

# RELIABILITY-BASED OPTIMAL INSPECTION FOR FRACTURE-CRITICAL STEEL BRIDGE MEMBERS

by

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**ABSTRACT**

To prevent members in steel bridges from fatigue failure, one usually needs to perform frequent periodic bridge inspections and employ detailed inspection methods. This is especially true for fracture-critical members or details. Carrying out these inspections puts a large burden on a transportation agency's bridge maintenance budget. A systematic reliability-based method for inspection scheduling is proposed to yield the most economical inspection strategy for steel bridges that, at the same time, guarantees an acceptable safety level through the planned service life. A methodology is presented for evaluating the fatigue reliability of a specified detail classified according to AASHTO fatigue categories. A Miner's rule approach is employed to evaluate the fatigue reliability. The inspection scheduling problem is modeled as an optimization problem with a well-defined objective function, that includes the total expected cost of inspection, repair and failure formulated on the basis of an event tree framework, and appropriate constraints in inspection intervals and minimum (target) structural reliability. An optimal inspection-scheduling plan can thus be obtained for any specified fatigue details (fracture-critical details) in steel bridges. Examples presented demonstrate the advantage of the reliability-based optimal inspection scheduling in cost saving and structural reliability control over alternative periodic inspection plans. Two numerical examples for a steel bridge in Texas are presented to demonstrate the proposed reliability-based optimal inspection scheduling.

Keywords: Fracture-critical, steel bridges, reliability, optimization, inspection scheduling.

## INTRODUCTION

A reliability-based inspection scheduling procedure that can yield an optimal inspection schedule and maintain a specified safety level for fracture-critical members in steel bridges through the planned service life is presented. This procedure is based, in sequence, on a stress range analysis, a fatigue reliability analysis, and an optimization analysis. The objective of the stress range analysis is to provide an accurate estimate of the effective stress range for the identified member or detail that will be needed in the subsequent fatigue reliability analysis. Three analysis approaches for evaluating stress ranges in steel bridges are presented here: (a) spectrum analysis; (b) an assumed stress Rayleigh distribution analysis; and (c) a fatigue truck analysis. With an accurate effective stress range distribution representing the actual traffic status on a bridge, fatigue reliability analysis for the member can then be performed where a limit state function based on Miner's Rule (with an empirical S-N curve relation resulting from numerous fatigue test results) is used for all the details classified in the fatigue categories of the AASHTO Specifications (1). After acquiring the necessary information from the stress range and fatigue reliability analyses, an optimization problem is formulated with an objective function involving costs, and appropriate constraints on the inspection intervals and structural safety. The optimization, is for ease of interpretation, defined using an event tree approach. Finally, solution of the optimization problem yields the optimal inspection schedule. Numerical examples from a case study of a steel bridge in Texas are presented to demonstrate the proposed reliability-based optimal inspection scheduling procedure.

## STRESS RANGE ANALYSIS OF FATIGUE LOADINGS IN STEEL BRIDGES

The operating stress range for steel members or details in bridges is a key factor that directly affects the fatigue performance of members or details in steel bridges. Therefore, obtaining an accurate description of the effective stress range,  $S_{RE}$ , applied on the identified detail is very important in fatigue reliability analysis. Generally, any one of three methods: a stress spectrum analysis, a Rayleigh distribution analysis; or a fatigue truck analysis, may be applied to establish the effective stress range.

### Stress Spectrum Analysis

If stress range data on a desired detail are systematically collected and sufficiently representative of actual traffic loads, a stress range spectrum can be derived from such data. For variable-amplitude stress ranges on details, Schilling et al. (2) proposed an effective stress range  $S_{RE}$  to characterize a stress range spectrum and this value can be applied in Miner's Rule (3) for fatigue analysis. This effective stress range  $S_{RE}$  is defined as the root mean cube (RMC) of the collected stress range data:

$$S_{RE} = \left\{ \sum_{i=1}^n \gamma_i \cdot S_{R,i}^3 \right\}^{1/3} \quad (1)$$

where  $\gamma_i$  is the ratio of the number of  $S_{R,i}$  stress range amplitude cycles to the total number of cycles. Schilling (2) concluded that this definition of an effective stress range satisfactorily relates variable- and constant- amplitude results. Effective stress ranges,  $S_{RE}$  derived thus from a representative stress spectrum of a desired detail can be very accurate descriptions of the stresses for fatigue analysis of the detail. However, collecting stress data using strain gages is costly as well as difficult or restrictive for complex details. Another way to circumvent this problem is to convert the stress data collected from one detail to other uninstrumented details but this

conversion can lead to large errors. Hence, this stress spectrum analysis approach is suitable only for those details monitored by strain gages that can yield the necessary stress spectrum.

### Rayleigh Distribution Analysis

On the basis of the analysis of 51 sets of stress range spectrum data on bridges from 6 sources including Interstate and U.S. routes in semi-rural and metropolitan locations, Schilling et al. (2) showed that a Rayleigh distribution can provide a reasonable model for the stress range spectrum of details in steel bridges. The Rayleigh probability density function for the stress range,  $S_R$ , can be expressed as:

$$f_{S_{RE}}(s_{RE}) = \left( \frac{s_{RE}}{S_{R0}^2} \right) \cdot \exp \left[ -\frac{1}{2} \left( \frac{s_{RE}}{S_{R0}} \right)^2 \right]; \quad \text{where } S_{R0} = \sqrt{\frac{2}{\pi}} E(S_{RE}) \quad (2)$$

The mean stress range effect of  $(S_{RE})^B$  can then be computed in closed form and applied in a Miner's rule format as follows:

$$E(S_{RE}^B) = (\sqrt{2} S_{R0})^B \Gamma \left( \frac{B}{2} + 1 \right) = \sum_{i=1}^n \gamma_i \cdot S_{R,i}^3 \quad (3)$$

### Fatigue Truck Analysis

Another convenient way to evaluate  $S_{RE}$  for a desired detail is to apply an "effective" fatigue truck (representative in a fatigue sense of all the actual trucks on the bridge) in a structural analysis to obtain an effective stress range  $S_{RE}$ . According to the Weight-In-Motion (WIM) data research from FHWA (4) in 1981, an HS15 (weight = 54 kips) fatigue truck was proposed and has been adopted in the AASHTO Specifications (1). Due to the different volume and characteristics of current traffic from that of the 1980s, this definition of an HS15 fatigue truck suggested in the AASHTO Specifications needs to be re-examined. Weight-In-Motion (WIM) data collected directly from the traffic site, especially without the knowledge of drivers, can help to establish an improved definition of the effective fatigue truck. The effective gross vehicle weight,  $GVW_E$ , calculated from the RMC of the load spectrum from WIM data can be taken as the weight of the fatigue truck.

$$GVW_E = \left\{ \sum_{i=1}^n \alpha_i \cdot GVW_i^3 \right\}^{1/3} \quad (4)$$

where  $\alpha_i$  is the relative likelihood of trucks with gross vehicle weight,  $GVW_i$  in the overall truck load spectrum. One can take this value of  $GVW_E$  as the weight of a new fatigue truck and then distribute this weight to the fatigue truck in a similar manner as is done with the HS15 effective fatigue truck in the AASHTO Specifications. A more detailed study on axle spacings can help and is a topic of ongoing study. With such information, one could apply WIM data to simulate actual truck passages along a bridge. The moment range,  $M_R$ , resulting from each truck can be obtained from a moment influence line analysis using standard structural analysis programs. An effective moment range,  $M_{RE}$ , can then be computed:

$$M_{RE} = \left\{ \sum_{i=1}^n \alpha_i \cdot M_{R,i}^3 \right\}^{1/3} \quad (5)$$

where  $M_{R,i}$  is the moment range resulting from the truck with weight,  $GVW_i$ . The effective of fatigue truck configuration in terms of axle spacings and weights, then, is that which results in the moment range,  $M_{RE}$ , on the same bridge and where the total axle weights equal  $GVW_E$ .

Suitable axle spacings and axle weights for the fatigue truck can then be derived. After applying an effective fatigue truck as a live load on the bridge with an appropriate impact factor ( $I$ ) and a distribution factor ( $DF$ ), an influence line analysis may be performed and the effective stress range,  $S_{RE}$ , can be obtained (see Schilling (5), (6)).

### FATIGUE RELIABILITY ANALYSIS FOR FRACTURE-CRITICAL MEMBERS

The objective here is to apply reliability theory to evaluate the safety of fracture-critical members (details) under fatigue loadings in their service lives. The AASHTO fatigue analysis approach is presented for details classified according to AASHTO fatigue categories. After defining a target reliability, a minimum acceptable level for structural safety, the actual reliability for the chosen fracture-critical detail may be compared with this target to provide information for subsequent inspection schedules.

#### Target Reliability Index ( $\beta_{min}$ )

The target reliability index,  $\beta_{min}$ , is defined as the minimum safety level approved and accepted by all or most of the individuals in a specific application. In our problem, this value can be applied as a standard, against which one can measure the safety of the identified detail or member. The target reliability index,  $\beta_{min}$ , can be given in terms of the inverse of the cumulative distribution function of a standard Gaussian random variable,  $\Phi^{-1}(\cdot)$ , and a maximum acceptable probability of failure,  $P_F$ :

$$\beta_{min} = \Phi^{-1}(1 - P_F) \quad (6)$$

#### AASHTO Fatigue Reliability Analysis

In the AASHTO Specifications, empirical S-N curve relations were established from fatigue tests conducted in the 1970s to prevent design details in steel bridges from fatigue failure. Eight categories, designated as A to E' in the specifications, are tabulated to classify details in steel bridges and to provide information for the S-N curve relation that can be expressed as:

$$N = A \cdot S_R^{-3} \quad (7)$$

where  $N$  is the number of constant-amplitude stress cycles of range,  $S_R$ , applied on the specified detail that cause failure, and  $A$  is a parameter that can be obtained from fatigue tests (each fatigue category has a different value of  $A$ ). Thus, for any detail, one can obtain either the expected fatigue life  $N$  in terms of number of stress cycles under a design stress range, or a limit stress range for desired service life.

Because of randomness in the actual traffic loadings, stress ranges are not of constant amplitude. Therefore, Miner (3) proposed an empirical rule to evaluate the fatigue damage,  $D$ , under variable-amplitude stress ranges as follows:

$$D = \sum \frac{n_i}{N_i} \geq \Delta \quad (8)$$

where  $D$  is Miner's damage accumulation index,  $n_i$  is the actual number of cycles with constant stress range amplitude,  $S_{R,i}$ , and  $N_i$  is the number of cycles that the detail can sustain under the same constant stress range,  $S_{R,i}$ . Fatigue damage of the specified detail occurs when  $D$  exceeds  $\Delta$ . The parameter,  $\Delta$ , is around 1.0 for metallic materials based on Miner's observations. Combining the S-N curve relation with Miner's rule, we have:

$$D = \sum \frac{n_i}{N_i} = \sum \frac{\gamma_i \cdot N}{A \cdot S_{R,i}^{-3}} = \frac{N}{A} \sum [\gamma_i \cdot S_{R,i}^3] = \frac{N}{A} S_{RE}^3 \geq \Delta \quad (9)$$

where  $N$  is the total number of stress cycles at all stress levels,  $\gamma_i$  is the ratio of  $n_i$  to  $N$ , and  $S_{RE}$  is the effective stress range calculated, for example, from the RMC (root mean cube) of the stress spectrum for the specified detail.

When  $D$  equals  $\Delta$  in Eq. (9), the critical number of stress cycles  $N_c$  to fatigue failure under the variable-amplitude loading with effective stress range,  $S_{RE}$ , can be represented as:

$$N_c = \frac{A \cdot \Delta}{S_{RE}^3} \quad (10)$$

The fatigue property,  $A$ , of a specific detail may be modeled as a lognormal random variable as was shown based on test data by Fisher et al. (7). It is assumed here that for each fatigue category,  $A$  followed a lognormal distribution with a mean value,  $\mu$ , and coefficient of variation, COV or  $\delta$ , as shown in Table 1. In addition, Wirsching and Chen (8) studied the test data reported by Miner (3) and found that  $\Delta$  may also be modeled by a lognormal distribution with a mean value of 1.0 and a COV of 0.30. Since  $A$  and  $\Delta$  are random variables, according to Eq. (10),  $N_c$  is also a random variable. Similar to the treatment by Zhao et al. (9), we propose a limit state function  $g(\mathbf{X})$  for fatigue reliability analysis, which is defined in terms of  $N_c$  (the number of cycles to failure for the detail) and  $N$  (the actual number of cycles to which the detail is subjected):

$$g(\mathbf{X}) = N_c - N = \frac{A \cdot \Delta}{S_{RE}^3} - N \quad (11)$$

where  $g(\mathbf{X}) < 0$  implies failure. Note that the number of stress cycles,  $N$ , is computed from truck passages. With knowledge of the number of stress cycles per truck passage ( $C$ ) and the Average Daily Truck Traffic (ADTT), the number of stress cycles,  $N$ , can be related to the number of years of operation,  $Y$ :

$$N(Y) = 365 \times C \times \text{ADTT} \times Y \quad (12)$$

The fatigue failure event of a detail can then be defined as

$$g(\mathbf{X}) = N_c - N(Y) \leq 0 \quad (13)$$

The probability of fatigue failure for the detail in question can then be related to a reliability index,  $\beta$ , and evaluated as:

$$P_F = P(g(\mathbf{X}) \leq 0) = \Phi(-\beta) \quad (14)$$

If the random variables  $A$  and  $\Delta$  are assumed to follow lognormal distributions, the reliability index  $\beta$  can be directly expressed as:

$$\beta = \frac{(\lambda_\Delta + \lambda_A) - 3 \ln S_{RE} - \ln(N)}{\sqrt{\zeta_\Delta^2 + \zeta_A^2}} \quad (15)$$

where the parameters,  $\lambda_\Delta$ ,  $\lambda_A$ ,  $\zeta_\Delta$ , and  $\zeta_A$  are given in terms of the mean ( $\mu$ ) and the coefficient of variation ( $\delta$ ) of  $A$  and  $\Delta$  as follows:

$$\lambda_A = \ln(\mu_A) - \frac{\zeta_A^2}{2}, \quad \lambda_\Delta = \ln(\mu_\Delta) - \frac{\zeta_\Delta^2}{2}; \quad \zeta_A = \sqrt{\ln(1 + \delta_A^2)}, \quad \zeta_\Delta = \sqrt{\ln(1 + \delta_\Delta^2)} \quad (16)$$

Note that  $S_{RE}$  in Eq. (15) can be evaluated from a stress spectrum analysis, a Rayleigh distribution analysis, or a fatigue truck analysis as described earlier. This AASHTO reliability analysis approach can be applied to any detail classified by AASHTO fatigue category.

## OPTIMAL INSPECTION SCHEDULING

Currently, scheduling of inspections to prevent steel bridges from fatigue failure is based on a two-year periodic pattern required by the Federal Highway Administration (FHWA) and the responsible engineer's experience. However, every steel bridge has its own specific geometric shape, design philosophy, and traffic condition, and even on a single bridge, details may be classified into any of eight fatigue categories and might experience quite different levels of stress ranges. The different fatigue category and the stress range can result in different fatigue lives for each detail. Hence, a specific fixed inspection interval schedule may not meet the inspection demands of all types of fatigue details in a steel bridge. Besides, a periodic inspection schedule will lead to a fixed number of inspections over the service life of the bridge. This number of inspections may be more than the demands for some fatigue details and less for others. If the cost of an inspection is high, such as is the case for a FCM (Fracture-Critical Member) inspection which is usually expensive to perform and causes other traffic inconveniences, a greater number of inspection times will increase the budget burden for the transportation agency. Thus, the present strategy of inspection scheduling may be not only uneconomical but also inadequate from a safety point of view. A method of inspection scheduling for steel bridges based on reliability theory and optimization that can yield a balanced solution that takes into consideration both economical and safety aspects is proposed.

Reliability-based inspection scheduling has been applied in many areas of engineering. Thoft-Christensen and Sorensen (10), Madsen (11) and Sorensen et al. (12) applied such reliability-based inspection strategies to offshore structures. Frangopol et al. (13) utilized such inspection strategies for reinforced concrete bridges. For corrosion problems in steel girder bridges, Sommer et al. (14) proposed a reliability-based strategy for inspection scheduling. The general approach in all such applications is to formulate the inspection scheduling problem as an optimization problem that seeks to minimize a cost function (the objective function) by adjusting inspection times within appropriate constraints and simultaneously maintaining safety constraints as well.

In our problem, let us first consider a single fatigue detail. The cost function for this detail is composed of the cost of inspections, repairs and structure failure during the service life. An event tree analysis that simulates all the possible scenarios after every inspection during the service life can be formulated. Fatigue reliability results obtained using the AASHTO approach are applied and transformed into appropriate probabilistic form for the event tree analysis.

### Event Tree Analysis

After every inspection of a fatigue detail, possible actions of "repair" and "no repair" are enumerated to construct the event tree for this detail over the service life. An example of an event tree, which is similar to one suggested by Frangopol et al. (13), is shown in Fig. 1. From the year  $T_0$  when the detail is assumed to come into use till the year  $T_1^-$  when the first inspection is about to be performed, the detail is assumed not to have had any repair; so, a single horizontal branch can represent the status of the detail from  $T_0$  to  $T_1^-$ . The time spent during inspection is assumed to be negligibly short. After the first inspection at  $T_1^+$ , a repair decision has to be made according to the inspection results for the fatigue detail. If a crack is detected and the crack size is over the size limit defined to be that needing repair, a repair action is assumed to be implemented immediately. The time spent during repair is also assumed to be negligibly short in this study but this assumption can easily be relaxed. If no crack is detected or the crack size can be tolerated (i.e., it is smaller than a size that warrants repair), no repair action should be taken.

Hence, the original single horizontal branch modeling the fatigue detail's status at  $T_1$  is bifurcated into two branches; one branch models the detail status after repair from  $T_1^+$  to  $T_2^-$  (the time just preceding the second inspection) and the other branch models the status without any repair from  $T_1^+$  to  $T_2^-$  (in Fig. 1, a branch designated "1" represents a repair action and a branch designated "0" represents a no-repair action). Both branches are bifurcated again at  $T_2^+$  just after the second inspection is completed to model the repair status of the detail. Continuing onward, each branch in the event tree will be bifurcated again and again immediately after every inspection is completed until the service life,  $T_f$ , is reached. Therefore, the event tree simulates all the possible repair realizations in all of the branches during the planned service life. If  $N$  inspections are performed during the service life of a detail,  $2^N$  branches will be generated at the end of the event tree, implying that  $2^N$  possible scenarios will happen in the future. Note that in the optimization problem to be formulated, we will be attempting to determine the number  $N$  of inspections and the times of these inspections,  $T_1, T_2, \dots, T_N$ .

The fatigue reliability, modeled as a decreasing function,  $\beta(T)$ , obtained from the AASHTO fatigue analysis for the desired fatigue detail will be greatly affected by the repair realizations after each inspection. In this study, an assumption is made that the detail after repair will be as good as new. Thus, the fatigue reliability at every inspection time point will be raised (or updated) to the same level as that at the starting point,  $T_0$  if a repair action is taken (i.e., on all "1" branches). Fatigue reliability patterns for all of the  $2^N$  branches are related to repair realizations for the detail. Fig. 1 shows a schematic representation of the fatigue reliability patterns for every branch in a typical event tree when  $N$  equals 2. Note that the "as good as new" assumption may be modified to conditions where either the detail is "not as good as new" or is "better than new" when sufficient data are available for the repair procedure and the altered reliability of the repaired detail. Both the subsequent reliability curve following the repair and the associated costs might in general change for these assumptions relative to the "as good as new" case but such changes are easy to implement.

### Probability of Repair $P_R$

The decision whether or not to repair that needs to be made after every inspection of a detail can be interpreted in a probabilistic form. The probability of repair,  $P_R$ , will be employed to describe the decision made after every inspection. This probability may be thought to be the same as the probability of first observation of a crack in the identified detail. An associated limit state function,  $H(\mathbf{X})$ , similar to the one employed for fatigue failure in the AASHTO approach described earlier, may be defined as follows:

$$H(\mathbf{X}) = N_{0.75c} - N = 0.75N_c - N = \frac{0.75A \cdot \Delta}{S_{RE}^3} - N \quad (17)$$

where  $N$  is the number of stress cycles to which the detail is subjected, and  $N_{0.75c}$  is equal to 75 percent of the critical number of stress cycles for fatigue failure of the detail. Choosing 75 percent of the critical number of stress cycles to correspond to observation of the first crack in fatigue tests was found to be acceptable by Fisher et al (7). The probability of repair for the detail in question can then be related to an index,  $\gamma$ , and evaluated as:

$$P_R = P(H(\mathbf{X}) \leq 0) = \Phi(-\gamma) \quad (18)$$

Considering the event tree branches, it is found that the probability of repair at every inspection, i.e.  $P_R(T_i)$ , depends on the elapsed time since the last repair. It is assumed that the probabilities of repair at the various inspection times are statistically independent, i.e.,  $P_R(T_i)$  and  $P_R(T_j)$  are statistically independent for  $i$  not equal to  $j$ . The probability,  $P(B_i)$ , of a branch,  $B_i$ ,

that includes  $N_R$  repairs can be expressed as the product of  $N_R$  probability of repair terms and  $(N - N_R)$  probability of non-repair terms.

With the help of this event tree approach, and branch probabilities, we can now define the cost of inspections, the cost of repairs, the cost associated with failure for a specified detail. These costs will then be employed in the objective function for the optimization problem.

### Cost of Inspections

Let  $K_I$  represent the cost of a single inspection of the specified detail. Then, the total cost of inspections over the service life,  $C_I$ , can be represented as:

$$C_I = \sum_{i=1}^N K_I \quad (19)$$

### Cost of Repairs

Let  $K_R$  represent the cost of a single repair of the specified detail. Because of the different repair realizations of the detail as given by the event tree, the cost of repair at the time  $T_i$  is the product of  $K_R$  and  $E[R_i]$ , the expected number of repairs at  $T_i$ . Let  $R_i$  denote the repair event at time,  $T_i$  and  $B_j^i$  denote branch  $j$  of the event tree at time,  $T_i$ . The expected number of repairs at  $T_i$  can be expressed as:

$$E[R_i] = \sum_{j=1}^{2^{i-1}} P(R_i \cap B_j^i) \quad (20)$$

The total cost of repairs for the detail over the service life,  $C_R$ , can be represented as:

$$C_R = \sum_{i=1}^N K_R \cdot E[R_i] \quad (21)$$

### Cost of Failure

The cost of failure,  $C_F$ , is meant to represent the expected cost resulting from consequences of a failure. If the detail/member under consideration is fracture-critical, its failure could cause failure of the span where the detail is located or even failure of the entire steel bridge. Hence, the cost of failure should include the possible cost of rebuilding a span or the entire bridge, as appropriate, as well as costs due to lost use, injuries, fatalities, etc. – not all of which are easily and uncontroversially estimated. Nevertheless, all of these potential costs associated with a failure are summed to yield a quantity,  $K_F$ . The possibility of the failure consequence is the other term that should be included in the cost of failure. In addition, the scenarios created by the event tree should also be concerned in the evaluation of cost of failure. Let  $F$  denote the event that the detail in question fails and  $B_i$  denote branch  $i$  of the event tree. Then, the expected cost of failure for the specified detail over the service life may be defined as:

$$C_F = \sum_{i=1}^{2^N} \left\{ \frac{1}{T_f - T_0} \int_{T_0}^{T_f} K_F \cdot P(F \cap B_i) dT \right\} \quad (22)$$

Considering all the possible scenarios of the event tree for the detail, the reliability index  $E[\beta]$  can be represented as:

$$E[\beta] = -\Phi^{-1} \left( \sum_{j=1}^{2^N} P(F | B_j) \cdot P(B_j) \right) \quad (23)$$

Substituting Eq. (23) into Eq. (22), the cost of failure may finally be expressed as:

$$C_F = \frac{1}{T_f - T_0} \int_{T_0}^{T_f} K_F \cdot \Phi(-E[\beta]) dT \quad (24)$$

### Total Cost

With definitions of the cost of inspections, repairs, and failure for the specified detail, the total cost,  $C_T$ , may be represented as:

$$C_T = C_I + C_R + C_F \quad (25)$$

$$C_T = \left\{ \sum_{i=1}^N K_I \right\} + \left\{ \sum_{i=1}^N K_R \cdot E[R_i] \right\} + \left\{ \frac{1}{T_f - T_0} \int_{T_0}^{T_f} K_F \cdot \Phi(-E[\beta]) dT \right\} \quad (26)$$

### Constraints

The number of inspections,  $N$ , and the inspection times,  $T_1 \dots T_N$ , are variables for the optimization problem. One obvious constraint on the inspection times may be expressed as:

$$T_0 < T_1 < \dots < T_N < T_f \quad (27)$$

Usually, restrictions are placed on the time between inspections such that this inter-inspection interval is neither too large (upper bound,  $T_{max}$ ) nor too short (lower bound,  $T_{min}$ ). Such constraints on the inspection interval may be required by local and state transportation agencies. Hence, a second constraint on inspection times for the optimization problem is:

$$T_{min} \leq T_i - T_{i-1} \leq T_{max}, \quad i = 1 \dots N \quad (28)$$

It is also usually required to keep the safety (or reliability) above a certain level. This requirement can be achieved by defining a target reliability index,  $\beta_{min}$ , which is the minimum acceptable safety tolerance for the specified detail. Thus, a constraint to be included in our optimization problem may be defined as:

$$E[\beta(T_i)] \geq \beta_{min}, \quad i = 1 \dots (N+1) \quad \text{where } T_{N+1} = T_f \quad (29)$$

### Formulation of the Optimization Problem

In summary, the optimization problem for the inspection scheduling may be formulated as follows:

$$\min_{T_1 \dots T_N} C_T = \left\{ \sum_{i=1}^N K_I \right\} + \left\{ \sum_{i=1}^N K_R \cdot E[R_i] \right\} + \left\{ \frac{1}{T_f - T_0} \int_{T_0}^{T_f} K_F \cdot \Phi(-E[\beta]) dT \right\} \quad (30)$$

$$\begin{aligned} \text{s.t.} \quad & T_0 < T_1 < \dots < T_N < T_f \\ & T_{min} \leq T_i - T_{i-1} \leq T_{max}, \quad i = 1 \dots N \\ & E[\beta(T_i)] \geq \beta_{min}, \quad i = 1 \dots (N+1) \end{aligned}$$

Minimizing the total cost, a set of inspection times,  $T_i$ , may be found. In addition, changing the number of inspections,  $N$ , the total cost corresponding to the different number of inspections may be compared so as to finally yield the optimization solution.

## NUMERICAL EXAMPLES

### Problem Description:

The example bridge studied here is the 680-ft long Brazos River Bridge in Texas, which was built in 1972. Fig. 2 shows the layout of the Bridge as well as a magnified view of the selected fatigue detail, which is classified as a Category E detail as per the AASHTO Specifications. This detail located in the leftmost 150-ft span is analyzed. The stress range parameter,  $S_{R0}$ , applied on

this detail is 6.13 ksi as a Rayleigh distribution is assumed for the stress ranges (see Eq. (2)). The target reliability  $\beta_{min}$  for the detail is assumed to be 3.7, corresponding to a failure probability of approximately 1/10000. Additionally,  $T_{min} = 0.5\text{yr}$  and  $T_{max} = 2.0\text{yr}$  are taken to be constraints on the inter-inspection times. Two sets of relative costs of inspection, repair and failure: (i)  $K_I : K_R : K_F = 1 : 1.3 \times 10^2 : 4 \times 10^5$ ; and (ii)  $K_I : K_R : K_F = 1 : 2.6 \times 10^2 : 4 \times 10^5$  are considered for illustration of the optimal inspection scheduling. Other necessary information of fatigue reliability analysis and optimal inspection scheduling is listed in Tables 1. The number of stress cycles per truck passage,  $C$ , and the Average Daily Truck Traffic, ADTT, are taken to be 1 and 84, respectively (see Eq. (12)). The service life is taken to be 50 years.

## Results

After applying the AASHTO fatigue analysis approach, the fatigue reliability,  $\beta$ , of the specified detail over the service life is shown in Fig. 3. The target reliability level,  $\beta_{min}$ , of 3.7 is also shown in the figure. It can be seen that the fatigue reliability of the chosen detail is below the target reliability by the thirteenth year.

We will assume that in year 2002 (i.e., 30 years after 1972), no crack was found or that the crack in the detail was repaired to its original condition. Applying the optimization method proposed, for the relative costs of  $K_I : K_R : K_F = 1 : 1.3 \times 10^2 : 4 \times 10^5$ , it is found in Fig. 4 that the optimal inspection times for the next twenty years are eleven in number and the optimal inspection schedule for the next twenty years is demonstrated in Fig. 5. The inspection times in years are  $\mathbf{T} = (2.0, 4.0, 6.0, 8.0, 10.0, 12.0, 13.5, 14.0, 14.5, 15.0, 15.5)$ . Furthermore, comparing the optimal inspection schedule with the periodic schedule of two-year intervals, the total cost (162.7) of the optimal schedule is less than the total cost (168.9) of the periodic schedule. Furthermore, though the optimal schedule requires two more inspections than the two-year periodic schedule, these additional inspections and the short interval between inspections exceeds 42 years of age reduces the risk of the detail's failure. This fact can be confirmed by the reduced cost associated with failure (in the total cost of the optimal schedule). Therefore the optimal schedule clearly the preferred choice for inspecting this detail over its service life.

For the second case with relative costs of  $K_I : K_R : K_F = 1 : 2.6 \times 10^2 : 4 \times 10^5$ , nine inspection times for the next twenty years are needed and the optimal schedule of inspection times in years,  $\mathbf{T} = (1.7, 3.4, 5.4, 7.4, 9.4, 11.4, 13.4, 15.1, 17.1)$  is shown in Fig. 6. Though the number of inspections (nine) is the same as with the periodic schedule, the total cost (200.5) of the optimal schedule is still less than the total cost (211.7) of the periodic schedule.

The increase in the number of inspections,  $N$ , increases inspection and repair costs but typically decreases failure cost. The optimal result occurs at the number,  $N_{opt}$ , where the decrease in failure costs starts to become less than the increase in inspection and repair costs. Comparing the optimal schedules resulted from the cases studied, the lower repair cost example (case i) requires more inspections to reach the optimal point, but the higher repair cost example relatively demands that fewer inspections be performed to minimize costs and achieve the optimal point. Clearly, the relative costs of inspection, repair, and failure affect the optimization results in a very direct manner. Note that the effect of discounted rates is not considered in this example. However, Eqs. (19), (21), and (22) can include consideration for discounted rates in a straightforward manner as is demonstrated by Madsen (11) and Frangopol et al. (13).

## CONCLUSIONS

An optimal inspection scheduling procedure for fracture-critical members in steel bridges has been presented. The method builds upon a stress range analysis and a fatigue reliability analysis for the desired fracture-critical detail, and then applies information from these two analyses to formulate an optimization problem of inspection scheduling in accordance with a systematic event tree framework. Solution of the optimization problem yields the optimal inspection schedule for the detail. From the results of the presented numerical examples, it is seen that a periodic inspection schedule over the service life will not be the optimal (lowest-cost) schedule for some details. The reliability-based fatigue inspection strategy presented here yields the optimal inspection schedule. The optimization results are affected by the fatigue reliability as well as the relative costs of inspection, repair and failure. The influence of relative costs of inspection, repair and failure was illustrated in the numerical examples presented.

Reliability-based inspection scheduling offers a rational method for inspection and maintenance strategies of steel bridges. Applying this method by changing the upper and lower bounds on inspection intervals, bridge authorities can optimally allocate their maintenance budgets in a more efficient manner without compromising the safety of their bridges. This scheduling methodology may also be applied to other degrading civil infrastructure systems if the reliability, costs, and the related random variables affecting performance can be quantified.

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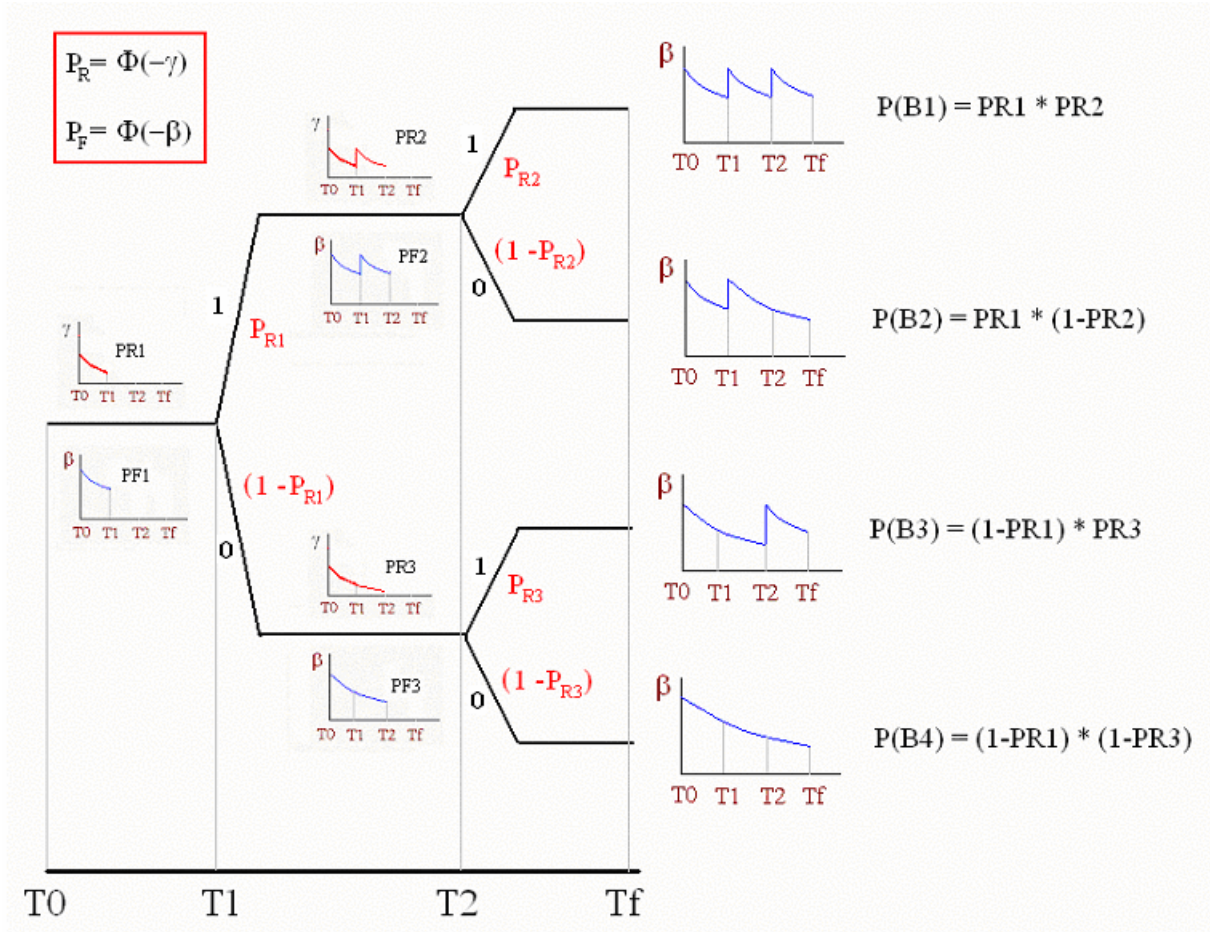
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**TABLE 1 Statistical Properties for Random Variables.**

<i>Variable</i>	<i>Type</i>	<i>Mean (<math>\mu</math>)</i>	<i>COV (<math>\delta</math>)</i>
$A$ (Category A), ksi <sup>3</sup>	lognormal	$8.25 \times 10^{10}$	0.72
$A$ (Category B), ksi <sup>3</sup>	lognormal	$2.51 \times 10^{10}$	0.44
$A$ (Category B'), ksi <sup>3</sup>	lognormal	$1.23 \times 10^{10}$	0.41
$A$ (Category C), ksi <sup>3</sup>	lognormal	$9.98 \times 10^9$	0.48
$A$ (Category D), ksi <sup>3</sup>	lognormal	$4.67 \times 10^9$	0.44
$A$ (Category E), ksi <sup>3</sup>	lognormal	$1.76 \times 10^9$	0.30
$A$ (Category E'), ksi <sup>3</sup>	lognormal	$7.86 \times 10^8$	0.41
$\Delta$	lognormal	1.0	0.3



**FIGURE 1** Representative Event Tree showing Inspection and Repair Realizations (similar to one defined by Frangopol et al (13)).

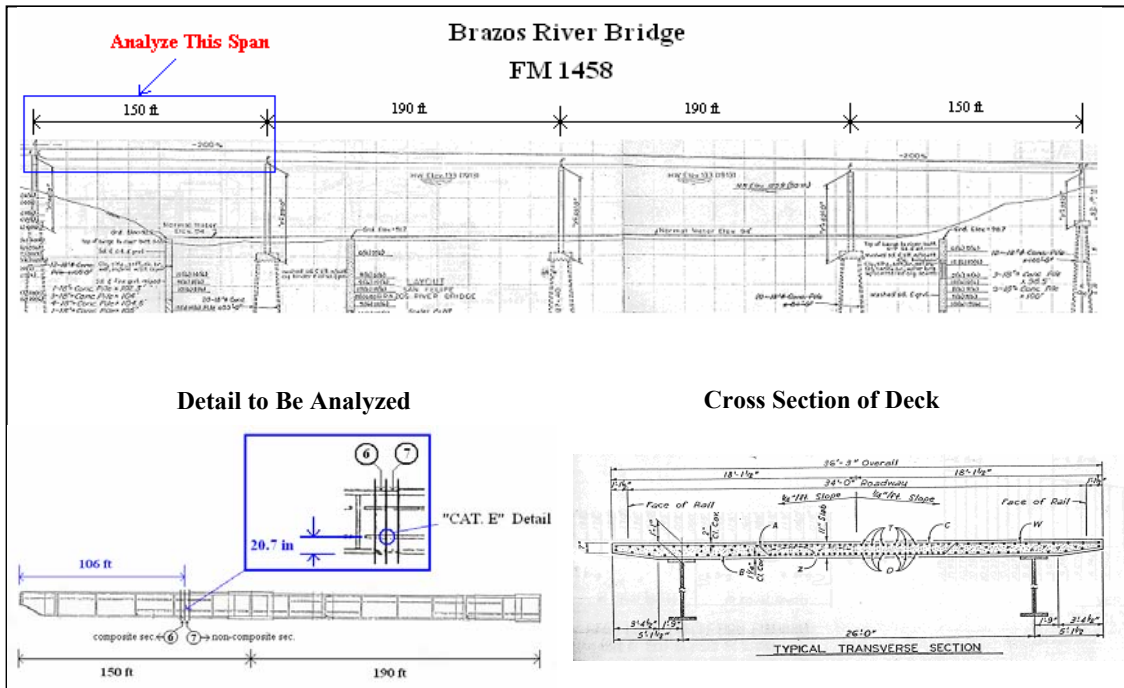
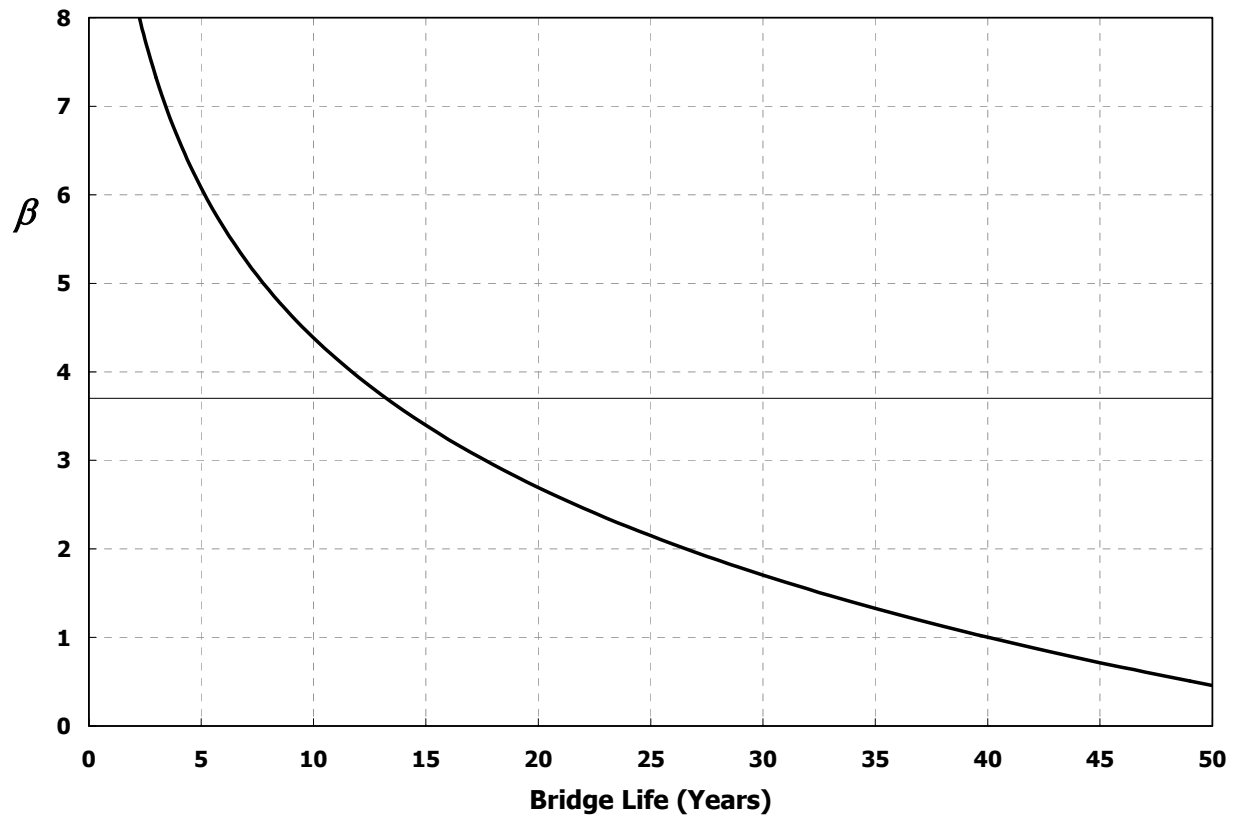


FIGURE 2 Brazos River Bridge.



**FIGURE 3 Fatigue Reliability in 50 Years.**

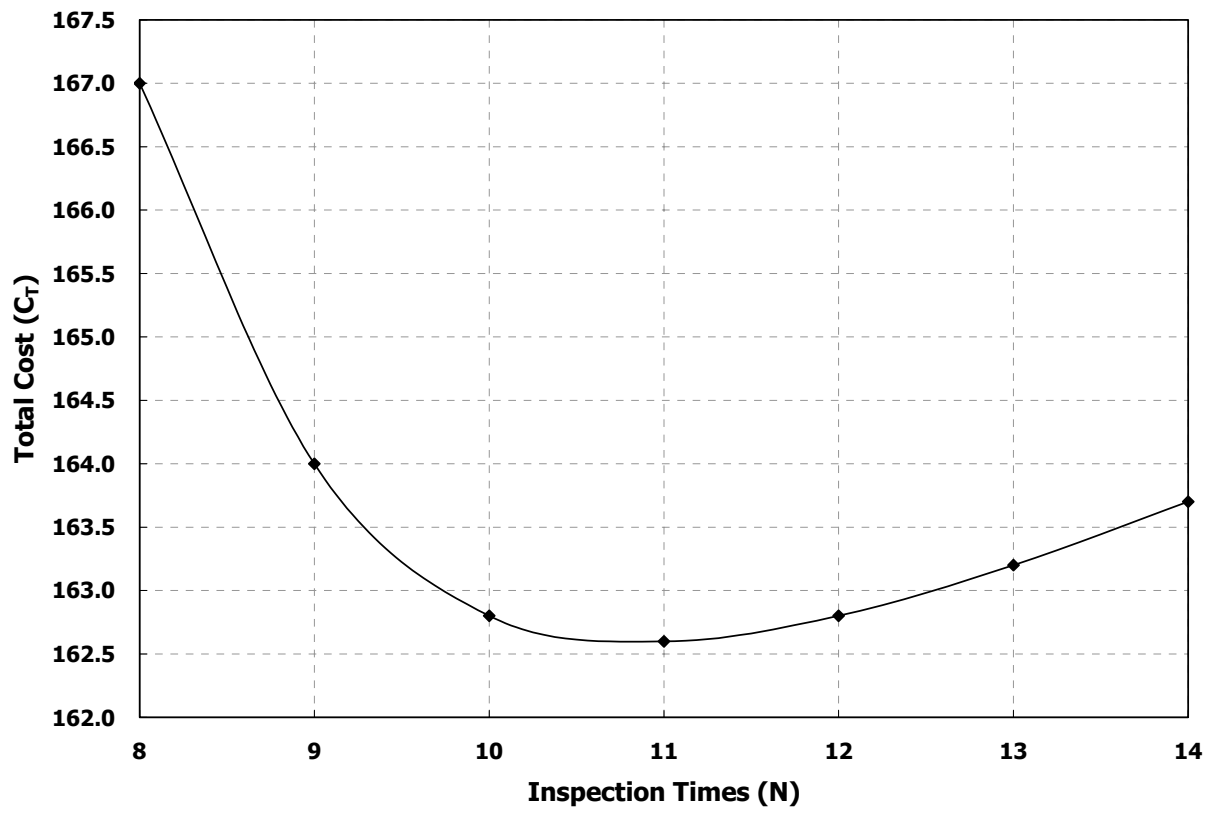
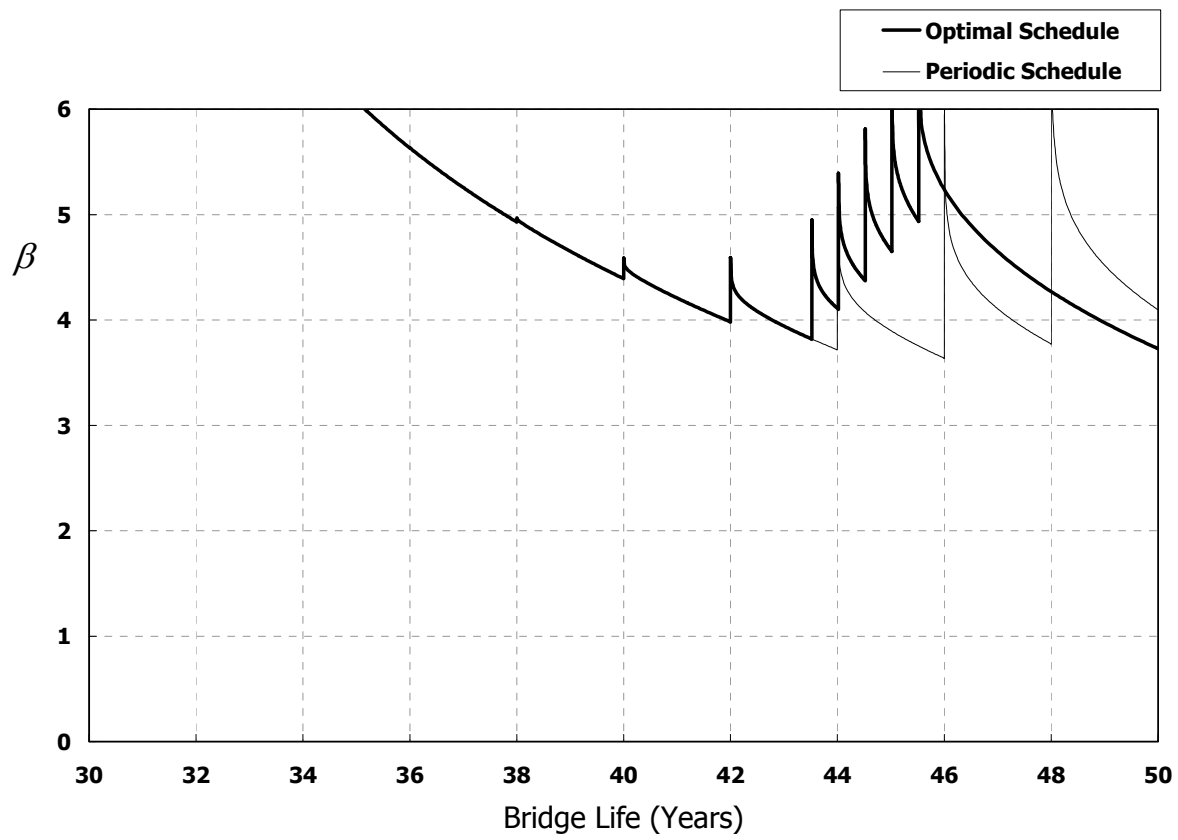
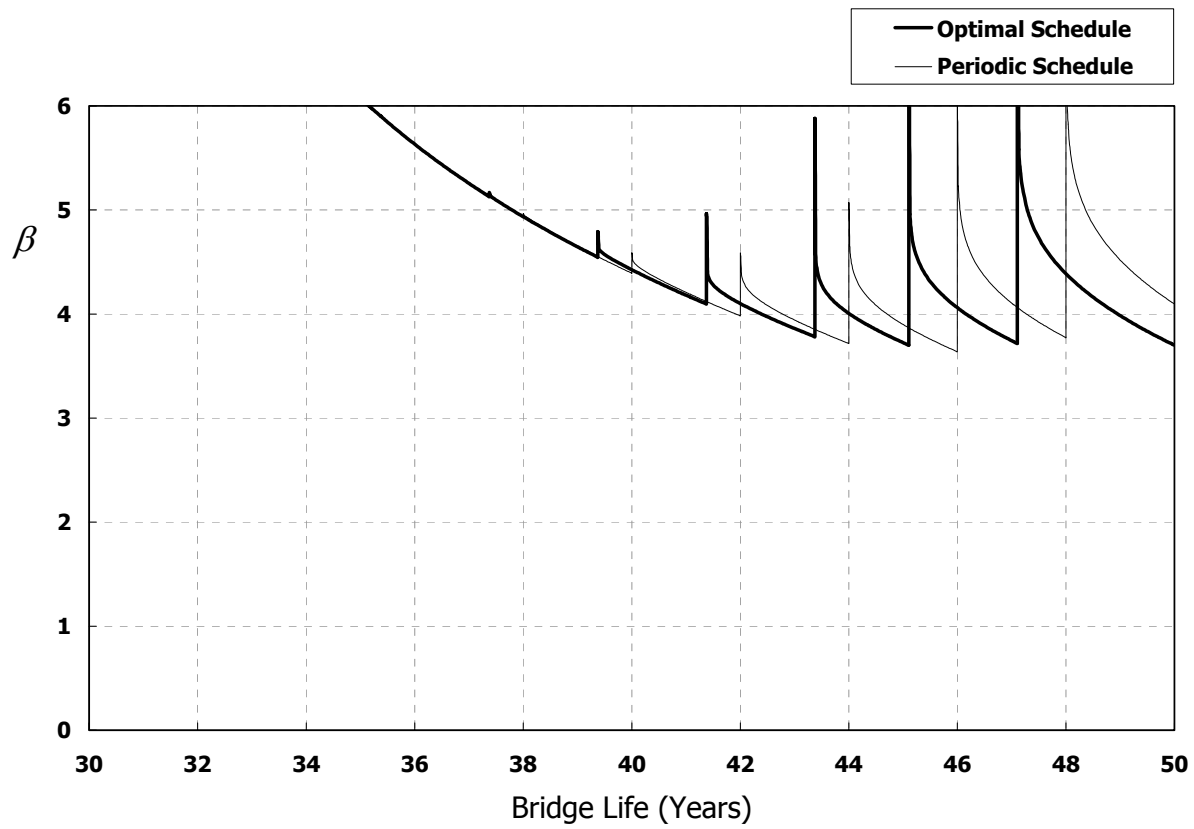


FIGURE 4 Inspection Times versus Total Cost for  $K_I : K_R : K_F = 1 : 1.3 \times 10^2 : 4 \times 10^5$ .



**FIGURE 5** Optimal Inspection Schedule for  $K_I : K_R : K_F = 1 : 1.3 \times 10^2 : 4 \times 10^5$ .



**FIGURE 6** Optimal Inspection Schedule for  $K_I : K_R : K_F = 1 : 2.6 \times 10^2 : 4 \times 10^5$ .