**CE 374K Hydrology Kinematic Wave Celerity Spring 2013**

The definition of the kinematic wave celerity, Ck, is

$$C\_{k}=\frac{dQ}{dA} (1)$$

Where Q is the discharge and A is the cross-sectional area of the flow at this discharge.

The velocity, V is defined by Manning’s Equation:

$$V=\frac{1.49}{n}\sqrt{S\_{0}}R^{2/3}$$

where n is the Manning roughness, So is the bed slope, and the hydraulic radius, R = A/P, the cross-sectional area divided by the wetted perimeter. The discharge is given by:

$$Q=VA=\frac{1.49}{n}\sqrt{S\_{0}}R^{2/3}A=\frac{1.49}{n}\sqrt{S\_{0}}\frac{A^{5/3}}{P^{2/3}}$$

Hence

$$C\_{k}=\frac{dQ}{dA}= \frac{d}{dA}\left(\frac{1.49}{n}\sqrt{S\_{0}}\frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}}\right)= \frac{1.49}{n}\sqrt{S\_{0}}\frac{d}{dA}\left(A^{\frac{5}{3}}P^{-\frac{2}{3}}\right) $$

$$thus C\_{k} =\frac{1.49}{n}\sqrt{S\_{0}}\left[\frac{5}{3}A^{\frac{2}{3}}P^{-\frac{2}{3}}+A^{\frac{5}{3}}\left(-\frac{2}{3}P^{-\frac{5}{3}}\right)\frac{dP}{dA}\right]$$

$$and C\_{k} =\frac{5}{3} \left[\frac{1.49}{n}\sqrt{S\_{0}} R^{2/3}\right]- \frac{2}{3}\left[\frac{1.49}{n}\sqrt{S\_{0}} R^{2/3} R \frac{dP}{dA}\right]$$

$$so C\_{k} =\frac{5V}{3} - \frac{2}{3}VR \frac{dP}{dA} (2)$$

Now, *for a very wide, rectangular channel*, the water spreads out to the entire width as soon as there is a little depth in the channel, and the depth, y, is small when compared to width, B, then perimeter P = B+2y ~ B for all values of A. Then dP/dA ~ 0 and the kinematic wave celerity is given by

$$C\_{k} =\frac{5V}{3} (3)$$

In this case, the wave celerity is significantly greater than the flow velocity. However, for channels of other shapes, the water spreads out gradually as the channel discharge rises and the wetted perimeter increases significantly as the channel fills, so dP/dA > 0. Then, the second term in Eq (2) comes into play and since V, R, dP/dA are all positive functions, it follows that the solution to Eq (1) given by Eq (3) is a limiting case that gives the upper limit of kinematic wave celerity. Its actual value as given by Eq (2) drops below that limit, and depending on the circumstances, it could even occur that Ck < V. This shows that the speed with which flood waves travel down channels could be either less than or greater than the flow velocity but in any case will not be greater than 5V/3.