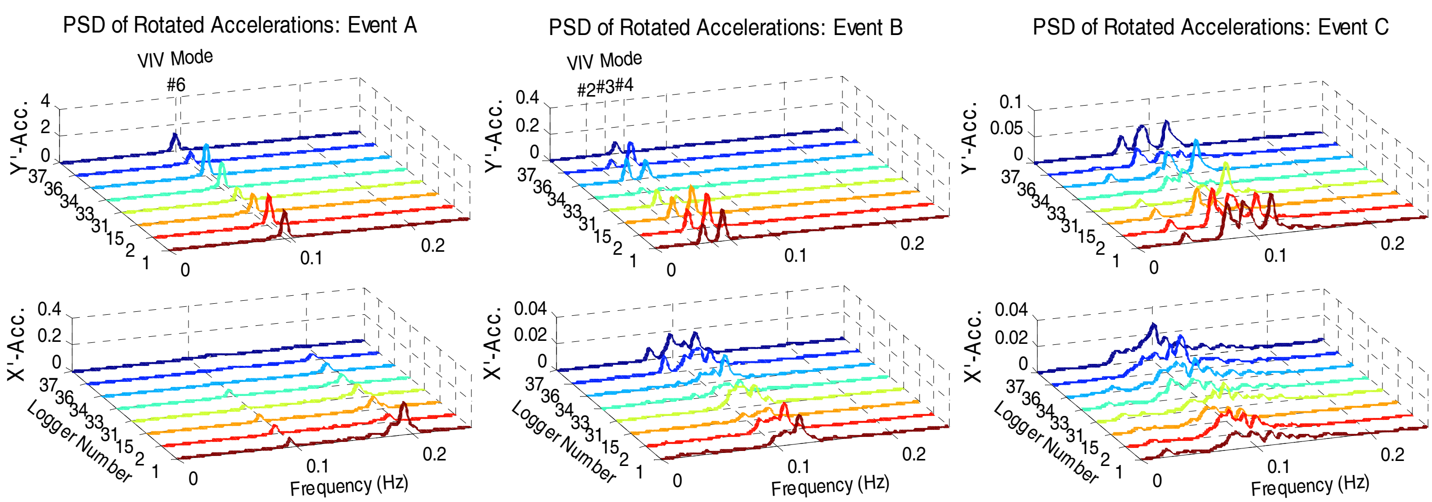
$$\Phi^T \boldsymbol{C}_v \Phi = \Lambda\,; \quad \boldsymbol{C}_v \Phi = \Phi\Lambda \qquad (1)$$

yielding eigenvalues, $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$, where $\lambda_1 > \lambda_2 > \ldots > \lambda_N$, and corresponding eigenvectors, $\Phi = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_N\}$.



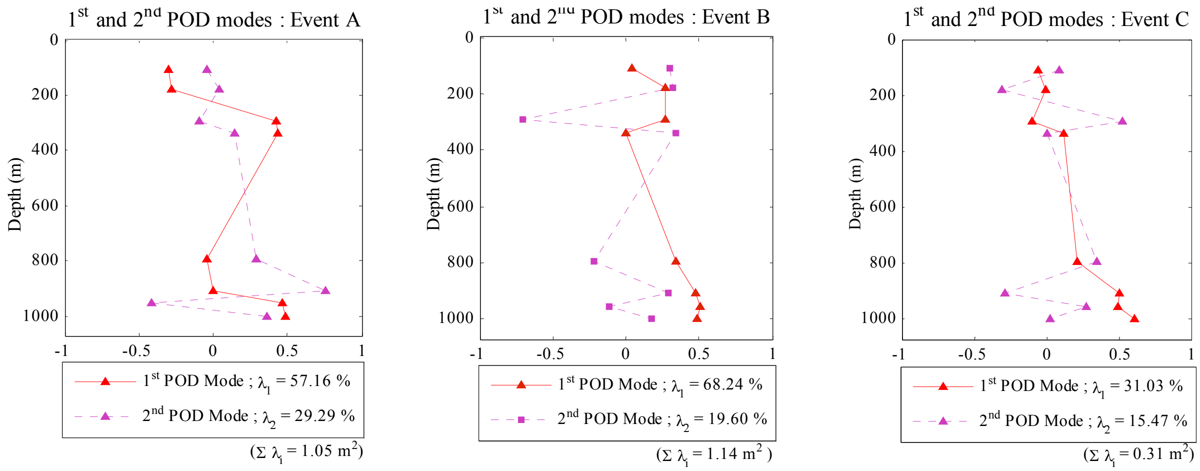
(a) (b)

Figure 1: (a) Rotation of X and Y coordinates through α˚; and (b) Example Welch spectra for acceleration at a single logger in the rotated Y-direction for different angles, α.



(a) (b) (c)

Figure 2: Examples of Welch spectra for rotated accelerations at eight loggers in the cross-flow and in-line directions considering three events: (a) a single-mode VIV event (A); (b) a multi-mode VIV event (B); and (c) a non-VIV event (C).



(a) (b) (c)

Figure 3: First two POD mode shapes and associated energies of the rotated displacement in the cross-flow direction for three events: (a) a single-mode VIV event (A); (b) a multi-mode VIV event (B); and (c) a non-VIV event (C).

It is now possible to rewrite the original N correlated processes, $\boldsymbol{V}(t)$, in terms of N uncorrelated scalar subprocesses, $\boldsymbol{Z}(t) = \{z_1(t), z_2(t), \ldots, z_N(t)\}^T$ such that

$$\boldsymbol{V}(t) = \boldsymbol{\Phi} \cdot \boldsymbol{Z}(t) = \sum_{j=1}^{N} \boldsymbol{\phi}_j z_j(t) \tag{2}$$

where $\boldsymbol{\phi}_j$ represents the j^{th} POD mode shape corresponding to the j^{th} generalized coordinate or scalar subprocess, $z_j(t)$. The energy associated with each subprocess, $z_j(t)$, may be described in terms of its variance, λ_j. A reduced-order representation, $\hat{\boldsymbol{V}}(t)$, may also be reconstructed by including only the first M POD modes and associated generalized coordinates as follows:

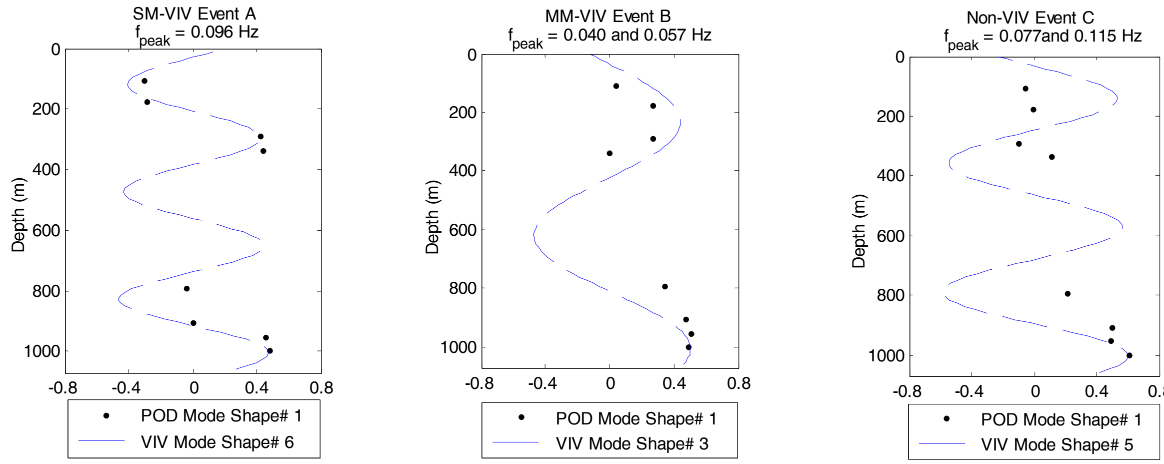
$$\hat{\boldsymbol{V}}(t) = \sum_{j=1}^{M} \boldsymbol{\phi}_j z_j(t) \text{ , where } M < N. \tag{3}$$

In the present study, $\boldsymbol{V(t)}$ will represent cross-flow riser displacement random processes (following twice-integrated accelerations obtained using the rotation procedure described) at eight elevations over the riser length depth, so that the number of POD modes, N, is equal to eight.

POD techniques are employed here to help empirically identify predominant energetic spatial vibration modes or patterns of riser motion. Other applications of POD for empirically examining modal response in structural dynamics include a study by Feeny (2002). In the present study, POD techniques are employed to capture the predominant energetic spatial modes in the rotated acceleration data or the rotated displacement data obtained by double integration, as shown in Fig 3. It can be clearly seen that the dominant POD mode shapes, especially the first mode, extracts locked-in modes very well spatially despite the poor spatial resolution of loggers along the riser's length. The single mode lock-in for Event A and the double mode lock-in for Event B were clear in the power spectra of Fig. 2. In Fig. 3, these two events are distinct from Event C (a non-VIV event) by the fact that the first few extracted POD modes there have dominant, high energies – indeed for Events A and B, the first mode carries roughly 57 and 68 percent, respectively, of the entire riser motion's spatio-temporal field variance. That there is no lock-in or preferred spatial pattern in Event C is evidenced by the relatively smaller contribution from the first few POD modes there (only 46% of the field energy comes from the first two). The POD eigenvalue, λ_j, associated with the energy in mode j is presented as a percentage of the total energy in Fig. 3. Note that the larger motions along the riser for the two VIV events are indicated by the higher sum of the POD eigenvalues (total energy). It is important to emphasize that the POD modes should resemble the appropriate riser natural modes but they are extracted using an empirical procedure that works with second-moment statistics of the data, and not by using spectral procedures. With better spatial resolution of loggers, it is conceivable that POD shapes would help to confirm analytically predicted shapes for a riser. The distribution in energies in these shapes (POD modes) could likewise confirm separately obtained energy contributions in narrow band peaks of power spectra in single- or multi-mode VIV events.

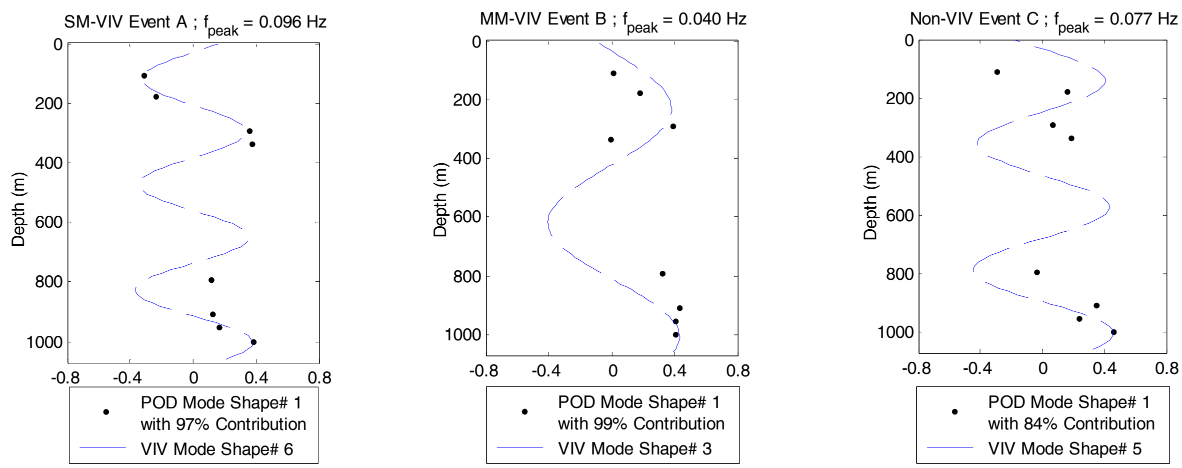
Notwithstanding the limitations of spatial resolution in the logger locations, Fig. 4 suggests that there is fairly good consistency of the first POD mode shape with the appropriate VIV mode for lock-in of the pertinent event. Since only one POD mode is shown and it does not account for the entire field energy, differences are due to unaccounted for energies in higher POD modes. Note that these POD-based empirical modes actually reflect preferred energy distributions in orthogonal basis functions and are optimal among all decomposition schemes.

An alternative approach for carrying out a POD analysis is to use cross-power spectral density matrix estimates of the input data at desired frequencies of interest instead of using covariance matrices (as was presented in Eq. 1). A similar decomposition procedure to that presented in Eqs. 1-3 can yield frequency-dependent eigenvalues and energies. In Fig. 5, the first POD mode shape and associated energy for the same three events as in Fig. 4 are presented, but here they are derived using cross-power spectral density matrices using data from all of the loggers. Mode decomposition is limited to the frequency where a peak in the power spectra was identified. It can be seen that the first POD mode shape is not very different from that based on the covariance matrix (Fig. 4); however, the first mode accounts for a significantly larger percent of energy indicating that at the peak (lock-in) frequency, the motion is well described by a shape resembling the first POD mode. The energy in the first POD mode for the non-VIV event (C) is significantly lower than for the two VIV events.



(a) (b) (c)

Figure 4: Examples of the first POD mode shape compared with a VIV mode shape for three events: (a) a single-mode VIV event (A); (b) a multi-mode VIV event (B); and (c) a non-VIV event (C).



(a) (b) (c)

Figure 5: Examples of the first POD mode shape (based on cross-power spectra at specified frequencies) compared with a VIV mode shape for three events: (a) a single-mode VIV event (A); (b) a multi-mode VIV event (B); and (c) a non-VIV event (C).

CONCLUSIONS

By employing spectral methods, it is possible to identify events associated with VIV lock-in from acceleration measurements, but it is not as straightforward to extract response modal shapes of riser motion. On the other hand, without conducting a modal analysis, it is possible to estimate dominant energetic spatial modes of response using Proper Orthogonal Decomposition (POD). POD can also be used as an approximate indicator of VIV occurrence by considering the proportion of the total riser motion energy derived from the first few dominant POD modes. This is because with VIV events, one usually finds greater energy contributions captured by the first POD mode, than with non-VIV events.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance of BP in providing the riser acceleration field data used in this study.

REFERENCES

Feeny, BF (2002). "On Proper Orthogonal Co-ordinates as Indicators of Modal Activity," *J. Sound & Vibration*, Vol. 255, No. 5, pp. 805-817.

Kaasen, KE, Lie, H, Solaas, F, and Vandiver, JK (2000). "Norwegian A Deepwater Program: analysis of Vortex-Induced Vibrations of Marine Risers Based on Full-Scale Measurements," *Proceedings of the Annual Offshore Technology Conference*, Vol. 2, pp. 565-575.

Lumley, JL (1970). "Stochastic Tools in Turbulence," Academic Press, New York.

Saranyasoontorn, K, and Manuel, L, "Low-Dimensional Representations of Inflow Turbulence and Wind Turbine Response Using Proper Orthogonal Decomposition," *Journal of Solar Energy Engineering*, Transactions of the ASME, November 2005.

Vandiver, JK, and Jong, J-Y (1987). "The Relationship between In-line and Cross-flow Vortex-Induced Vibration of Cylinders", *J Fluids and Structures*, Vol. 1, pp. 381-399.

Welch, PD (1967). "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms," *IEEE Trans. Audio Electroacoust.*, Vol. AU-15, pp. 70-73.